# SNS COLLEGE OF TECHNOLOGY

**An Autonomous Institution**

**Coimbatore-35**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

### 19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

### UNIT 3 – FIR FILTER DESIGN

TOPIC – FIR Filter Design using Windowing Techniques

# WINDOWING TECHNIQUES OF FIR FILTERS

- The windows are finite duration sequences used to modify the impulse response of the FIR filters in order to reduce the ripples in the pass band and stop band and also to achieve the desired transition from pass band and stop band
- The FIR filter design starts with desired frequency response $H_d(e^{j\omega})$. The desired impulse response $h_d(n)$ is obtained by taking inverse Fourier transform of $H_d(e^{j\omega})$. The desired impulse response will be an infinite duration sequence
- On multiplying finite duration window sequence with infinite duration impulse response with modified sample, which is used to design FIR filter
- **Types of Windowing Techniques: Rectangular Window, Hanning Window, Hamming Window and Blackman Window**

**Features of Hamming Window spectrum:**

1. The main-lobe width is equal to 8**π**/ N
2. The maximum side-lobe magnitude is -41dB
3. The side-lobe magnitude remains constant with increasing ω

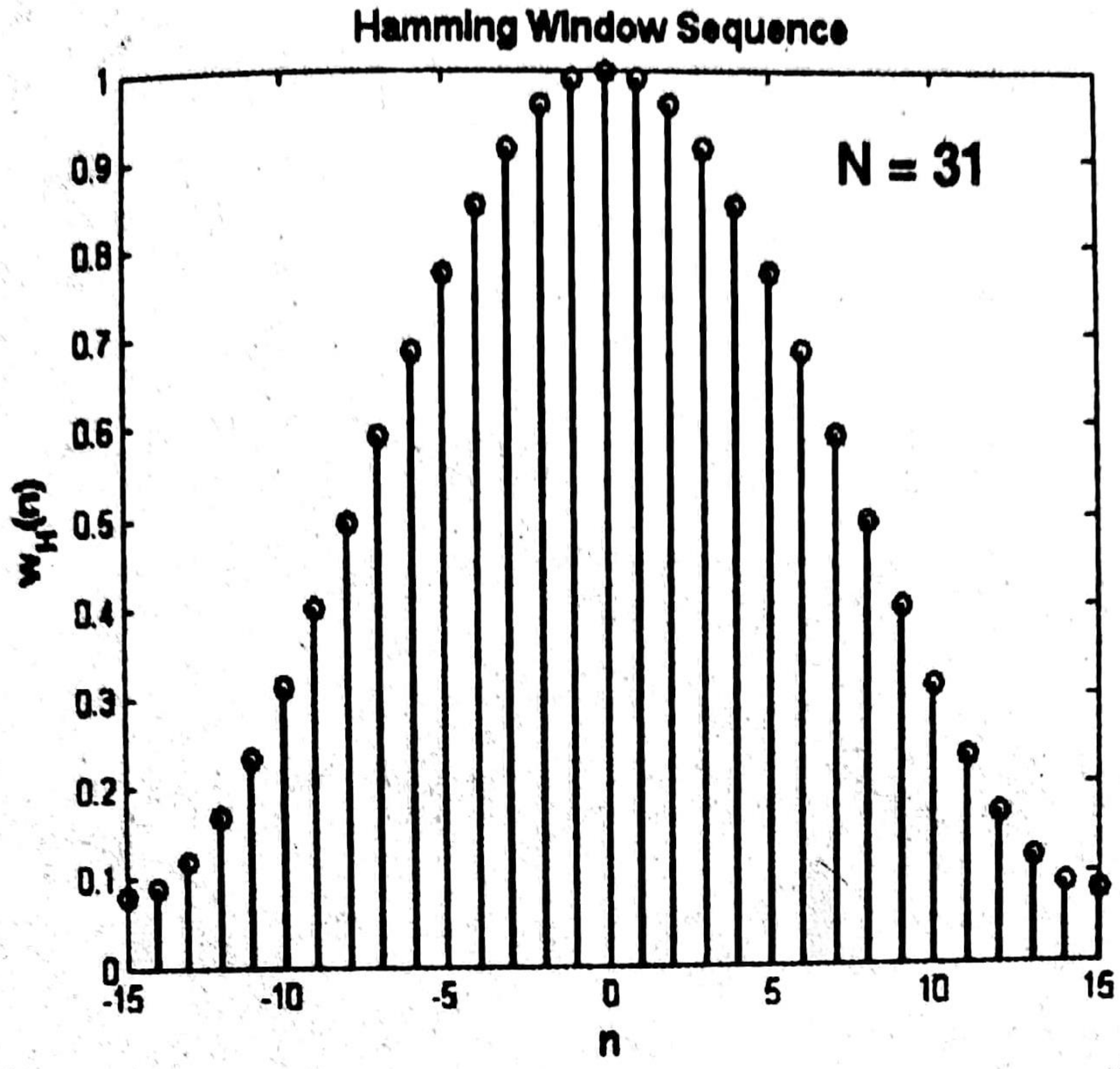
$$w_H(n) = 0.54 + 0.46 \cos \frac{2\pi n}{N-1} \quad ; \quad \text{for } n = -\frac{N-1}{2} \text{ to } +\frac{N-1}{2}$$

