Example 6.13 ***For the above example, calculate the end thrust on the gear.***

Given Data : Refer Example 6.12.

To find : End thrust on the gear.

☺ ***Solution :*** We know that the end thrust or axial load on the gear,

$$F_a = F_t \times \tan\beta = \frac{P}{v} \times \tan\beta$$

$$= \frac{15 \times 10^3}{7.59} \times \tan 15° = \mathbf{529.54\ N} \quad \textbf{Ans.}$$

Example 6.14 ***A compressor running at 360 r.p.m. is driven by a 140 kW, 1440 r.p.m. motor through a pair of 20° full depth helical gears having helix angle of 25°. The centre distance is approximately 400 mm. The motor pinion is to be forged steel and the driven gear is to be cast steel. Assume medium shock conditions. Design the gear pair.***

Given Data : N_2 = 360 r.p.m ; P = 40 kW ; N_1 = 1440 r.p.m. ; ϕ = 20° ; β = 25° ; a = 400 mm.

To find : Design the helical gear pair.

☺ ***Solution :*** Since the materials for pinion and gear are different, first we have to evaluate $[\sigma_{b1}]\, y_1'$ and $[\sigma_{b2}]\, y_2'$ to find out the weaker element.

$$\text{Gear ratio, } i = \frac{N_1}{N_2} = \frac{1440}{360} = 4$$

$$\text{Assume } z_1 = 20$$

$$\therefore \quad z_2 = i \times z_1 = 4 \times 20 = 80$$

Virtual number of teeth :

$$z_{v1} = \frac{z_1}{\cos^3 \beta} = \frac{20}{\cos^3 25°} \approx 27 \text{; and}$$

$$z_{v2} = \frac{80}{\cos^3 25°} \approx 108$$

Given that the pinion is to be forged steel and the gear is to be cast steel. Therefore consulting Table 5.3, the following steels are selected.

Pinion – Forged steel, and

Gear – Grade 1 *i.e.*, CS 65 cast steel

For pinion : From Table 5.4,

$$[\sigma_{b1}] = 112 \text{ N/mm}^2, \text{ for forged steel; and}$$

$$\text{Form factor, } y_1' = 0.154 - \left(\frac{0.912}{z_{v1}}\right), \text{ for } 20° \text{ full depth}$$

$$= 0.154 - \left(\frac{0.912}{27}\right) = 0.1202$$

$$\therefore \quad [\sigma_{b1}]\, y_1' = 112 \times 0.1202 = 13.465 \text{ N/mm}^2$$

For gear : From Table 5.4, $[\sigma_{b2}] = 105 \text{ N/mm}^2$, for cast steel ; and

$$\text{Form factor, } y_2' = 0.154 - \left(\frac{0.912}{z_{v2}}\right), \text{ for } 20° \text{ full depth}$$

$$= 0.154 - \left(\frac{0.912}{108}\right) = 0.1455$$

$$\therefore \quad [\sigma_{b2}]\, y_2' = 105 \times 0.1455 = 15.28 \text{ N/mm}^2$$

We find $[\sigma_{b1}]\, y_1' < [\sigma_{b2}]\, y_2'$, *i.e.*, the pinion is weaker. Thus, we have to design the ***pinion only.***

1. Material selection : Pinion – 40 Ni 2 Crl Mo28 ; and Gear – Grade 1 cast steel

2. $z_1 = 20$ and $z_2 = 80$. ... (Already calculated)

3. Calculation of module : Since the centre distance is given, we need not to equate F_s and F_d to calculate the module. The module can be calculated using the relation

$$a = \left(\frac{m_n}{\cos \beta}\right) \times \left(\frac{z_1 + z_2}{2}\right)$$

$$400 = \left(\frac{m_n}{\cos 25°}\right)\left(\frac{20+80}{2}\right)$$

or Normal module, $m_n = 7.25$ mm

From Table 5.8, the nearest higher standard normal module is 8 mm.

4. Calculation of b, d and v :

✓ Face width (b) : $b = 10\ m_n = 10 \times 8 = 80$ mm.

