



SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 1 – DISCRETE FOURIER TRANSFORM

TOPIC – Circular Convolution



EMPATHY



1

- Convolution of Periodic signal using Linear convolution is repetitive

2

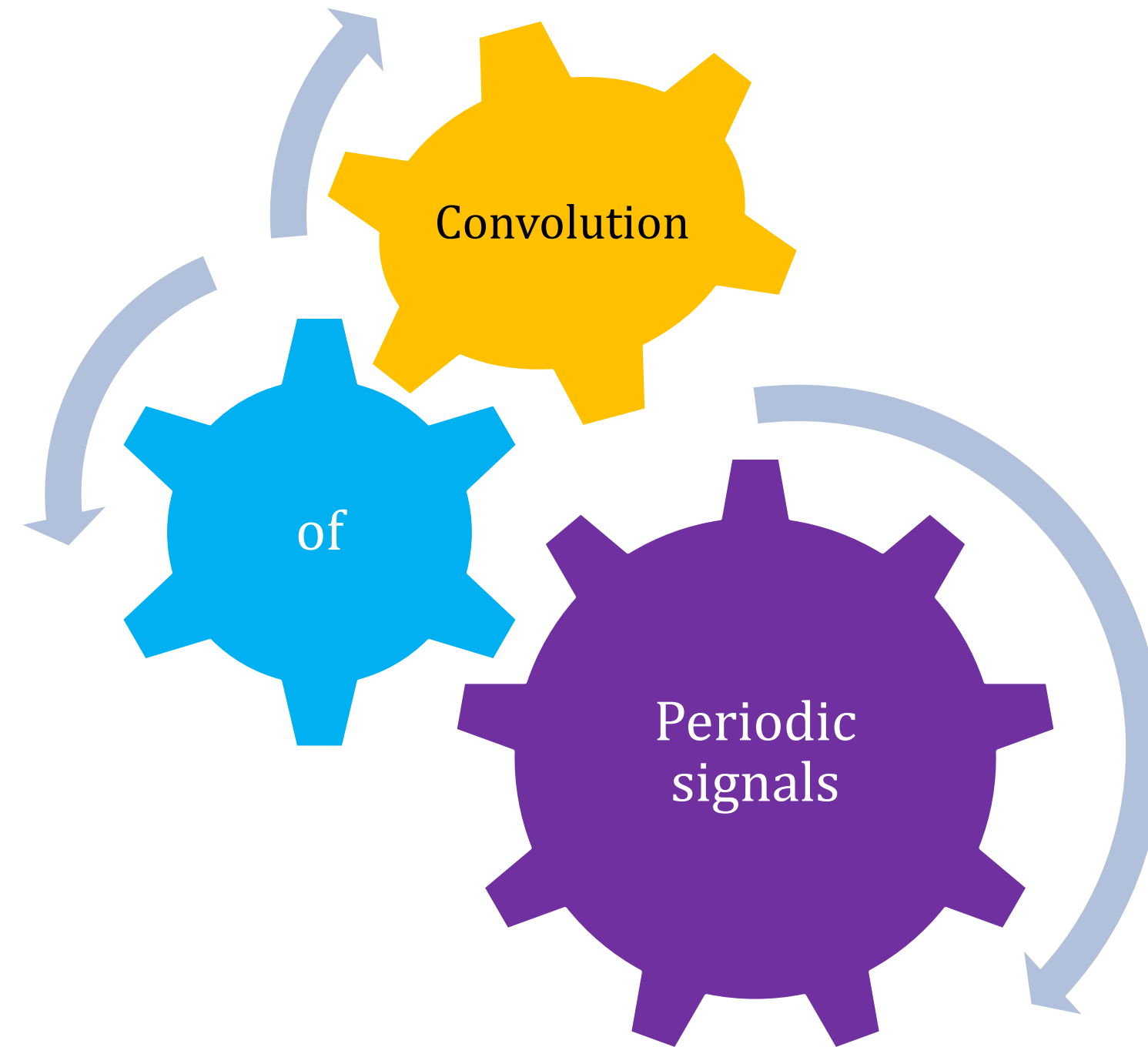
- Convolution takes lot of time

3

- Utilization of DFT for Convolution

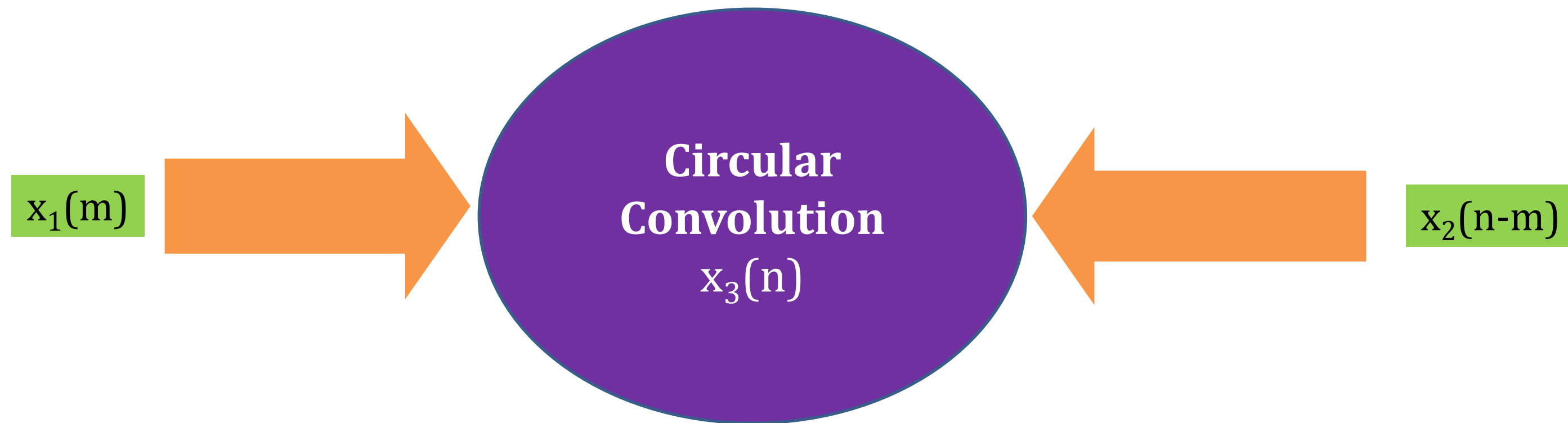


DEFINE



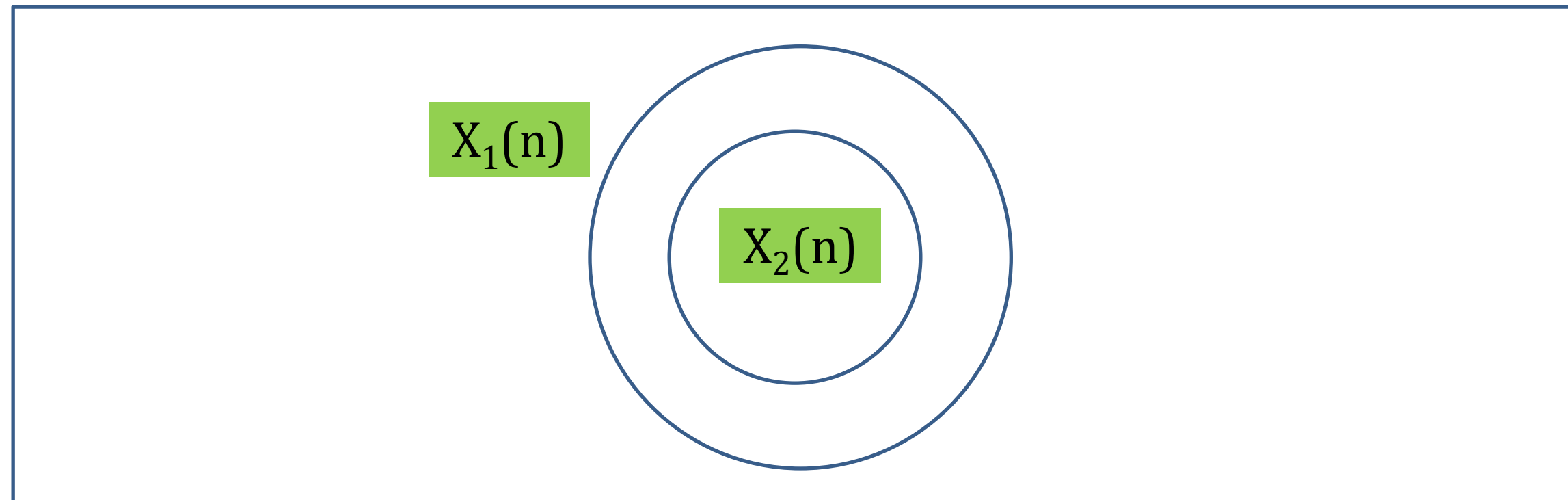


METHOD





NEED FOR CONVOLUTION



- To find the output of a system with input and impulse response of the system linear convolution is used
- Circular Convolution can be applied for periodic signals and to obtain linear Convolution through Circular padding of zeros are done



CIRCULAR CONVOLUTION



- The circular convolution of two periodic discrete time sequences

$X_1(n)$ and $X_2(n)$ with periodicity of N sample is defined as

