

Gauss Jacobi Method :

①. Let the system of simultaneous eqns. be

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

The diagonal elements should be dominant, so that the iteration process can be applied.

This system of eqns. can also be written as,

$$x = \frac{1}{a_1} (d_1 - b_1 y - c_1 z)$$

$$y = \frac{1}{b_2} (d_2 - a_2 x - c_2 z)$$

$$z = \frac{1}{c_3} (d_3 - a_3 x - b_3 y)$$

Let us assume $x_0 = y_0 = z_0 = 0$

1st Iteration :

$$x^{(1)} = \frac{1}{a_1} (d_1 - b_1 y^{(0)} - c_1 z^{(0)})$$

$$y^{(1)} = \frac{1}{b_2} (d_2 - a_2 x^{(0)} - c_2 z^{(0)})$$

$$z^{(1)} = \frac{1}{c_3} (d_3 - a_3 x^{(0)} - b_3 y^{(0)})$$

2nd Iteration :

$$x^{(2)} = \frac{1}{a_1} (d_1 - b_1 y^{(1)} - c_1 z^{(1)})$$

$$y^{(2)} = \frac{1}{b_2} (d_2 - a_2 x^{(1)} - c_2 z^{(1)})$$

$$z^{(2)} = \frac{1}{c_3} (d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

This process is repeated till the difference between two consecutive approximations is negligible.

Q. Solve the following eqns. by using Jacobi's Iteration method.

$$30x - 2y + 3z = 75$$

$$x + 17y - 2z = 48$$

$$x + y + 9z = 15$$

Soln.

$$|30| > |1| + |3|$$

$$|17| > |1| + |2|$$

$$|9| > |1| + |1|$$

Since the diagonal elements are dominant, the iteration process is applied here.

The given system can be written as,

$$x = \frac{1}{30} (75 + 2y - 3z)$$

$$y = \frac{1}{17} (48 - x + 2z)$$

$$z = \frac{1}{9} (15 - x - y)$$

Let us assume $x_0 = y_0 = z_0 = 0$

Iteration	$x = \frac{1}{30} (75 + 2y - 3z)$	$y = \frac{1}{17} (48 - x + 2z)$	$z = \frac{1}{9} (15 - x - y)$
1	2.5	2.8235	1.6667
2	2.5216	2.8726	1.0752
3	2.5840	2.8017	1.0673
4	2.5655	2.7971	1.0683
5	2.5796	2.7983	1.0708
6	2.5795	2.7978	1.0691
7	2.5796	2.7976	1.0692
8	2.5796	2.7976	1.0692

∴ The solution is, $x = 2.5796$
 $y = 2.7976$
 $z = 1.0692$

②. Solve the following system of eqns. by using Gauss Jacobi method.

$$10x + 2y + z = 9 \quad \dots (1)$$

$$x + 10y - z = -22 \quad \dots (2)$$

$$-2x + 3y + 10z = 22 \quad \dots (3)$$

Soln:

$$|10| > |2| + |1|$$

$$|10| > |1| + |-1|$$

$$|10| > |-2| + |3|$$

Since the diagonal elements are dominant, the iteration process is applied here.

The given system can be written as,

$$x = \frac{1}{10} (9 - 2y - z)$$

$$y = \frac{1}{10} (-22 - x + z)$$

$$z = \frac{1}{10} (22 + 2x - 3y)$$

Let us assume $x_0 = y_0 = z_0 = 0$

Iteration	$x = \frac{1}{10} (9 - 2y - z)$	$y = \frac{1}{10} (-22 - x + z)$	$z = \frac{1}{10} (22 + 2x - 3y)$
1	0.9	-2.2	2.2
2	1.12	-2.07	3.04
3	1.072	-2.098	3.045
4	0.9971	-1.9955	3.0024
5	0.9989	-1.9995	2.9981
6	1.000	-2.0001	2.9996
7	1.0000	-2.0000	3.0000
8	1.0000	-2.0000	3.0000
9	1.0000	-2.0000	3.0000
10	1.0000	-2.0000	3.0000

∴ The soln. is $x = 1.0000$

$$y = -2.0000$$

$$z = 3.0000$$