



SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 3 – FIR FILTER DESIGN

TOPIC – Linear Phase FIR Filter



FIR FILTERS



- **Finite Impulse Response (FIR) Systems:** Unit sample response (or) Impulse response $h(n)$ has finite no. of terms
- **Finite Impulse Response (FIR) Filters:** The filters designed by considering all the finite samples of impulse response
- The specification of a digital filter will be desired frequency response $H_d(e^{j\omega})$. The desired impulse response $h_d(n)$ of the digital filter can be obtained by taking inverse Fourier transform $H_d(e^{j\omega})$. The $h_d(n)$ will be an infinite duration discrete time signal defined for all values of n in the range $-\infty$ to $+\infty$



FIR FILTERS



- The transfer function $H(z)$ of the digital filter is obtained by taking Z transform of impulse response. Since $h_d(n)$ is an infinite duration signal, the transfer function obtained from $h_d(n)$ will have infinite terms, which cannot be realized or implemented in a digital system
- Therefore. Finite number of samples $h_d(n)$ are selected to form the impulse response, $h(n)$ of the filter.
- The transfer function $H(z)$ is obtained by taking Z transform of finite sample impulse response $h(n)$. The filters designed by using finite samples of impulse response are called Finite Impulse Response Filters.



ADVANTAGES & DISADVANTAGES OF FIR FILTERS



- **Advantages:** FIR filters with exactly linear phase can be easily designed
- Efficient realizations of FIR filter exist as both recursive and nonrecursive structures
- FIR filters realized nonrecursively, i.e., by direct convolution are always stable
- Roundoff noise, which is inherent in realizations with finite precision arithmetic can easily be made small for nonrecursive realization of FIR filters
- **Disadvantages:** The duration of the impulse response should be large to adequately approximate sharp cutoff filter. Hence a large amount of processing is required to realize such filters when realized via slow convolution
- The delay of linear phase FIR filters need not always be an integer no. of samples.
This non-integral delay can lead to problems in signal processing applications



STEPS IN DESIGNING FIR FILTER



- Choose an ideal (desired) frequency response, $H_d(e^{j\omega})$
- Take inverse Fourier transform of $H_d(e^{j\omega})$ to get $h_d(n)$ or sample $H_d(e^{j\omega})$ at finite number of points (N – Point) to get $H(k)$
- If $h_d(n)$ is determined then convert the infinite duration $h_d(n)$ to a finite duration $h(n)$ or if $H(k)$ is determined then take N-Point inverse DFT to get $h(n)$.
- Take Z transform of $h(n)$ to get $H(z)$, Where $H(z)$ -transfer function of the digital filter
- Choose a suitable structure and realize the filter
- Verify the design, In order to verify the design, determine the actual frequency response $H(e^{j\omega})$ of the filter, by letting $z = e^{j\omega}$ in $H(z)$ and sketch the magnitude response $|H(e^{j\omega})|$



LTI SYSTEM AS FREQUENCY SELECTIVE FILTERS



- The frequency response $H(e^{j\omega})$ is a complex quantity,

$$H(e^{j\omega}) = |H(e^{j\omega})| \angle H(e^{j\omega}) = C e^{-j\alpha\omega}$$

$$\text{where, } |H(e^{j\omega})| = C$$

Magnitude

$$\angle H(e^{j\omega}) = -\alpha\omega$$

Phase

- Magnitude of frequency response is constant and its phase is a linear function of frequency. If the phase function of frequency response of a filter is linear function of frequency, then the filter is called Linear phase filter