$$y(n) = x(n) * h(n) = \sum_{k=0}^{N-1} x(k) \cdot h(n-k) \quad x_3(n) = \sum_{m=0}^{N-1} x_1(m) x_2[((n-m))_N]$$

- If $x(n)$ and $h(n)$ two finite duration signals with length M and P respectively then the length of $y(n) = x(n) * h(n)$ is **$N=M+P-1$** samples



CIRCULAR CONVOLUTION



- If $x(n)$ and $h(n)$ two periodic signals with period N then the length of $y(n) = x(n) \circledast h(n)$ is also N
- The convolution of two periodic signal is also periodic and is circular convolution.

$$y(n) = x(n) \circledast h(n)$$

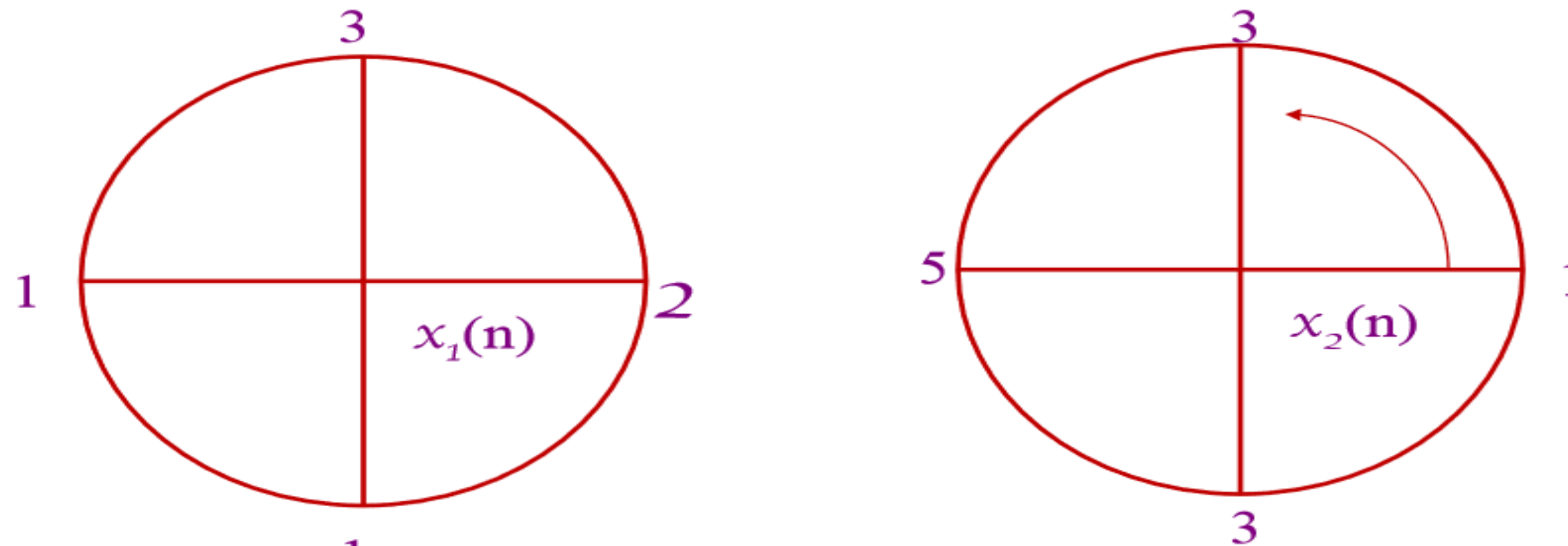
- Linear Convolution can be obtained by circular convolution by changing the length of both signals $x(n)$ and $h(n)$ to N by zero padding



CIRCULAR CONVOLUTION



- Compute the circular convolution using time domain approach for the following sequence: $X_1(n) = \{2,3,1,1\}$ and $X_2(n) = \{1,3,5,3\}$