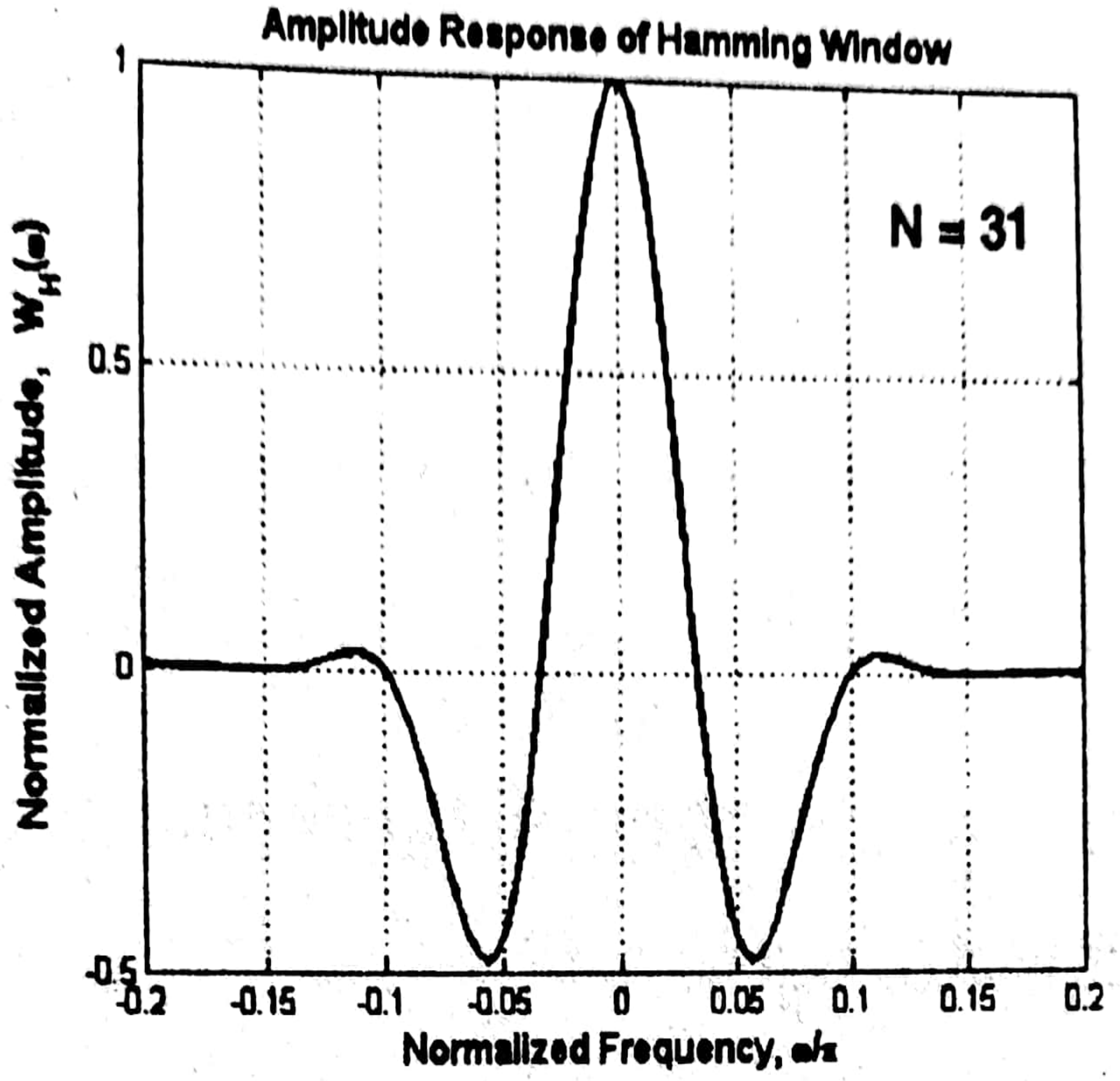
$$= 0 \quad ; \quad \text{other } n$$

$$w_H(n) = 0.54 - 0.46 \cos \frac{2\pi n}{N-1} \quad ; \quad \text{for } n = 0 \text{ to } N-1$$

