③ Design a linear phase FIR highpass filter using hamming window with a cutoff frequency $w_c = 0.8\pi$ rad/sam and $N=7$.

Soln: Symmetric Impulse response with symmetry condition $h(N-1-n) = h(n)$

The desired ideal frequency response $H_d(e^{jw})$ for FIR HPF

$$H_d(e^{jw}) = \begin{cases} e^{-jw\alpha} & ; -\pi \le w \le -w_c \ \& \ w_c \le w \le \pi \\ 0 & ; \text{otherwise} \end{cases}$$

The desired impulse response $h_d(n)$ is obtained by taking inverse Fourier transform of $H_d(e^{jw})$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw})\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-w_c} e^{-jw\alpha} e^{jwn}\, dw + \frac{1}{2\pi}\int_{w_c}^{\pi} e^{-jw\alpha} e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-w_c} e^{jw(n-\alpha)}\, dw + \frac{1}{2\pi}\int_{w_c}^{\pi} e^{jw(n-\alpha)}\, dw$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-w_c} + \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{w_c}^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-jw_c(n-\alpha)}}{j(n-\alpha)} - \frac{e^{-j\pi(n-\alpha)}}{j(n-\alpha)}\right] + \frac{1}{2\pi}\left[\frac{e^{j\pi(n-\alpha)}}{j(n-\alpha)} - \frac{e^{jw_c(n-\alpha)}}{j(n-\alpha)}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}}{2j} - \frac{e^{jw_c(n-\alpha)} - e^{-jw_c(n-\alpha)}}{2j}\right]$$

$$h_d(n) = \frac{\sin\pi(n-\alpha) - \sin w_c(n-\alpha)}{\pi(n-\alpha)}$$ for all n, except $n=\alpha$

Scanned with CamScanner

when $n = \alpha$;

$$h_d(n) = \lim_{(n-\alpha) \to 0} \frac{\sin \pi(n-\alpha) - \sin w_c(n-\alpha)}{\pi(n-\alpha)}$$

$$= \frac{1}{\pi}\left[\lim_{(n-\alpha)\to 0} \frac{\sin \pi(n-\alpha)}{(n-\alpha)} - \lim_{n-\alpha \to 0} \frac{\sin w_c(n-\alpha)}{(n-\alpha)}\right]$$

Using L' Hopital rule

$$= \frac{1}{\pi}(\pi - w_c) \Rightarrow 1 - \frac{w_c}{\pi} \qquad \lim_{\theta \to 0} \frac{\sin A\theta}{\theta} = A$$

The impulse response $h(n)$ of FIR filter is obtained by multiplying $h_d(n)$ by window sequence.

Hamming Window sequence:

$$w_H(n) = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & ; \text{for } n = 0 \text{ to } N-1 \\ 0 & ; \text{otherwise.} \end{cases}$$

$$\therefore h(n) = h_d(n)\, W_H(n)$$

$$h(n) = \frac{\sin \pi(n-\alpha) - \sin w_c(n-\alpha)}{\pi(n-\alpha)}\left[0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)\right]$$

for $n \neq \alpha$

$$h(n) = \left[1 - \frac{w_c}{\pi}\right]\left[0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)\right] \quad \text{for } n = \alpha$$

Given that $N = 7$, $w_c = 0.8\pi$ rad/sam

$$\therefore \alpha = \frac{N-1}{2} = \frac{7-1}{2} = 6/2 = 3 \qquad ; N-1 = 6$$

calculate $h(n)$ for $n = 0$ to 6

$h(n)$ satisfies the symmetry condition $h(N-1-n) = h(n)$; calculate $h(n)$ for $n = 0$ to 3

$$\therefore h(n) = \frac{-\sin w_c(n-3)}{\pi(n-3)}\left[0.54 - 0.46\cos\frac{n\pi}{3}\right] \quad \text{for } n \neq 3$$

Scanned with CamScanner

$\therefore\ h(n) = \left[1 - \frac{w_c}{\pi}\right]\left[0.54 - 0.46 \cos \frac{n\pi}{3}\right]$ for $n=3$

when $n=0$;

$$h(0) = \frac{-\sin(0.8\pi(0-3))\left[0.54 - 0.46\cos\frac{0\times\pi}{3}\right]}{\pi(0-3)} = -0.0081$$

when $n=1$;

$$h(1) = \frac{-\sin(0.8\pi(1-3))\left[0.54 - 0.46\cos\frac{1\times\pi}{3}\right]}{\pi(1-3)} = 0.0469$$

when $n=2$;

$$h(2) = \frac{-\sin(0.8\pi(2-3))\left[0.54 - 0.46\cos\frac{2\times\pi}{3}\right]}{\pi(2-3)} = -0.1441$$

when $n=3$;