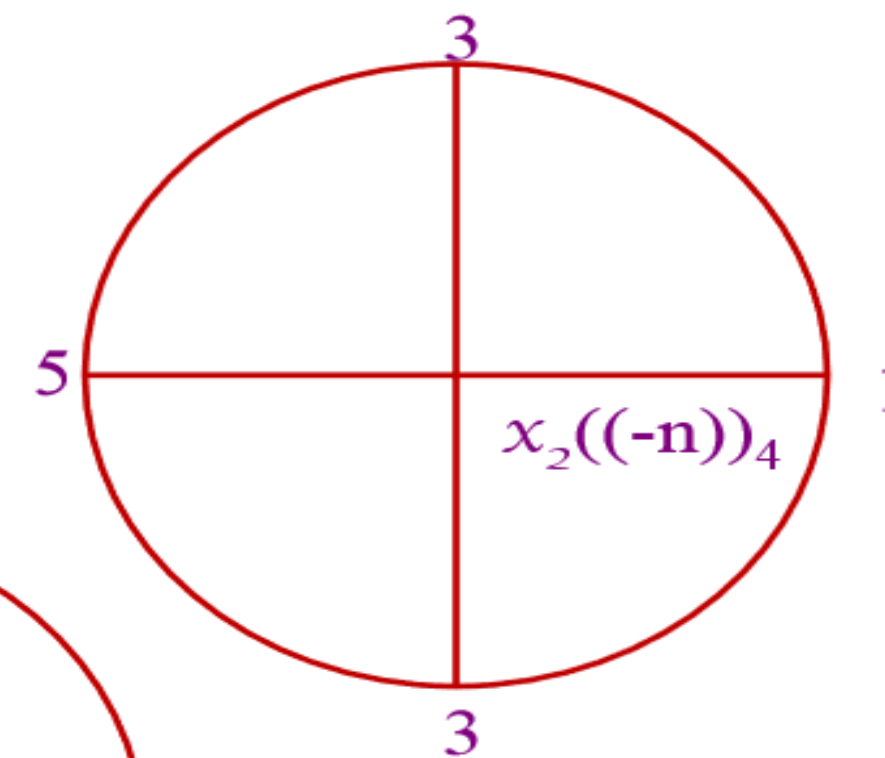
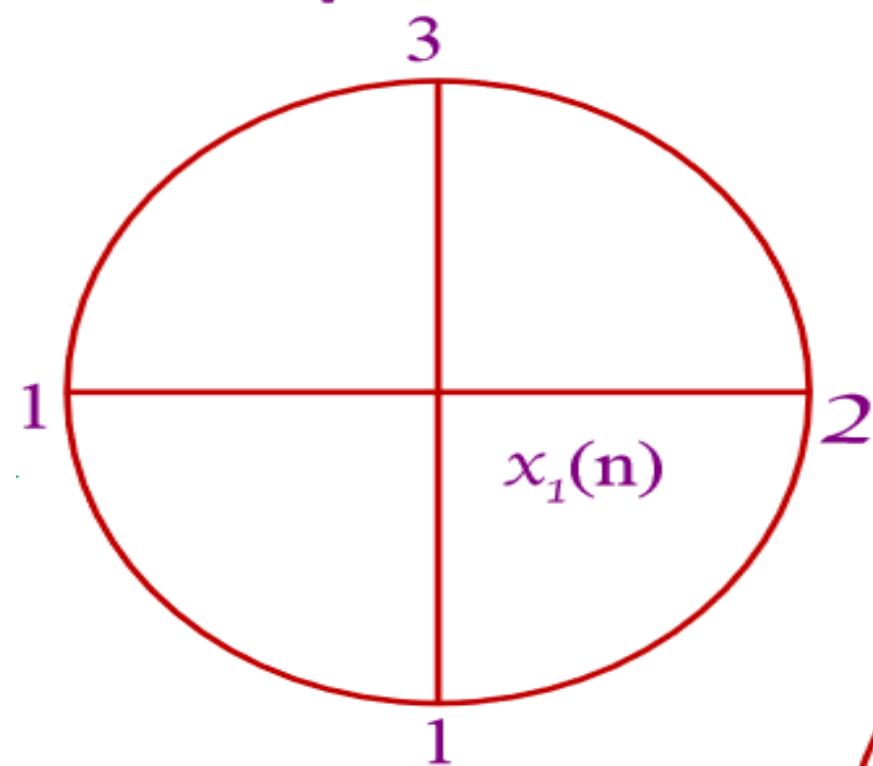
$$y(n) = \sum_{k=0}^{N-1} x_1(k) \cdot x_2(n-k) = \sum_{k=0}^3 x_1(k) \cdot x_2(n-k)$$