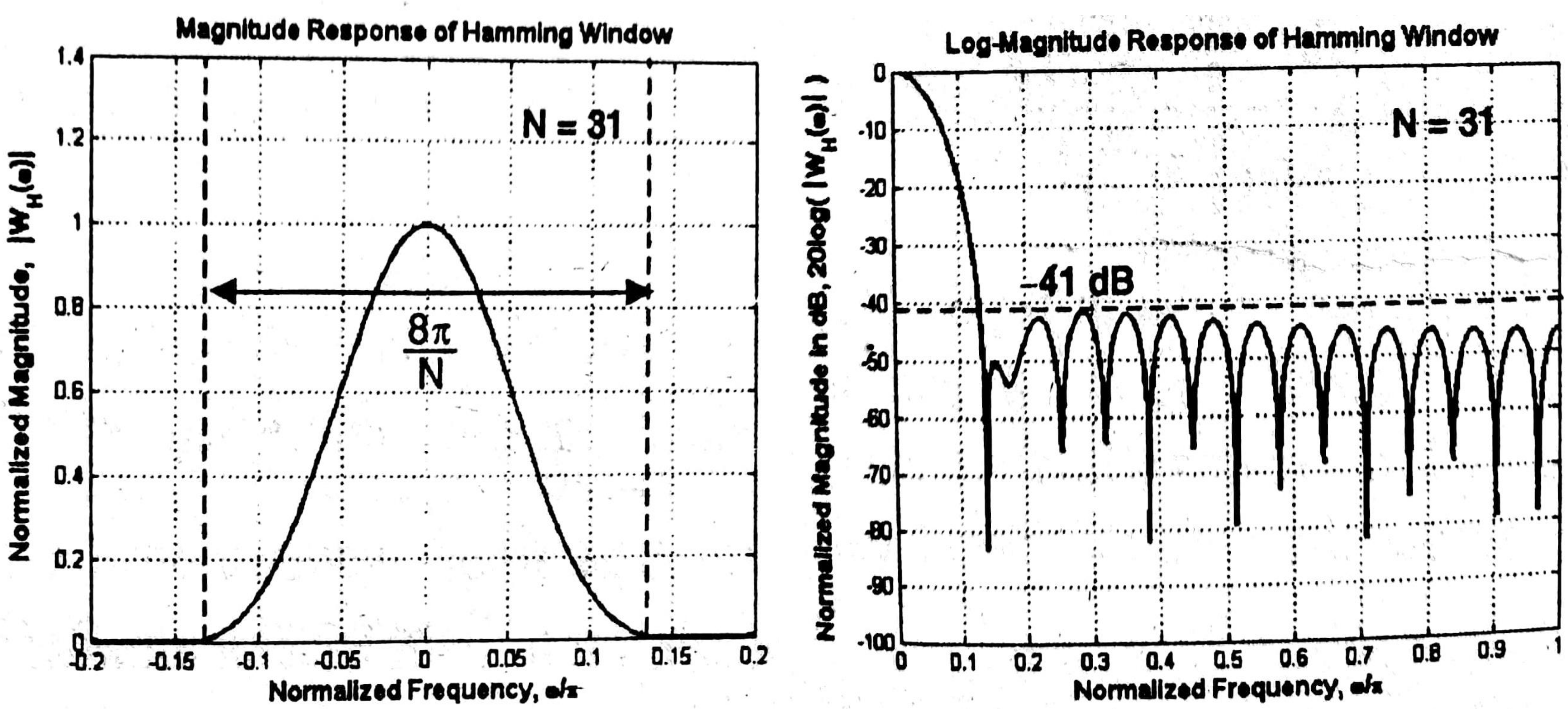
$$= 0 \quad ; \quad \text{other } n$$

# HAMMING WINDOW

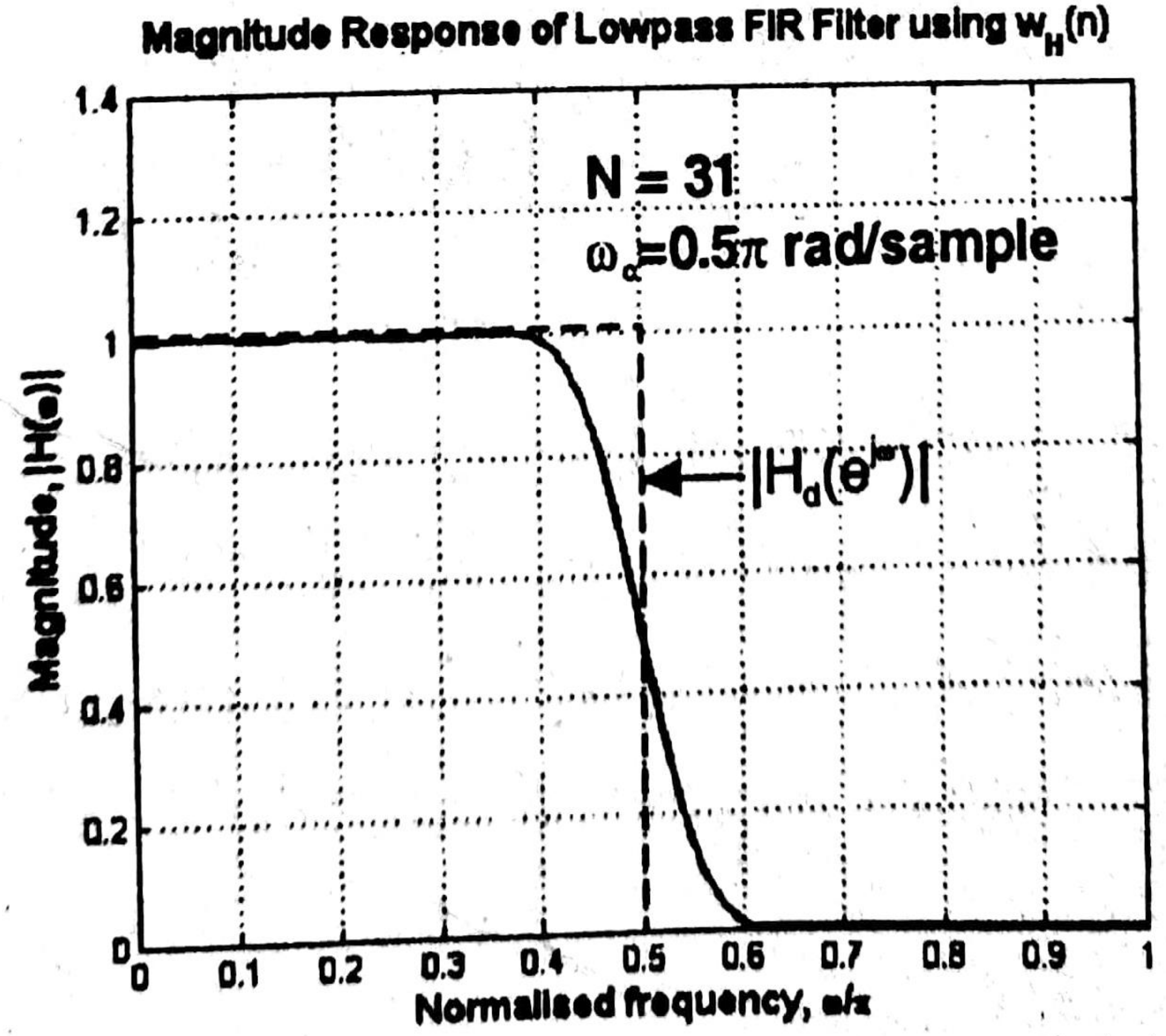


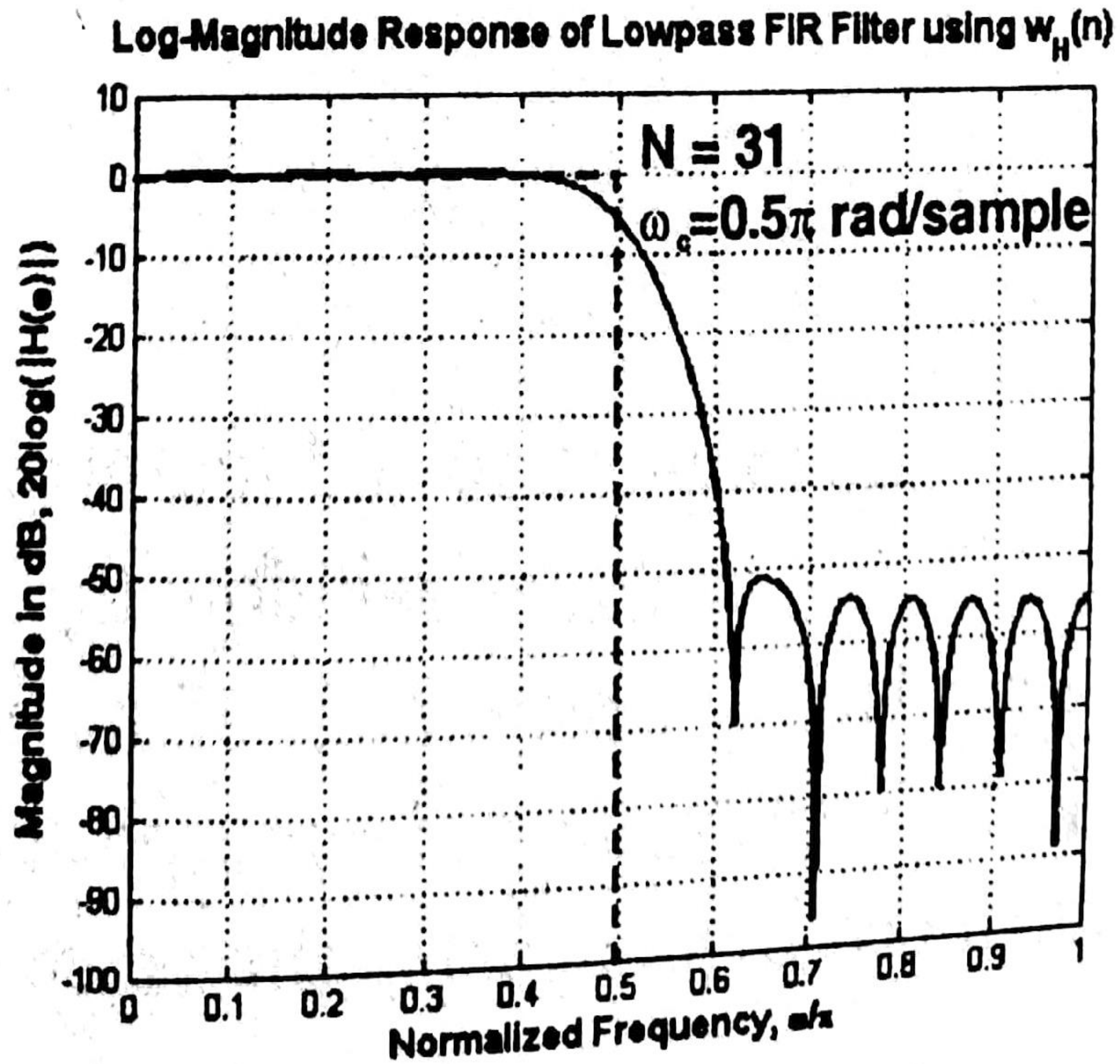



# HAMMING WINDOW



# HAMMING WINDOW



# BLACKMAN WINDOW

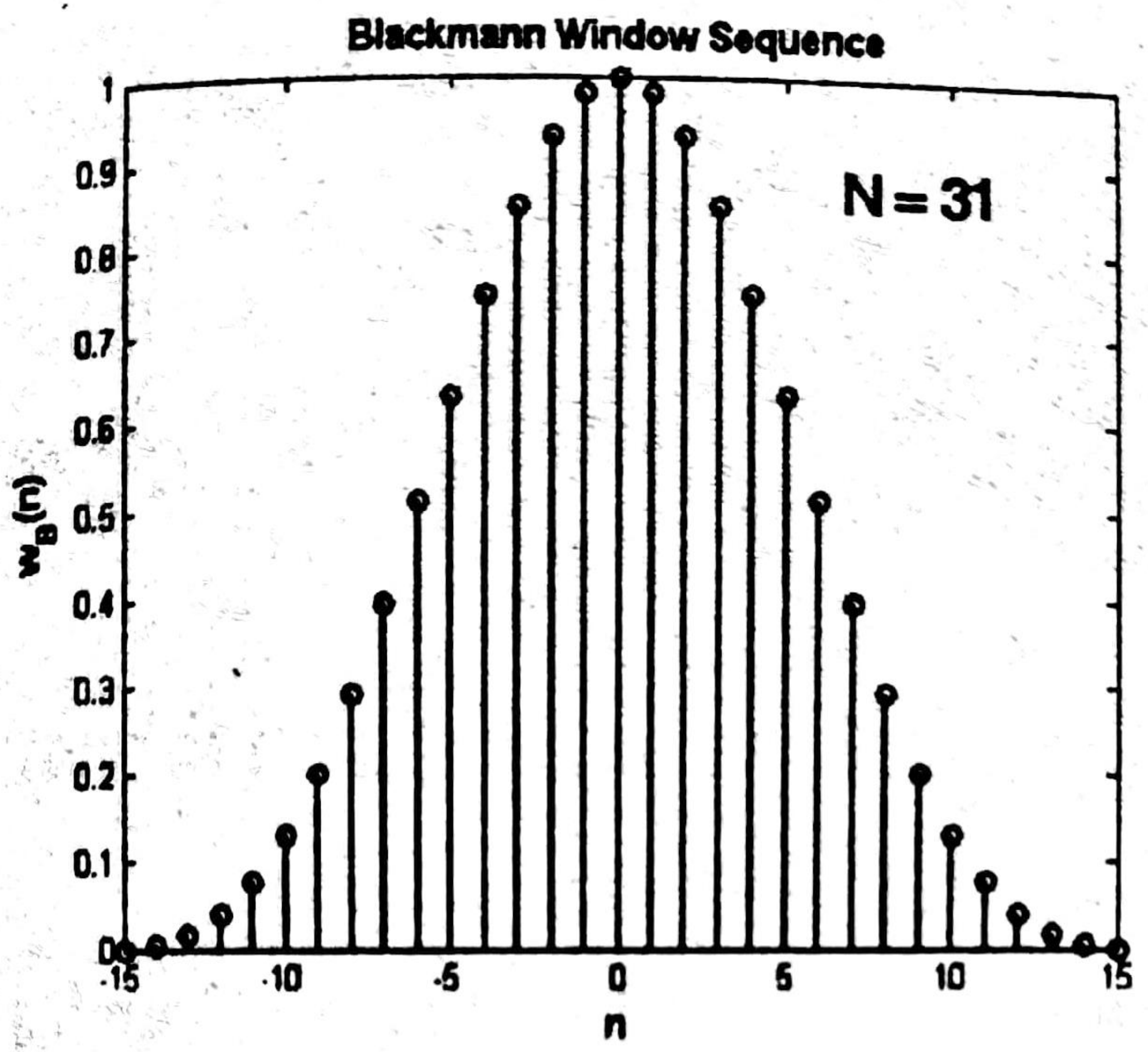