$$h(3) = \left[1 - \frac{0.8\pi}{\pi}\right]\left[0.54 - 0.46\cos\frac{3\times\pi}{3}\right] = 0.2$$

when $n=4$; $h(4) = h(6-4) = h(2) = -0.1441$

when $n=5$; $h(5) = h(6-5) = h(1) = 0.0469$

when $n=6$; $h(6) = h(6-6) = h(0) = -0.0081$

The transfer function $H(z)$ of FIR high pass filter is gien by

$$H(z) = z\{h(n)\} = \sum_{n=0}^{N-1} h(n) z^{-n} \Rightarrow \sum_{n=0}^{6} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(2)z^{-4} + h(1)z^{-5} + h(0)z^{-6}$$

$$= h(0)\left[1+z^{-6}\right] + h(1)\left[z^{-1}+z^{-5}\right] + h(2)\left[z^{-2}+z^{-4}\right] + h(3)z^{-3}$$

$$H(z) = -0.0081\left[1+z^{-6}\right] + 0.0469\left[z^{-1}+z^{-5}\right] - 0.1441\left[z^{-2}+z^{-4}\right] + 0.2z^{-3}.$$

Scanned with CamScanner

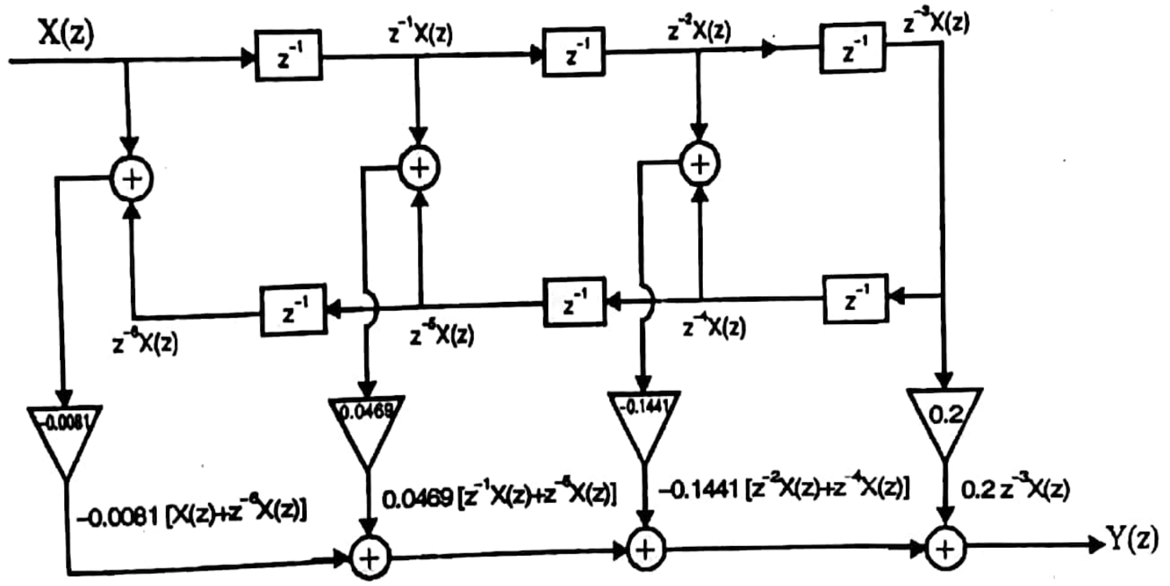
structure :-

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = -0.0081\left[1+z^{-6}\right] + 0.0469\left[z^{-1}+z^{-5}\right] - 0.1441\left[z^{-2}+z^{-4}\right] + 0.2z^{-3}.$$

$$\therefore Y(z) = -0.0081\left[X(z) + z^{-6}X(z)\right] + 0.0469\left[z^{-1}X(z) + z^{-5}X(z)\right] - 0.1441\left[z^{-2}X(z) + z^{-4}X(z)\right] + 0.2z^{-3}X(z).$$

Linear phase structure of FIR high pass filter :-



Frequency Response :-

$$|A(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\left(\frac{N-1}{2}\right)} 2h\left(\frac{N-1}{2}-n\right)\cos\omega n$$

$$\therefore A(\omega) = h(3) + \sum_{n=1}^{3} 2h(3-n)\cos\omega n$$

$$= h(3) + 2h(2)\cos\omega n + 2h(1)\cos 2\omega + 2h(0)\cos 3\omega$$

$$= 0.2 + 2\times(-0.1441)\cos\omega + 2\times 0.0469\cos 2\omega + 2\times(-0.0081)\cos 3\omega.$$

$$A(\omega) = 0.2 - 0.2882\cos\omega + 0.0938\cos 2\omega - 0.0162\cos 3\omega$$

Scanned with CamScanner