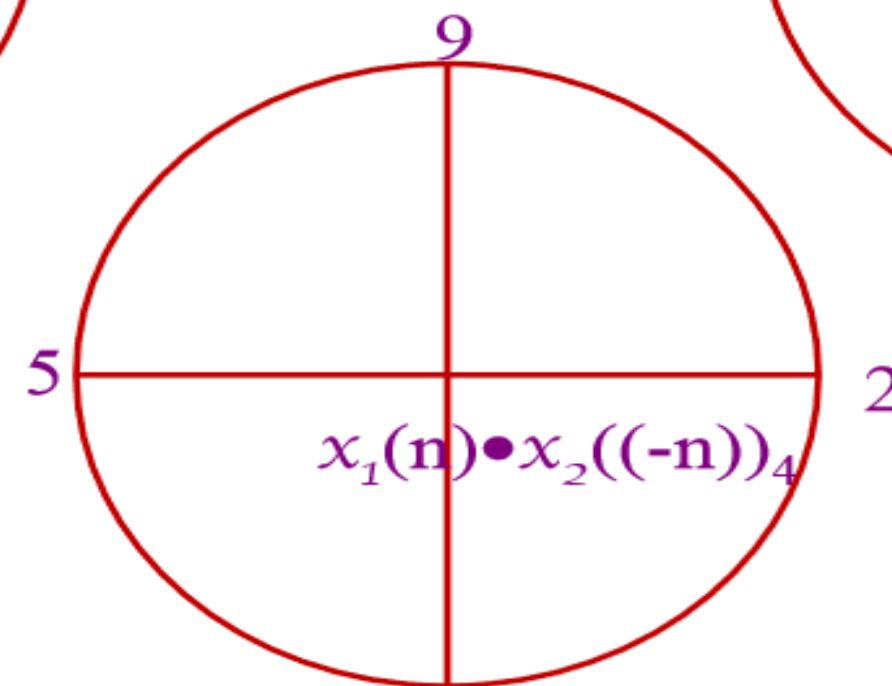
CIRCULAR CONVOLUTION



$$y(0) = \sum_{k=0}^3 x_1(k) \cdot x_2(-k)$$



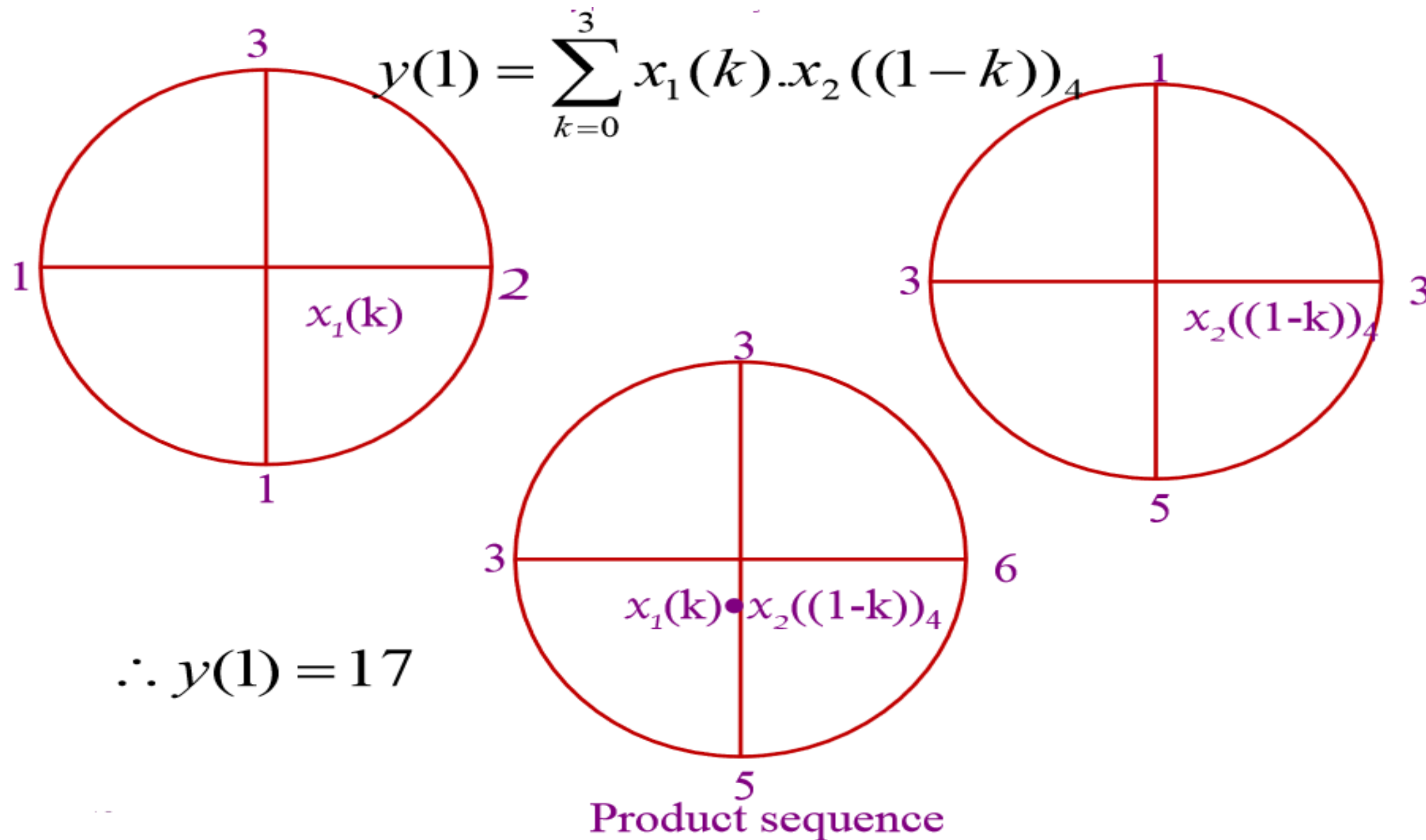
$$\therefore y(0) = 19$$



Product sequence

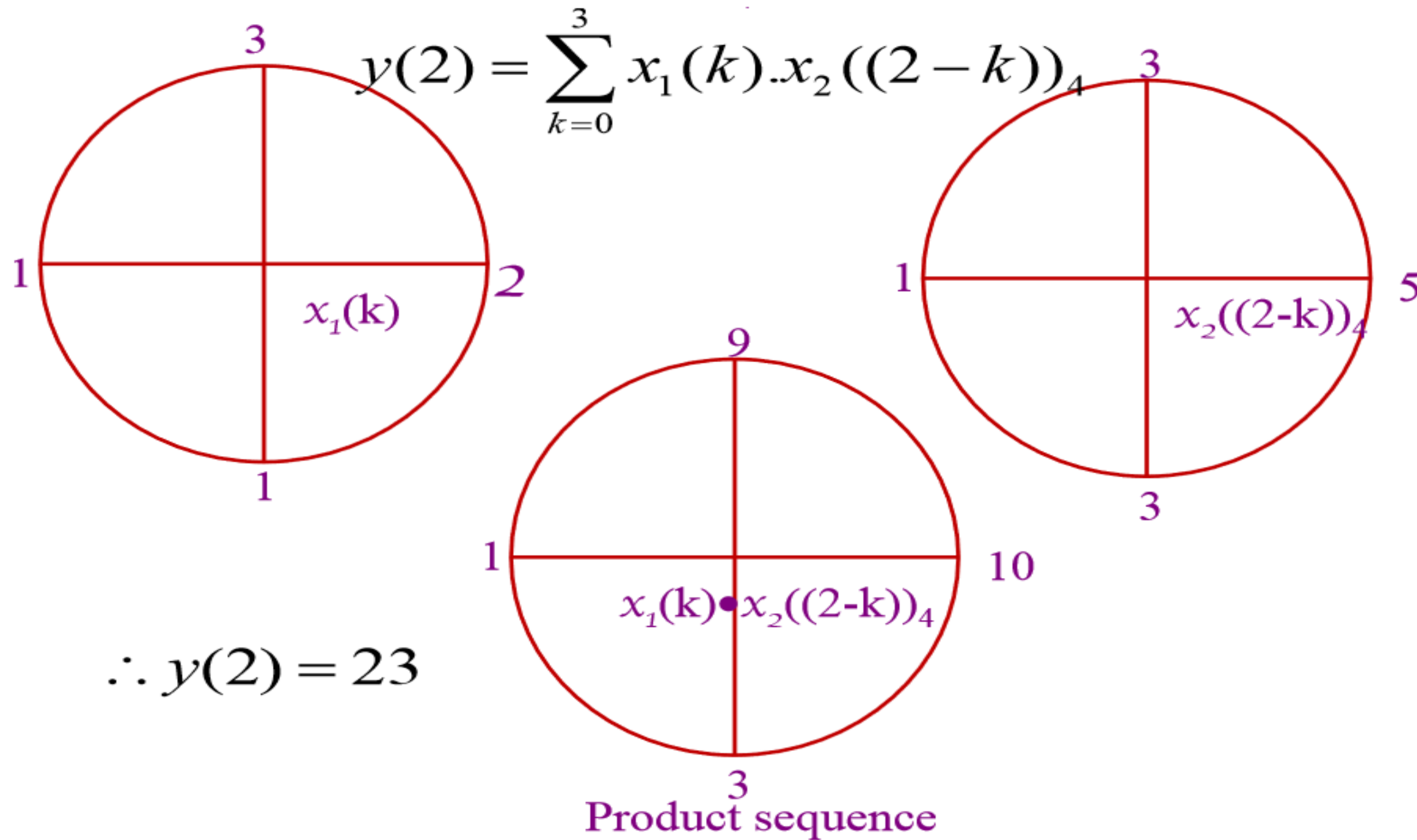


CIRCULAR CONVOLUTION



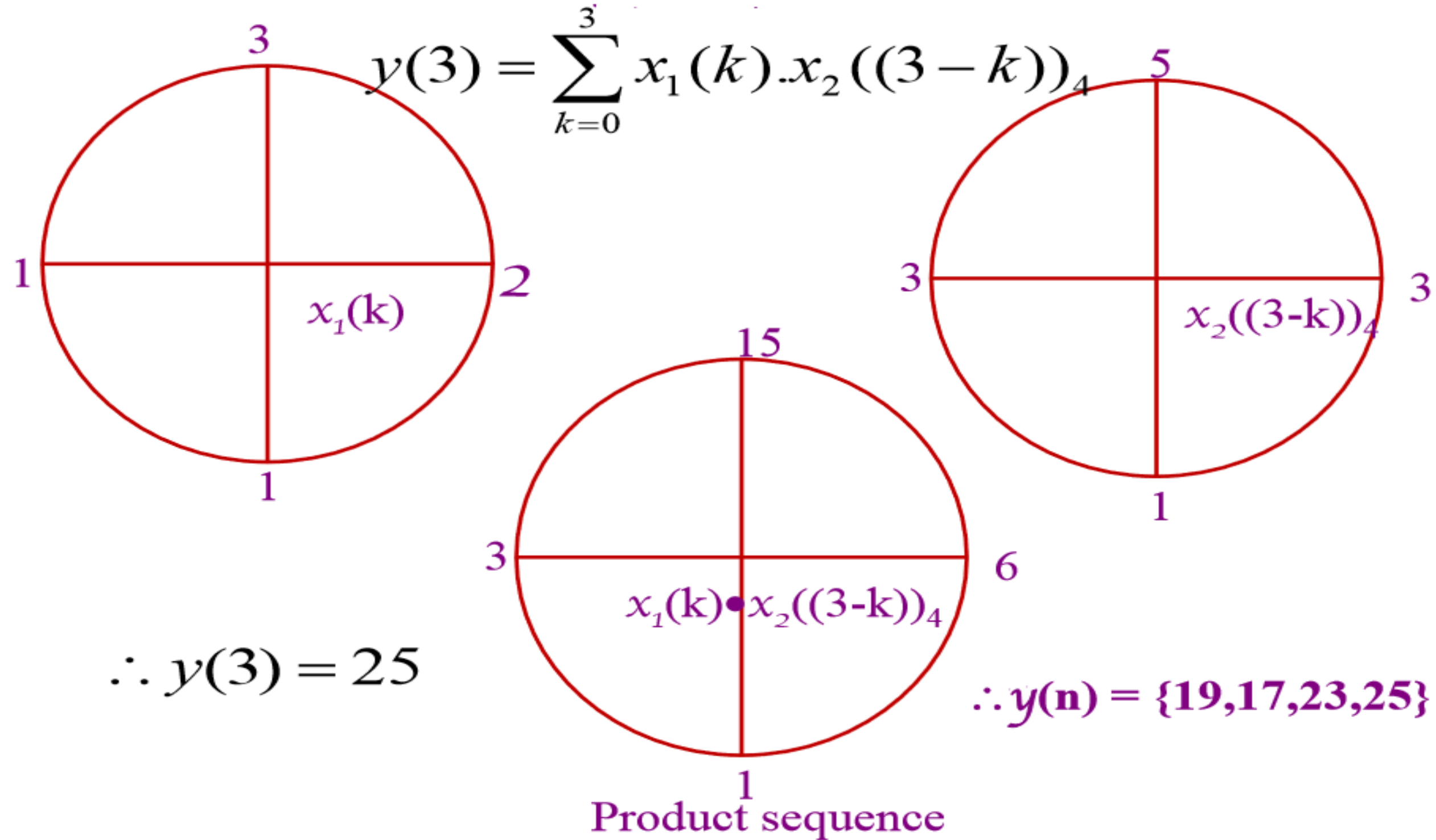


CIRCULAR CONVOLUTION





CIRCULAR CONVOLUTION





CIRCULAR CONVOLUTION – MATRIX APPROACH