✓ Pitch circle diameter (d_1) : $d_1 = \dfrac{m_n}{\cos\beta} \times z_1 = \dfrac{8}{\cos 25°} \times 20 = 176.54$ mm

✓ Pitch line velocity (v) : $v = \dfrac{\pi\, d_1\, N_1}{60} = \dfrac{\pi \times 176.54 \times 10^{-3} \times 1440}{60} = 13.31$ m/s

5. Calculation of beam strength (F_s) :

We know that,
$$F_s = \pi \cdot m_n \cdot b \cdot [\sigma_b] \cdot y'$$
$$= \pi \times 8 \times 80 \times 112 \times 0.1202 = 27067.76 \text{ N}$$

6. Calculation of accurate dynamic load (F_d) :

We know that,
$$F_d = F_t + \frac{21\, v\, (cb \cdot \cos^2\beta + F_t)\cos\beta}{21\, v + \sqrt{cb\cos^2\beta + F_t}}$$

where $F_t = \dfrac{P}{v} = \dfrac{140 \times 10^3}{13.31} = 10518.4$ N

c = Deformation factor, from Tables 5.7(a) and (b).

= $11860\, e$, for steel and steel, 20° full depth, from Table 5.7(a).

e = 0.038 mm, for m_n upto 8 and carefully cut gears, from Table 5.7(b).

∴ $c = 11860 \times 0.038 = 450.68$ N/mm

Then,
$$F_d = 10518.4 + \frac{21 \times 13.31 \times 10^3\,(450.68 \times 80 \times \cos^2 25° + 10518.4)\cos 25°}{21 \times 13.31 \times 10^3 + \sqrt{450.68 \times 80 \times \cos^2 25° + 10518.4}}$$
$$= 46865.44 \text{ N}$$

7. Check for beam strength (or tooth breakage) : We find $F_d > F_s$. So the design is unsatisfactory. Since the difference between F_d and F_s is high, we can increase the face width by increasing the normal module from 8 mm to 9 mm.

Then Face width, $b = 10\ m_n = 10 \times 9 = 90$ mm ;

Pitch circle diameter, $d_1 = \dfrac{m_n}{\cos\beta} \times z_1 = \dfrac{9}{\cos 25°} \times 20 = 198.61$ mm ;

Pitch line velocity, $v = \dfrac{\pi\, d_1\, N_1}{60} = \dfrac{\pi \times 198.61 \times 10^{-3} \times 1440}{60} = 14.97$ m/s ;

$$\text{Tangential load, } F_t = \frac{P}{v} = \frac{140 \times 10^3}{14.97} = 9352 \text{ N ; and}$$

Expected error, e = 0.0205 mm, for m_n upto 9 and precision gears from Table 5.7(b).

$\therefore$ Deformation factor, $c = 11860\, e$

$$= 11860 \times 0.0205 = 243.13 \text{ N/mm}$$

Then the modified value of dynamic load is given by

$$F_d = 9352 + \frac{21 \times 14.97 \times 10^3\,(243.13 \times 90 \times \cos^2 25° + 9352)\cos 25°}{21 \times 14.97 \times 10^3 + \sqrt{243.13 \times 90 \times \cos^2 25° + 9352}}$$

$$= 34104.29 \text{ N}$$

The modified beam strength (F_s) value is given by

$$F_s = \pi \times 9 \times 90 \times 112 \times 0.1202 = 34257.64 \text{ N}$$

Now we find $\mathbf{F_s > F_d}$. It means the gear tooth has adequate beam strength and it will not fail by breakage. Thus ***the design is satisfactory.***

8. Calculation of limiting wear load (F_w) :

We know that $$F_w = \frac{d_1 \times b \times Q \times K_w}{\cos^2 \beta}$$

where Q = Ratio factor $= \frac{2i}{i+1} = \frac{2 \times 4}{4+1} = 1.6$; and

K_w = Load stress factor = 2.553 N/mm^2,

for steel hardened to 400 BHN, from Table 5.9.

$$\therefore \quad F_w = \frac{198.61 \times 90 \times 1.6 \times 2.553}{\cos^2 25°} = 88892.06 \text{ N}$$

9. Check for wear : We find $\mathbf{F_w > F_s}$. It means the gear tooth has adequate wear capacity and will not wear out. Thus ***the design is safe and satisfactory.***

10. Calculation of basic dimensions of pinion and gear : Refer Table 6.1.

✓ Normal module : m_n = **9 mm**

✓ Face width : b = **90 mm**

✓ Number of teeth : z_1 = **20** ; and z_2 = **80**

✓ Pitch circle diameter : d_1 = **198.61 mm** ; and

$$d_2 = \frac{m_n}{\cos \beta} \times z_2 = \frac{9}{\cos 25°} \times 80$$