LTI SYSTEM AS FREQUENCY SELECTIVE FILTERS



- In order to examine the linear and nonlinear phase characteristics, two delay functions are defined and they are **Phase delay** and **Group delay**

$$\text{Let, } \angle H(e^{j\omega}) = \theta(\omega)$$

$$\theta(\omega) = -\alpha\omega$$

$$\text{Phase delay, } \tau_p = -\frac{\theta(\omega)}{\omega}$$

$$\therefore \tau_p = -\frac{\theta(\omega)}{\omega} = -\frac{-\alpha\omega}{\omega} = \alpha$$

$$\text{Group delay, } \tau_g = -\frac{d}{d\omega}\theta(\omega)$$

$$\tau_g = -\frac{d}{d\omega}\theta(\omega) = -\frac{d}{d\omega}(-\alpha\omega) = \alpha$$



IDEAL FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS



- The filters are classified according to their frequency response characteristics. The ideal (desired) frequency response $H_d(e^{j\omega})$ of four major types of filters. They are Low pass, High pass, Band pass and Band stop filters
- The $H_d(e^{j\omega})$ is periodic, with periodicity of **0 to 2π** (or **$-\pi$ to π**). Also any analog frequency Ω will map (or can be converted) to frequency of digital system ω within the range **0 to 2π** (or **$-\pi$ to π**)
- Hence the frequency response of digital filters are defined in the interval **0 to 2π** (or **$-\pi$ to π**)



IDEAL FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS



**Ideal Frequency Response
of Low pass Filter $H_d(e^{j\omega})$**

$$\begin{aligned} H_d(e^{j\omega}) &= 0 && ; \text{ for } \omega = -\pi \text{ to } -\omega_c \\ &= C e^{-j\alpha\omega} && ; \text{ for } \omega = -\omega_c \text{ to } +\omega_c \\ &= 0 && ; \text{ for } \omega = +\omega_c \text{ to } +\pi \end{aligned}$$

**Ideal Frequency Response
of High pass Filter $H_d(e^{j\omega})$**

$$\begin{aligned} H_d(e^{j\omega}) &= C e^{-j\alpha\omega} && ; \text{ for } \omega = -\pi \text{ to } -\omega_c \\ &= 0 && ; \text{ for } \omega = -\omega_c \text{ to } +\omega_c \\ &= C e^{-j\alpha\omega} && ; \text{ for } \omega = +\omega_c \text{ to } +\pi \end{aligned}$$



IDEAL FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS



**Ideal Frequency Response
of Band pass Filter $H_d(e^{j\omega})$**

$$\begin{aligned} H_d(e^{j\omega}) &= 0 && ; \text{ for } \omega = -\pi \text{ to } -\omega_{c2} \\ &= C e^{-j\alpha\omega} && ; \text{ for } \omega = -\omega_{c2} \text{ to } -\omega_{c1} \\ &= 0 && ; \text{ for } \omega = -\omega_{c1} \text{ to } +\omega_{c1} \\ &= C e^{-j\alpha\omega} && ; \text{ for } \omega = +\omega_{c1} \text{ to } +\omega_{c2} \\ &= 0 && ; \text{ for } \omega = +\omega_{c2} \text{ to } +\pi \end{aligned}$$