$$y[n] = \begin{bmatrix} h(0) & h(N-1) & \dots & h(1) \\ h(1) & h(0) & \dots & h(2) \\ \vdots & \vdots & \dots & \vdots \\ h(N-1) & h(N-2) & \dots & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

e.g. $h(n) = \{2, 3, 1, 1\}$ and $x(n) = \{1, 3, 5, 3\}$

$$\therefore y[n] = \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

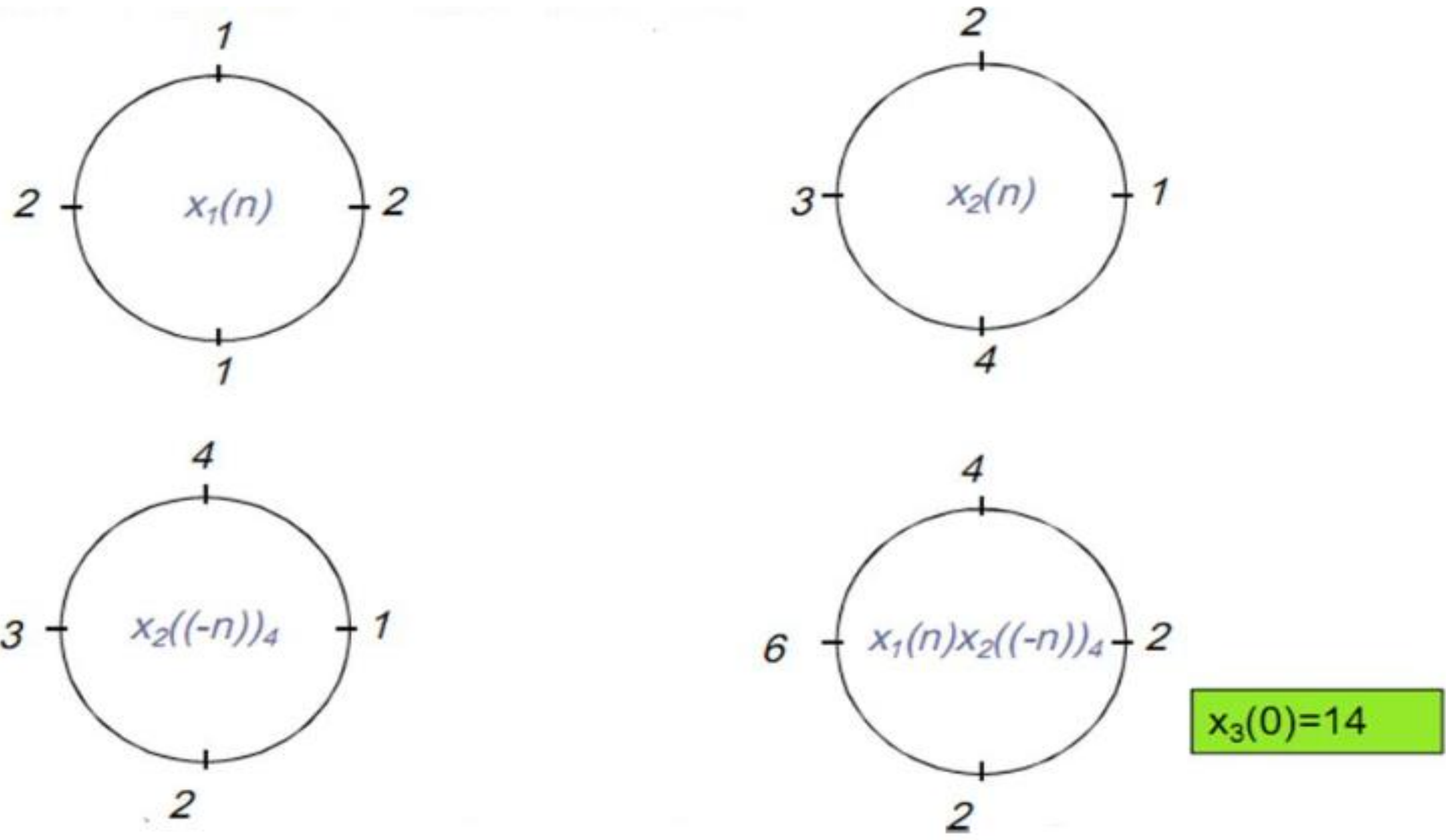
$$\therefore y[n] = \begin{bmatrix} 2+3+5+9 \\ 3+6+5+3 \\ 1+9+10+3 \\ 1+3+15+6 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \\ 23 \\ 25 \end{bmatrix}$$