**Features of Blackman Window spectrum:**

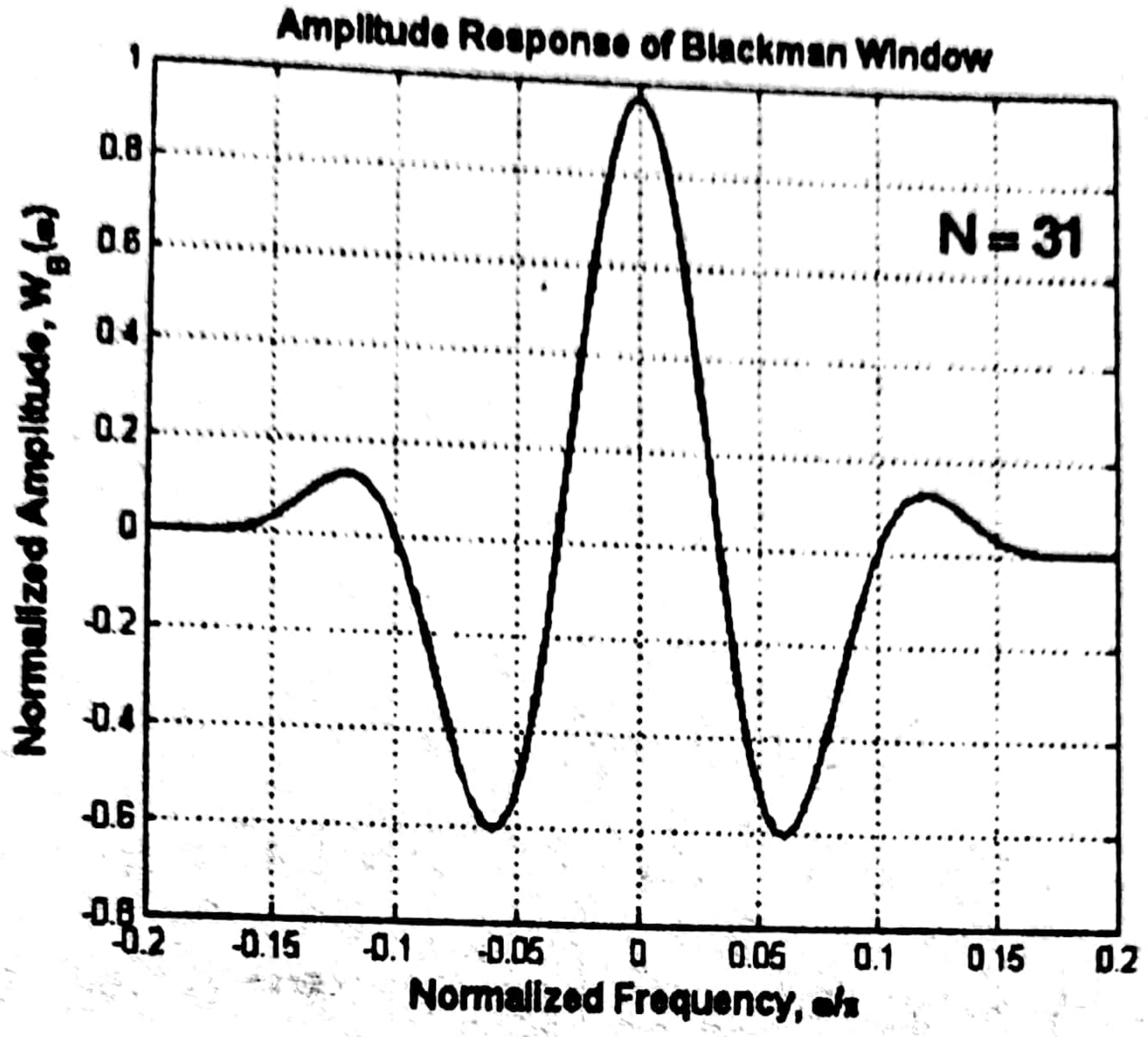
1. The main-lobe width is equal to $12\pi/N$
2. The maximum side-lobe magnitude is -58dB
3. The side-lobe magnitude decreases with increasing $\omega$

