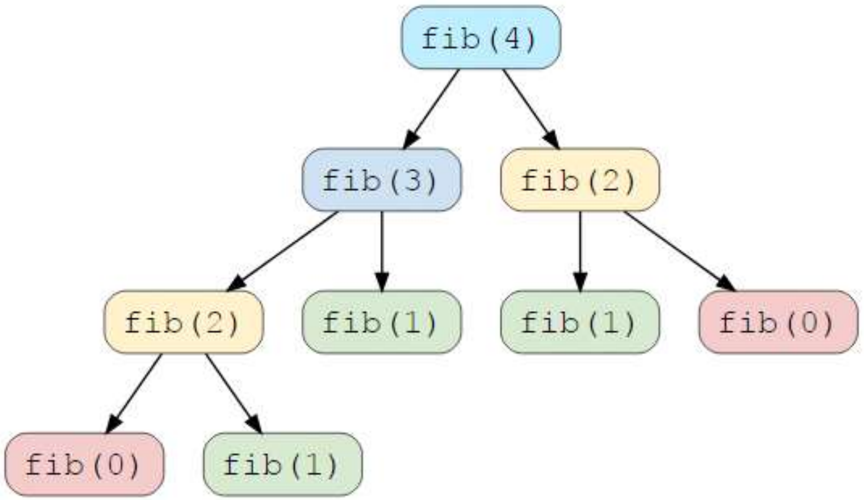
Unit III
Dynamic Programming and Greedy Technique

- Dynamic Programming
  - ***Computing a Binomial Coefficient***
  - Warshall’s algorithm
  - Floyd’s algorithm
  - Optimal Binary Search Trees
  - Knapsack Problem and Memory functions

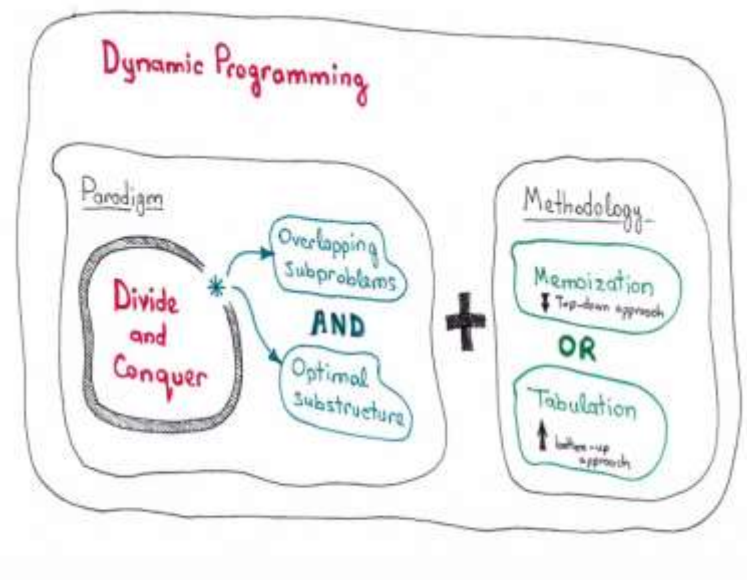
Dynamic Programming

- Dynamic programming – pblm □ similar sub problems □ reuse the solution
- **Characteristics**
  - Overlapping sub problems – solving same sub problems
  - Optimal substructure property – optimal solution can be built from sub problem
  - Example : Fibonacci series



Dynamic Programming

- **Methodology**
  - Top-down with memoization
    - Storing the result of already solved sub-problem is called memoization
  - Bottom-up with Tabulation
    - Sub-problems (bottom – up)