CIRCULAR CONVOLUTION

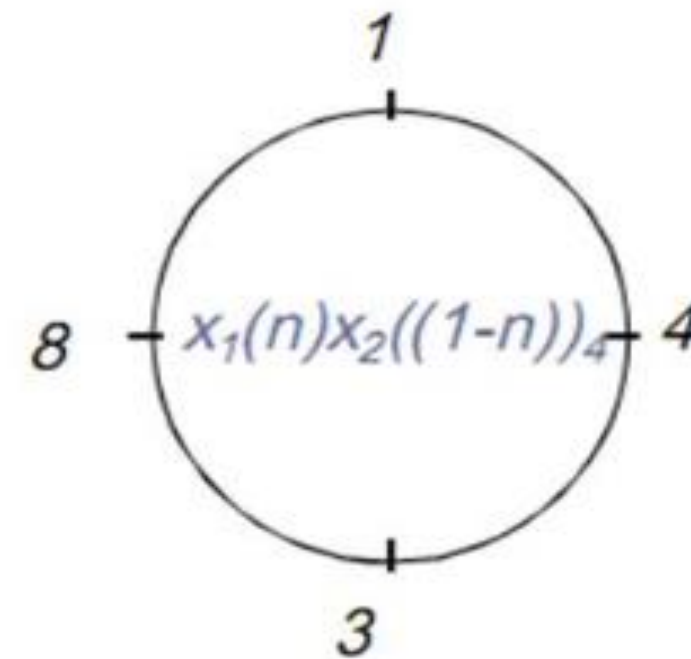
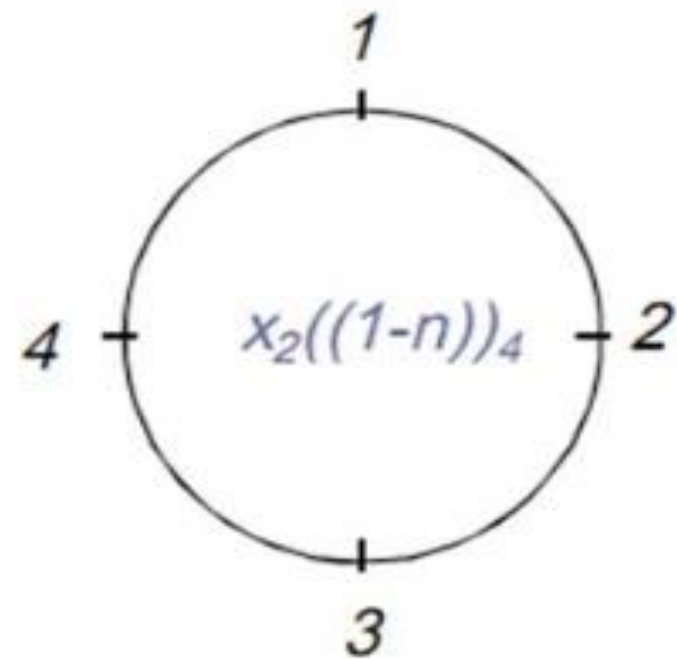
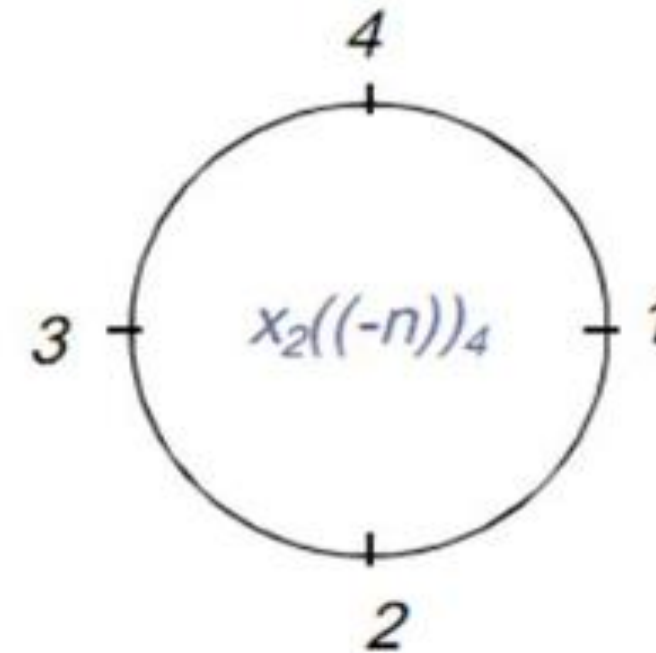
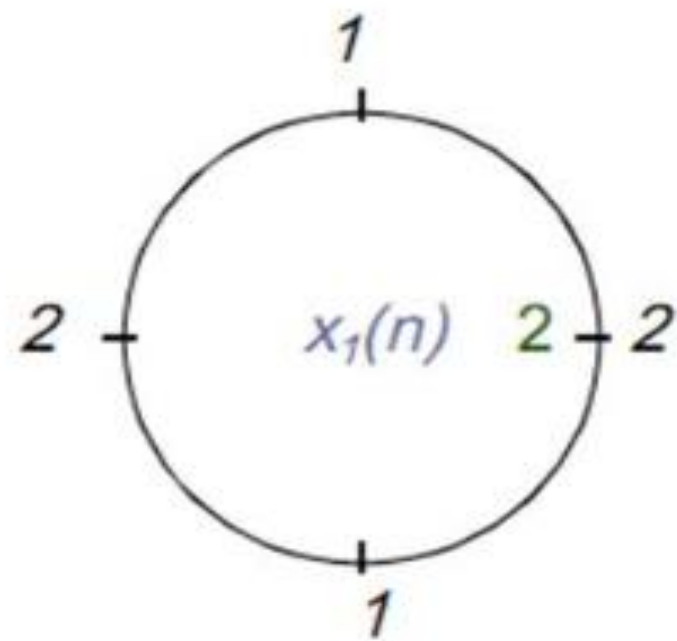


Perform circular convolution $X_1(n) = \{2, 1, 2, 1\}$ and $X_2(n) = \{1, 2, 3, 4\}$