$$w_B(n) = 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1} \quad ; \quad \text{for } n = -\frac{N-1}{2} \text{ to } +\frac{N-1}{2}$$
$$= 0 \quad ; \quad \text{other } n$$

$$w_B(n) = 0.42 - 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1} \quad ; \quad \text{for } n = 0 \text{ to } N-1$$
$$= 0 \quad ; \quad \text{other } n$$

# BLACKMAN WINDOW



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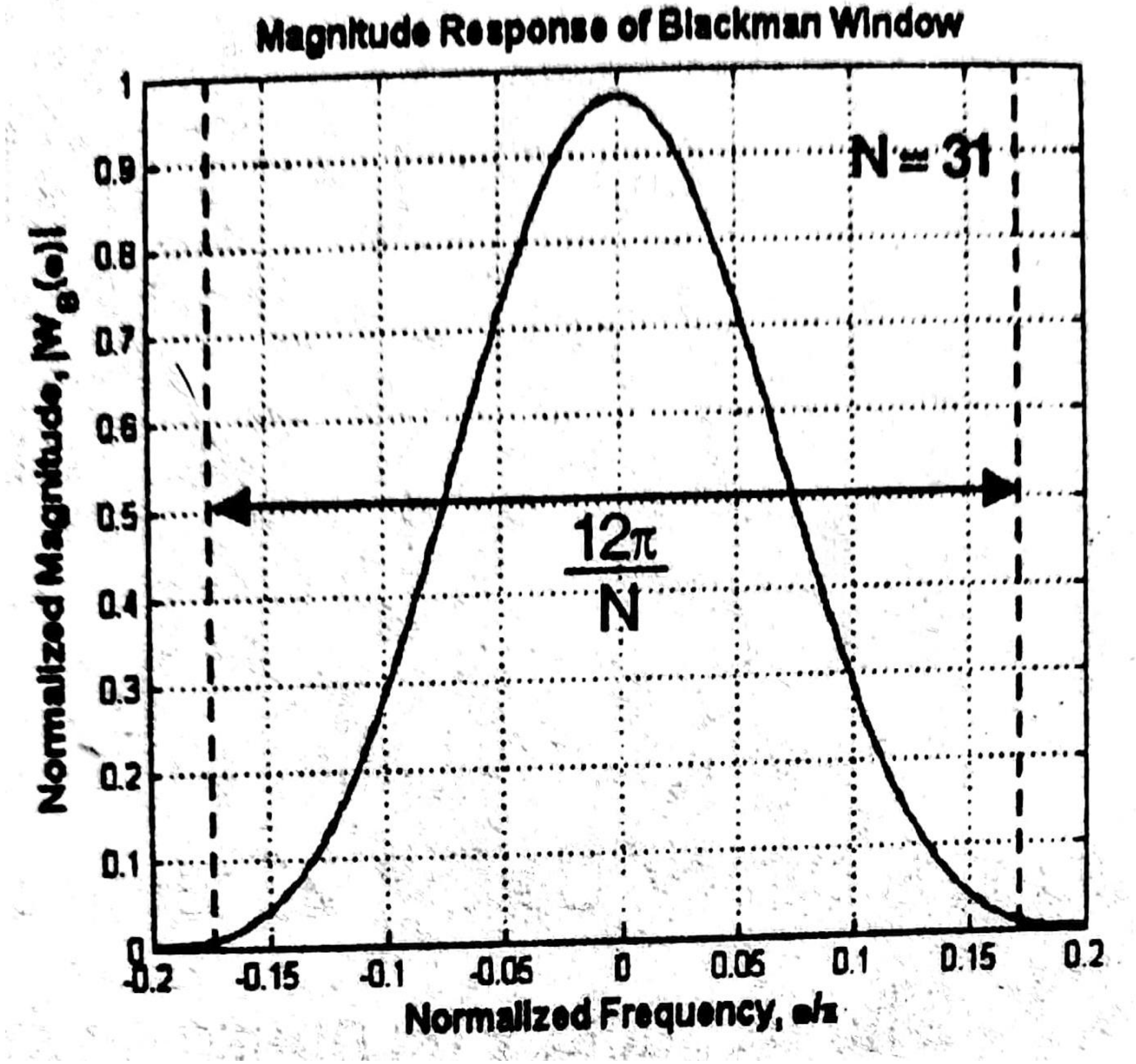