**Ideal Frequency Response
of Band stop Filter $H_d(e^{j\omega})$**

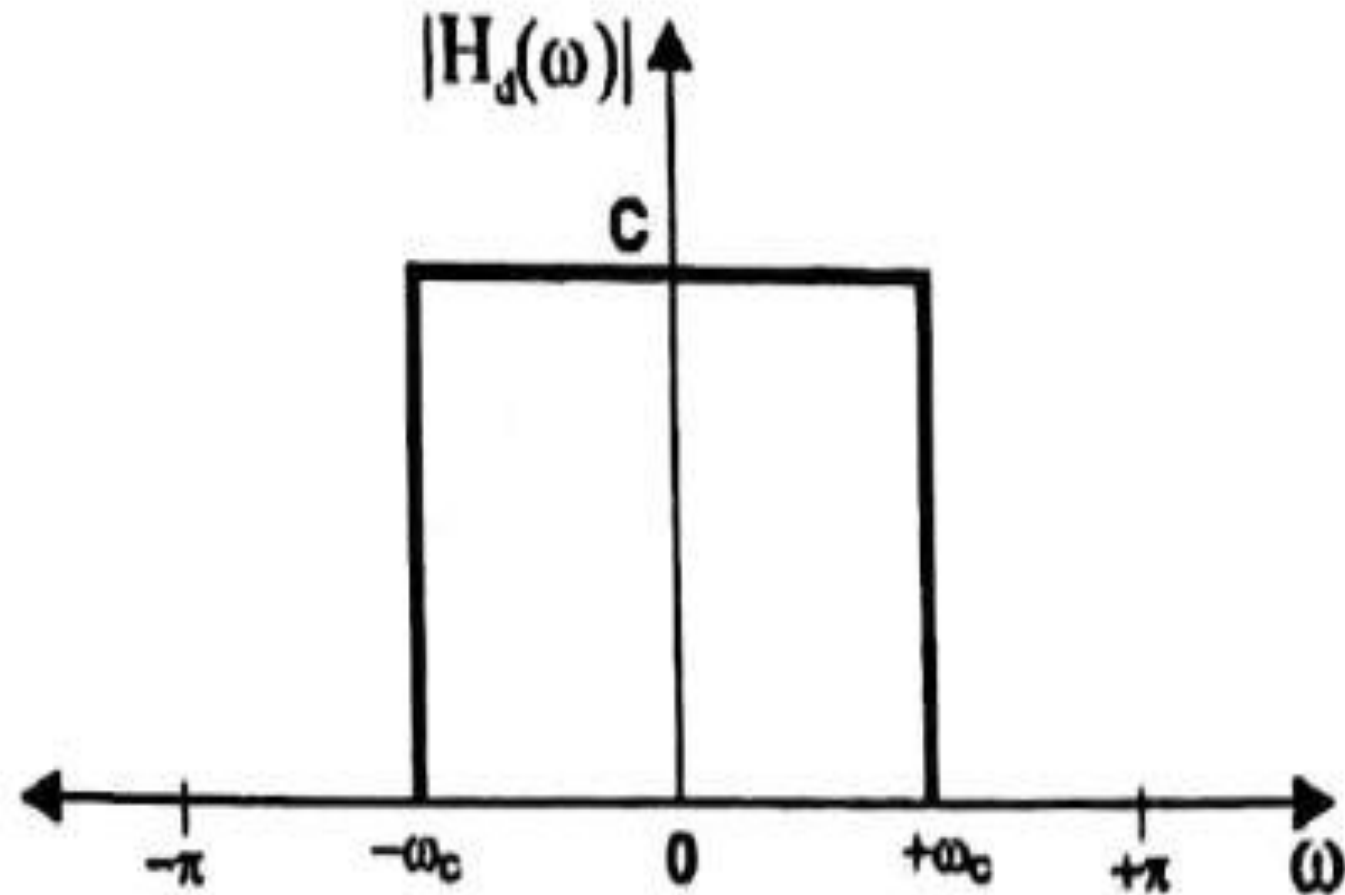
$$\begin{aligned} H_d(e^{j\omega}) &= C e^{-j\alpha\omega} && ; \text{ for } \omega = -\pi \text{ to } -\omega_{c2} \\ &= 0 && ; \text{ for } \omega = -\omega_{c2} \text{ to } -\omega_{c1} \\ &= C e^{-j\alpha\omega} && ; \text{ for } \omega = -\omega_{c1} \text{ to } +\omega_{c1} \\ &= 0 && ; \text{ for } \omega = +\omega_{c1} \text{ to } +\omega_{c2} \\ &= C e^{-j\alpha\omega} && ; \text{ for } \omega = +\omega_{c2} \text{ to } +\pi \end{aligned}$$