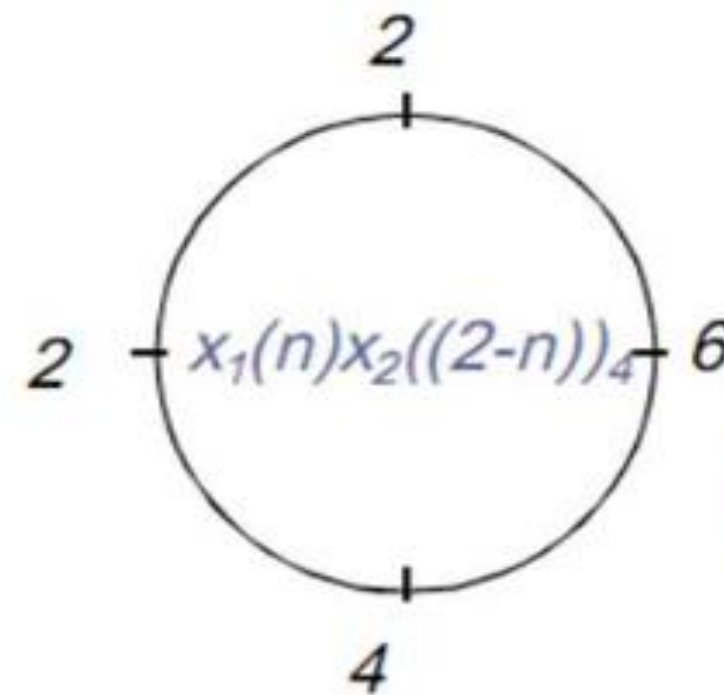
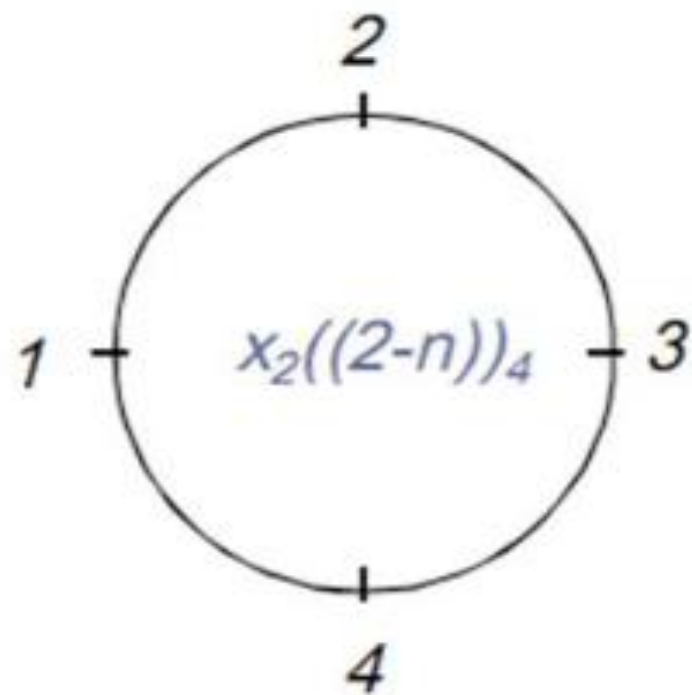
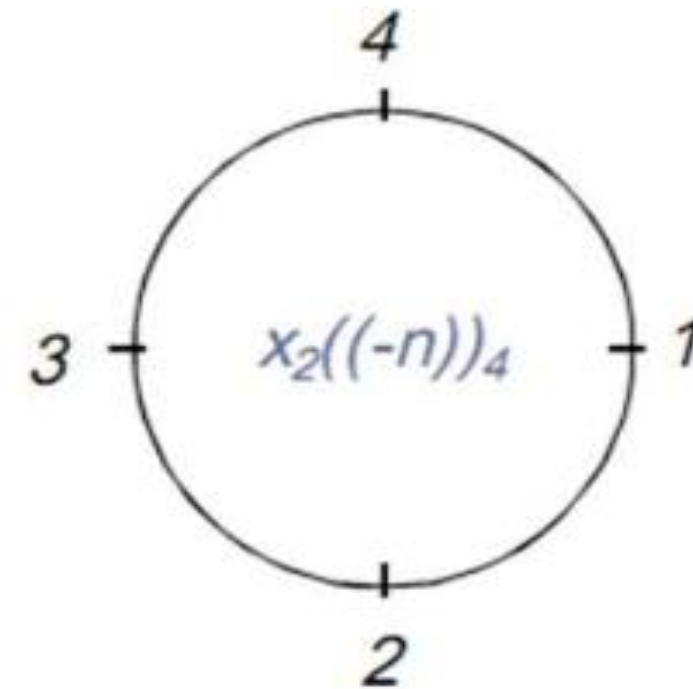
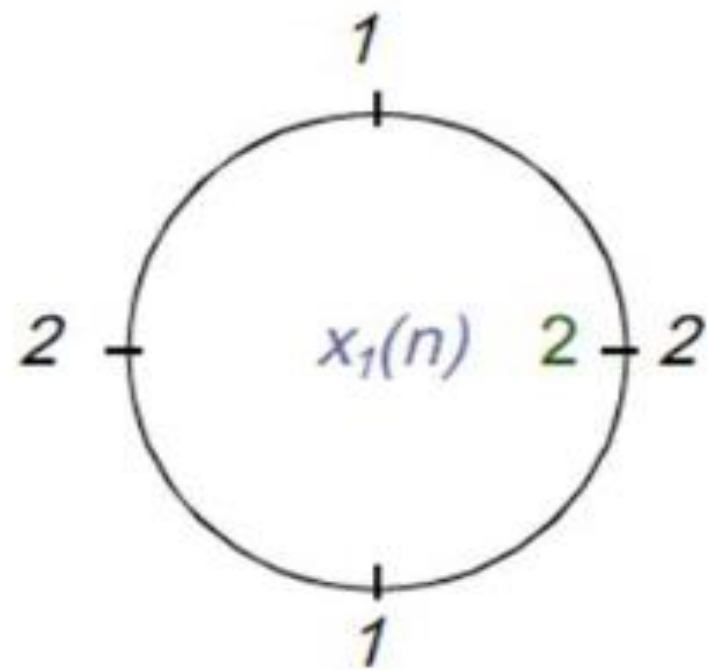
CIRCULAR CONVOLUTION