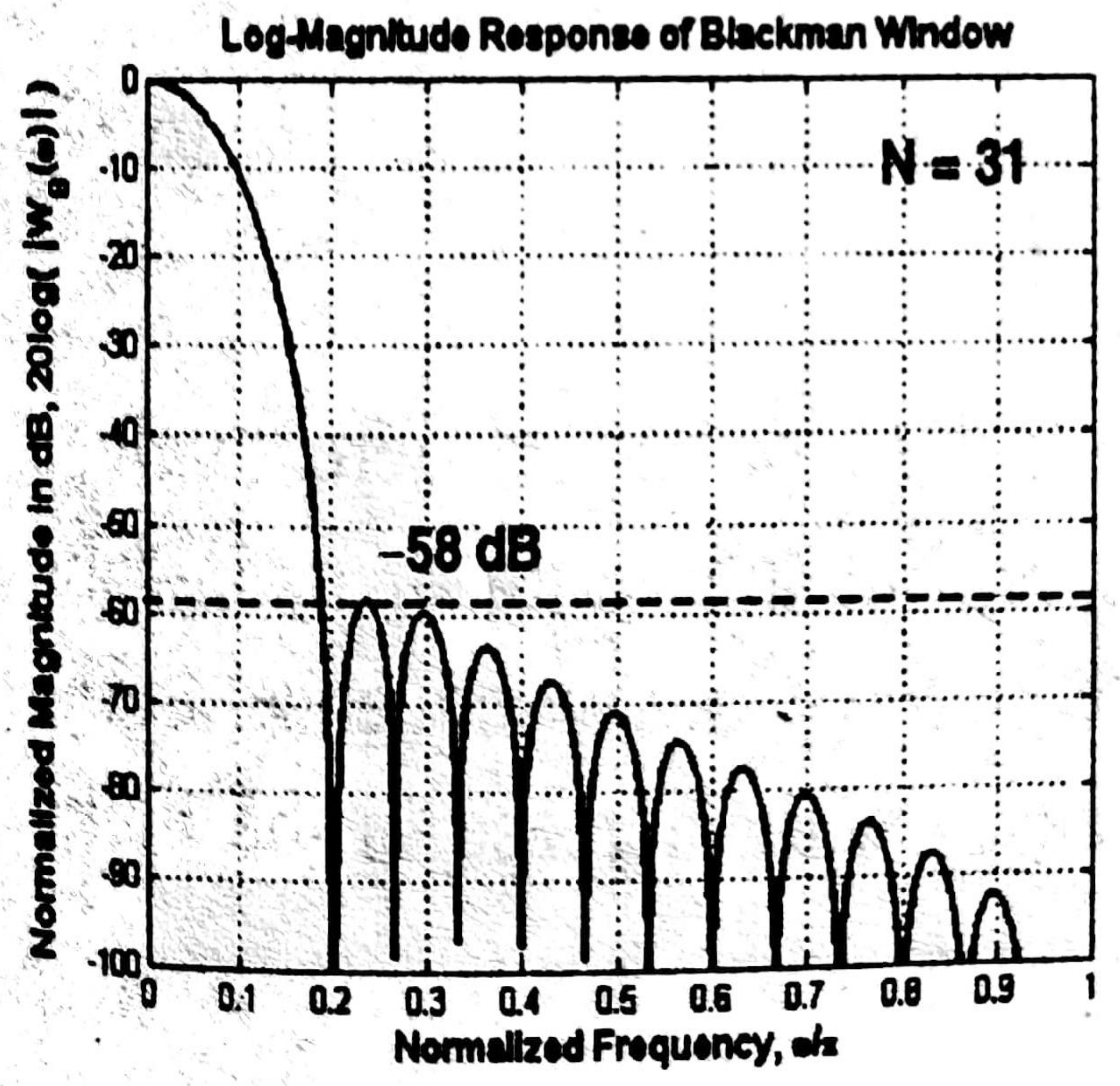

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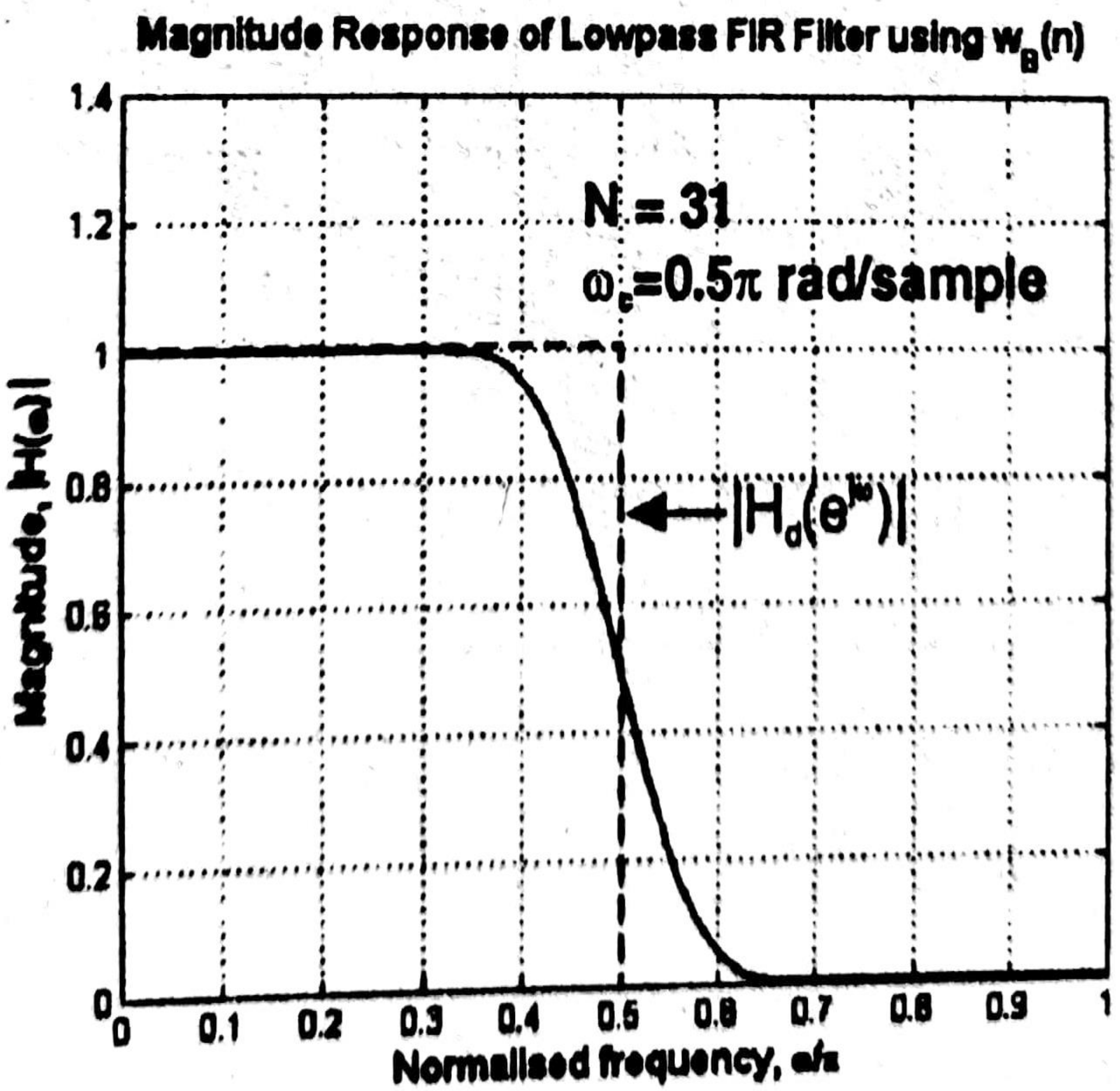