Difference between Divide and conquer and Dynamic Programming

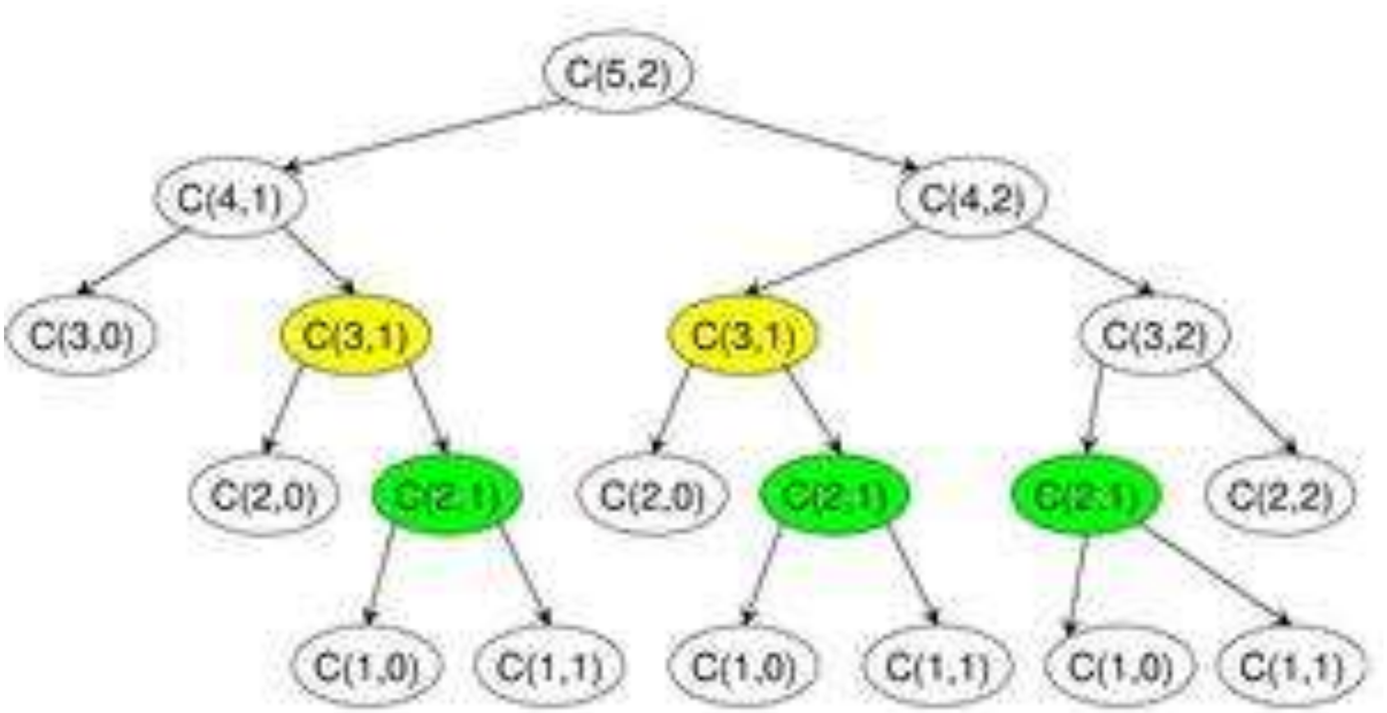
Divide and conquer	**Dynamic Programming**
Sub problems are not dependent on each other	Sub problems are dependent on each other
Doesn’t store the solution of sub-problem	Stores the solution of sub problem

Computing a Binomial Coefficient

- Binomial coefficient – computation of no of ways r items that can be chosen from n elements C($^{n}_{r}$)
- C(n, k) = n! / (n-k)! * k!
- C(n, k) = C (n-1, k-1) + C (n-1, k) , n>k, k>0
- C (n,0) = 1, C (n,n) = 1
- ***Example:***
- 1st formula : C (4,2) □ 4! /(2!) * 2! □ 24 / 4 □ 6
- 2nd formula : C(4,2) □ C(3,1) + C(3,2) □ ….. □ 6
- C (4,2) □ how many two combinations of elements can be picked from set of 4 elements
- Example: possibilities of 1,2,3,4 □ (1,2) (1,3) (1,4) (2,3) (2,4) (3,4)

Computing a Binomial Coefficient

- Example : C (5,2)
- C(n, k) = C (n-1, k-1) + C (n-1, k) , n>k, k>0
- C (n,0) = 1, C (n,n) = 1



Computing a Binomial Coefficient - Tabulation

	0	1	2	3	4	5	...	$(k-1)$	k
0	1								
1	1	1							
2	1	2	1						
3	1	3	3	1					
4	1	4	6	4	1				
5									
⋮									
k	1								1
⋮									
$(n-1)$	1							$C(n-1, k-1)$	$C(n-1,k)$
n	1								$C(n,k)$

Computing a Binomial Coefficient - Algorithm

Algorithm *Binomial*(n, k)

for $i \leftarrow 0$ **to** n **do** // fill out the table row wise

for $i = 0$ **to** $\min(i, k)$ **do**

if j==0 or j==i **then** $C[i, j] \leftarrow 1$ // IC

else $C[i, j] \leftarrow C[i-1, j-1] + C[i-1, j]$ // recursive relation

return $C[n, k]$

Computing a Binomial Coefficient - Analysis

- Cost of the algorithm – table
- Sum – 2 parts (upper and lower triangle)
- *A*(*n*, *k*) = sum for upper triangle + sum for the lower rectangle

$$A(n,k) = \sum_{i=1}^{k} \underbrace{\sum_{j=1}^{i-1} 1} + \sum_{i=k+1}^{n} \underbrace{\sum_{j=1}^{k} 1}$$

$$\Rightarrow \sum_{i=1}^{k} \left((i-1) - \cancel{1} + \cancel{1}\right) + \sum_{i=k+1}^{n} \left(k - \cancel{1} + \cancel{1}\right)$$

$$\Rightarrow \underbrace{\sum_{i=1}^{k} (i-1)} + \sum_{i=k+1}^{n} k$$

$$\Rightarrow \left[\underbrace{\sum_{i=1}^{k} i} - \underbrace{\sum_{i=1}^{k} 1}\right] + k \sum_{i=k+1}^{n} 1$$

$$\Rightarrow \frac{k(k+1)}{2} - \left(k - \cancel{1} + \cancel{1}\right) + k\left[n - (k+1) + 1\right]$$

Computing a Binomial Coefficient - Analysis

$$\Rightarrow \frac{k^2 + k}{2} - k + k\left[n - k - \cancel{1} + \cancel{1}\right]$$

$$\Rightarrow \frac{k^2 + k - 2k + 2\left(nk - k^2\right)}{2}$$

$$\Rightarrow \frac{k^2 - k + 2nk - 2k^2}{2}$$

$$\Rightarrow \frac{-k^2 - k + 2nk}{2}$$

$$\approx nk$$

$$\boxed{O(nk)}$$

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