$x_3(1)=16$



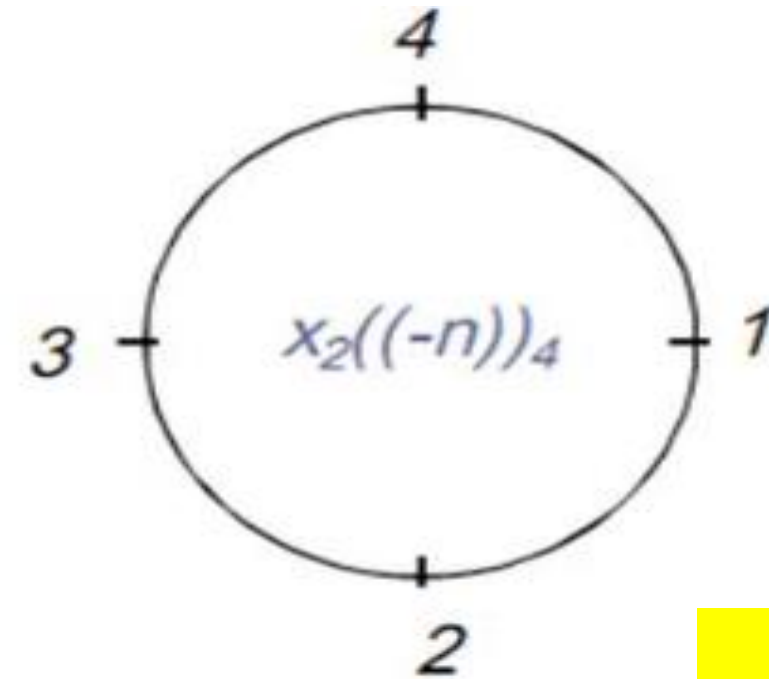
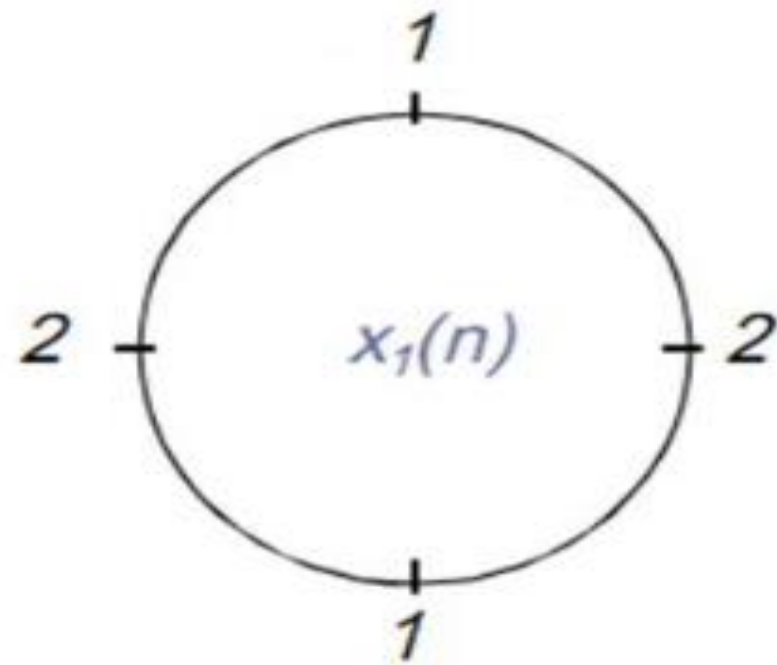
CIRCULAR CONVOLUTION



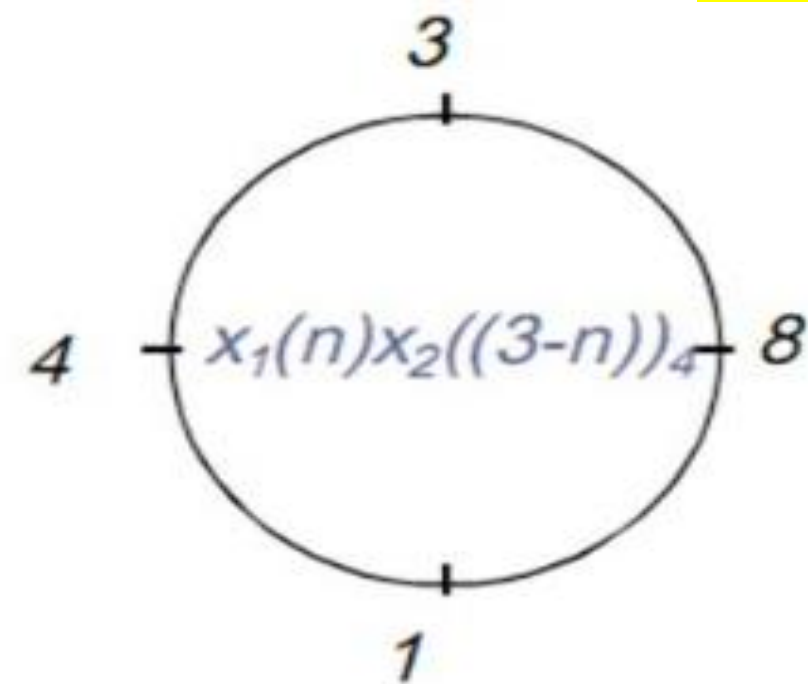
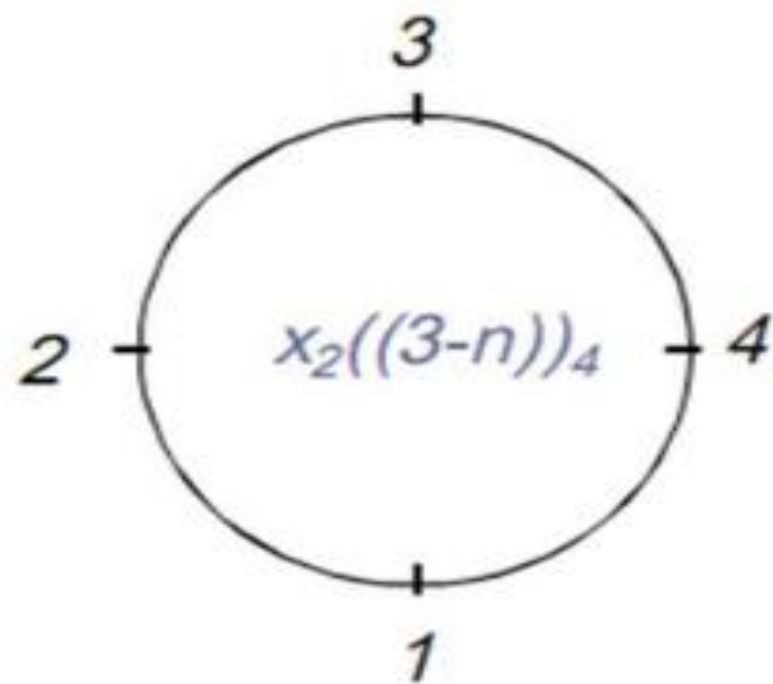
$x_3(2) = 14$



CIRCULAR CONVOLUTION



$$y(n) = \{14, 16, 14, 16\}$$



$$x_3(3) = 16$$



DIFFERENCE B/W LINEAR & CIRCULAR CONVOLUTION



S.No.	Linear Convolution	Circular Convolution
1	In case of convolution two signal sequences input signal $x(n)$ and impulse response $h(n)$ given by the same system, output $y(n)$ is calculated	Multiplication of two DFT s is called as circular convolution.
2	Multiplication of two sequences in time domain is called as Linear convolution	Multiplication of two sequences in frequency domain is called as circular convolution.
3	Linear Convolution of two signals returns $N-1$ elements where N is sum of elements in both sequences.	Circular convolution returns same number of elements that of two signals.
4	$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$	$x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2[((n-m))_N]$



ASSESSMENT



1. Define Circular Convolution.
2. If $x(n)$ and $h(n)$ two finite duration signals with length M and P respectively then the length of $y(n) = x(n) * h(n)$ is ----- samples
3. Mention some applications of Circular Convolution.
4. Determine circular convolution of $X_1(n) = \{2,1,2,1\}$ and $X_2(n) = \{1,2,3,4\}$
5. What is the difference between linear convolution and circular convolution.



THANK YOU