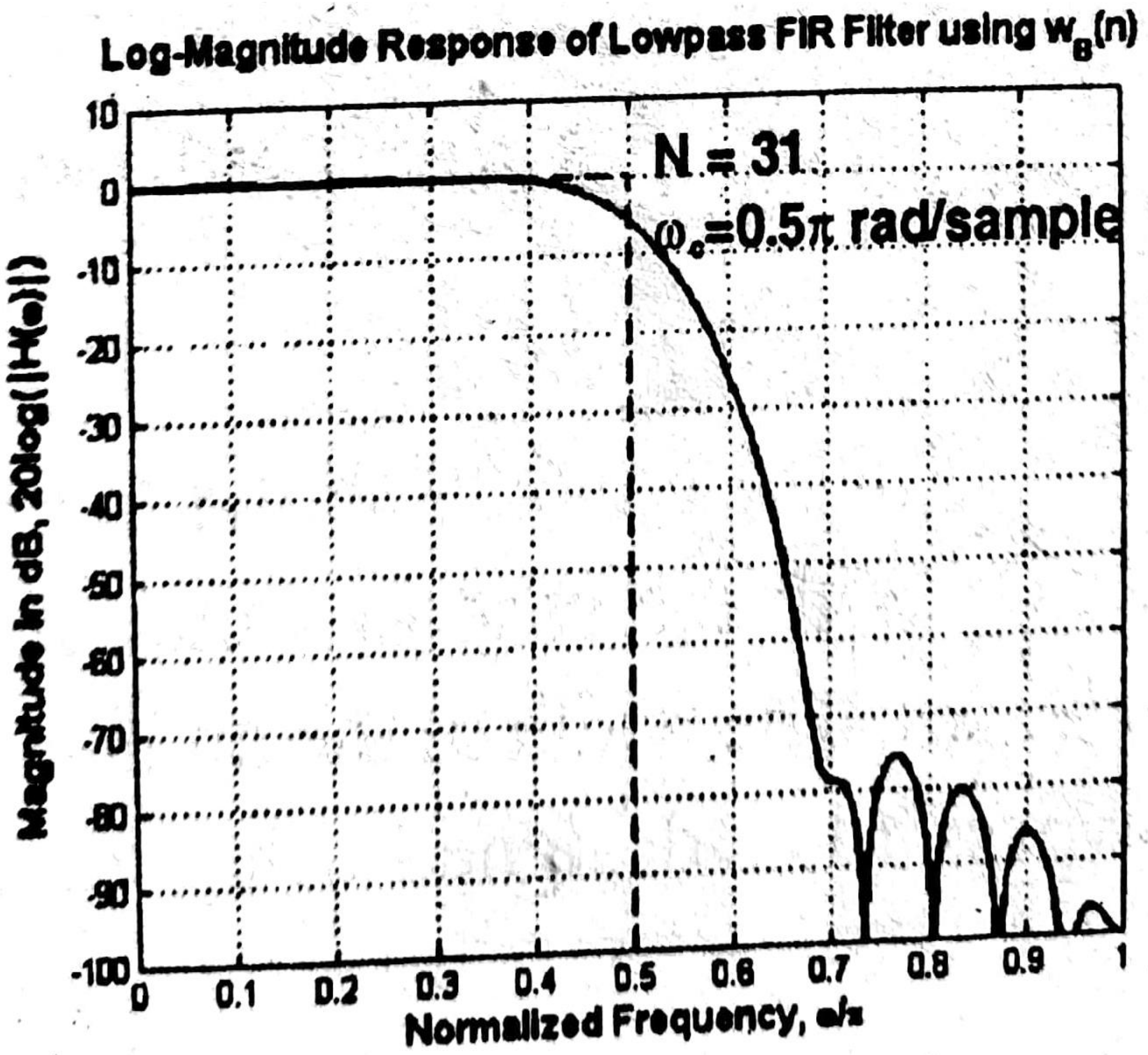

# FREQUENCY DOMAIN CHARACTERISTICS

| S.No. | Type of Window | Approximate width of main-lobe | Magnitude of first side -lobe |
|---|---|---|---|
| 1 | Rectangular | $4\pi/N$ | -13dB |
| 2 | Hanning | $8\pi/N$ | -31dB |
| 3 | Hamming | $8\pi/N$ | -41dB |
| 4 | Blackman | $12\pi/N$ | -58dB |

# IDEAL FREQUENCY RESPONSE FOR FIR FILTER DESIGN USING WINDOWS

**Low Pass**

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \quad -\omega_c \le \omega \le +\omega_c \\ 0 & ; \quad -\pi \le \omega < -\omega_c \\ 0 & ; \quad \omega_c < \omega \le \pi \end{cases}$$

**High Pass**

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \quad -\pi \le \omega \le -\omega_c \\ e^{-j\omega\alpha} & ; \quad \omega_c \le \omega \le \pi \\ 0 & ; \quad -\omega_c < \omega < +\omega_c \end{cases}$$

# IDEAL FREQUENCY RESPONSE FOR FIR FILTER DESIGN USING WINDOWS

**Band Pass**

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \quad -\omega_{c2} \le \omega \le -\omega_{c1} \\ e^{-j\omega\alpha} & ; \quad \omega_{c1} \le \omega \le \omega_{c2} \\ 0 & ; \quad -\pi \le \omega < -\omega_{c2} \\ 0 & ; \quad -\omega_{c1} < \omega < +\omega_{c1} \\ 0 & ; \quad \omega_{c2} < \omega \le \pi \end{cases}$$

**Band stop**

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \quad -\pi \le \omega \le -\omega_{c2} \\ e^{-j\omega\alpha} & ; \quad -\omega_{c1} \le \omega \le +\omega_{c1} \\ e^{-j\omega\alpha} & ; \quad \omega_{c2} \le \omega \le \pi \\ 0 & ; \quad -\omega_{c2} < \omega < -\omega_{c1} \\ 0 & ; \quad \omega_{c1} < \omega < \omega_{c2} \end{cases}$$

# DESIRED IMPULSE RESPONSE FOR FIR FILTER DESIGN USING WINDOWS

**Low Pass**

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\pi \le \omega < -\omega_c \text{ and } +\omega_c < \omega \le +\pi\right]$$

**High Pass**

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\omega\alpha} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\omega\alpha} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\omega_c < \omega < +\omega_c\right]$$

# DESIRED IMPULSE RESPONSE FOR FIR FILTER DESIGN USING WINDOWS

**Band Pass**

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_{c2}}^{-\omega_{c1}} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_{c1}}^{\omega_{c2}} e^{-j\omega\alpha} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\pi \le \omega < -\omega_{c2}\ ;\ -\omega_{c1} < \omega < +\omega_{c1} \text{ and } +\omega_{c2} < \omega \le +\pi\right]$$

**Band Stop**

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_{c2}} e^{j\omega\alpha} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{-\omega_{c1}}^{+\omega_{c1}} e^{-j\omega\alpha} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_{c2}}^{\pi} e^{-j\omega\alpha} e^{j\omega n} d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\omega_{c2} < \omega < -\omega_{c1} \text{ and } +\omega_{c1} < \omega < +\omega_{c2}\right]$$

# FIR FILTER DESIGN USING WINDOWS

## Symmetry Condition h(N-1-n) = h(n)

1. The specifications of digital FIR filter are,

(i) The desired frequency response $H_d(e^{j\omega})$ = **C** $\mathbf{e^{-j\alpha\omega}}$

Where C = Constant and $\boldsymbol{\alpha}$ **= N-1/2**

(i) The cutoff frequency $\omega_c$ for lowpass and high pass $\omega_{c1}$ and $\omega_{c2}$ for bandpass and bandstop filters.

(ii) The number of samples of impulse response N

2. Determine the desired impulse response $h_d(n)$ by taking inverse Fourier transform of the desired frequency response $H_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

3. Choose the desired window sequence w(n) defined for n= 0 to N-1. Multiply $h_d(n)$ with w(n) to get the impulse response h(n) of the filter. Calculate N-samples of the impulse response for n=0 to N-1

**Impulse Response** $h(n) = h_d(n) \times w(n)$ for n=0 to N-1

- The impulse response is symmetric with centre of symmetry at (N-1)/2 and so h(N-1-n) = h(n). It is sufficient if we calculate h(n) for n=0 to (N-1)/2

4. Take Z transform of the impulse response h(n) to get the transfer function H(z) of FIR Filter

$$H(z) = \mathcal{Z}\{h(n)\} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

5. Draw a suitable structure for realization of FIR filter.

# FIR FILTER DESIGN USING WINDOWS

## Symmetry Condition h(-n) = h(n)

1. The specifications of digital FIR filter are,

(i) The desired frequency response $H_d(e^{j\omega})$ = **C**

Where C = Constant ( C=1=Normalized Magnitude)

(i) The cutoff frequency $\omega_c$ for lowpass and high pass $\omega_{c1}$ and $\omega_{c2}$ for bandpass and bandstop filters.

(ii) The number of samples of impulse response N

2. Determine the desired impulse response $h_d(n)$ by taking inverse Fourier transform of the desired frequency response $H_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

# FIR FILTER DESIGN USING WINDOWS

3. Choose the desired window sequence w(n) defined for n= n= -(N-1)/2 to (N-1)/2. Multiply $h_d(n)$ with w(n) to get the impulse response h(n) of the filter. Calculate N-samples of the impulse response for n= n= -(N-1)/2 to (N-1)/2

**Impulse Response**

$h(n) = h_d(n) \times w(n)$ for n= n= -(N-1)/2 to (N-1)/2

- The impulse response is symmetric with centre of symmetry at n=0 and so h(-n) = h(n). It is sufficient if we calculate h(n) for n=0 to (N-1)/2

4. Take Z transform of the impulse response h(n) to get the transfer function H(z) of FIR Filter, $H_N(z)$

$$H_N(z) = \mathcal{Z}\{h(n)\} = \sum_{n=-\frac{N-1}{2}}^{+\frac{N-1}{2}} h(n) z^{-n}$$

# PROCEDURE FOR DIGITAL FIR FILTER BY FOURIER SERIES METHOD

5. Convert the noncausal transfer function, $H_N(z)$ to causal transfer function, H(z) by multiplying $H_N(z)\ Z^{-(N-1)/2}$

$$H(z) = z^{-\frac{N-1}{2}} \sum_{n=-\frac{N-1}{2}}^{+\frac{N-1}{2}} h(n) z^{-n}$$

**Transfer Function**

$$H(z) = z^{-\frac{N-1}{2}} \left[ h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right] \right]$$

6. Draw a suitable structure for realization of FIR filter

# COMPARISON OF RECTANGULAR & HAMMING WINDOW

| S.No. | Rectangular Window | Hamming Window |
|---|---|---|
| 1 | The width of the main-lobe in window spectrum is $4\pi/N$ | The width of the main-lobe in window spectrum is $8\pi/N$ |
| 2 | The maximum side-lobe magnitude in window spectrum is -13dB | The maximum side-lobe magnitude in window spectrum is -41dB |
| 3 | In window spectrum the side-lobe magnitude slightly decreases with increasing ω | In window spectrum the side - lobe magnitude remains constant |
| 4 | In FIR filter designed using rectangular window, the minimum stopband attenuation is 22dB | In FIR filter designed using hamming window, the minimum stopband attenuation is 51dB |

# COMPARISON OF HAMMING & HANNING WINDOW

| S.No. | Hamming Window | Hanning Window |
|---|---|---|
| 1 | The width of the main-lobe in window spectrum is $8\pi/N$ | The width of the main-lobe in window spectrum is $8\pi/N$ |
| 2 | The maximum side-lobe magnitude in window spectrum is -41dB | The maximum side-lobe magnitude in window spectrum is -31dB |
| 3 | In window spectrum the side - lobe magnitude remains constant | In window spectrum the side - lobe magnitude decreases with increasing $\omega$ |
| 4 | In FIR filter designed using hamming window, the minimum stopband attenuation is 51dB | In FIR filter designed using hanning window, the minimum stopband attenuation is 44dB |

# COMPARISON OF HAMMING & BLACKMAN WINDOW

| S.No. | Hamming Window | Blackman Window |
|---|---|---|
| 1 | The width of the main-lobe in window spectrum is 8**π**/N | The width of the main-lobe in window spectrum is 12**π**/N |
| 2 | The maximum side-lobe magnitude in window spectrum is -41dB | The maximum side-lobe magnitude in window spectrum is -58dB |
| 3 | In window spectrum the side - lobe magnitude remains constant with increasing ω | In window spectrum the side - lobe magnitude decreases rapidly with increasing ω |
| 4 | In FIR filter designed using hamming window, the minimum stopband attenuation is 51dB | In FIR filter designed using blackman window, the minimum stopband attenuation is 78dB |

# ASSESSMENT

1. The FIR filter design starts with desired frequency response $H_d(e^{j\omega})$. The desired impulse response $h_d(n)$ is obtained by taking -----------------
2. List the types of windowing techniques.
3. How to calculate desired impulse response $h_d(n)$
4. Compare rectangular window and hamming window.
5. Define Blackman window.
6. Summarize the features of hamming window spectrum.
7. The transfer function H(z) of FIR filter is defined as ------------

# THANK YOU