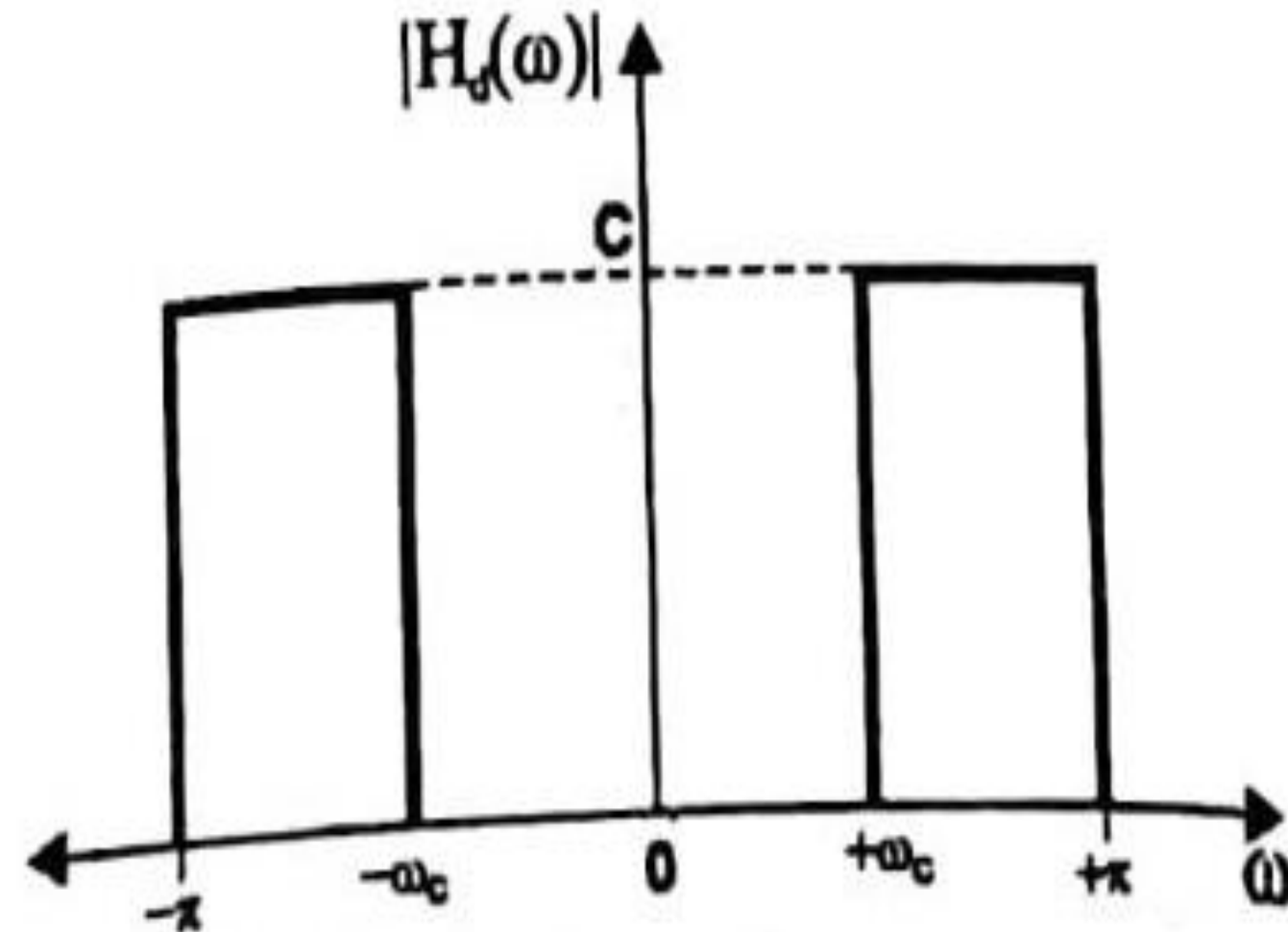
MAGNITUDE RESPONSE



IDEAL LOWPASS FILTER



IDEAL HIGHPASS FILTER



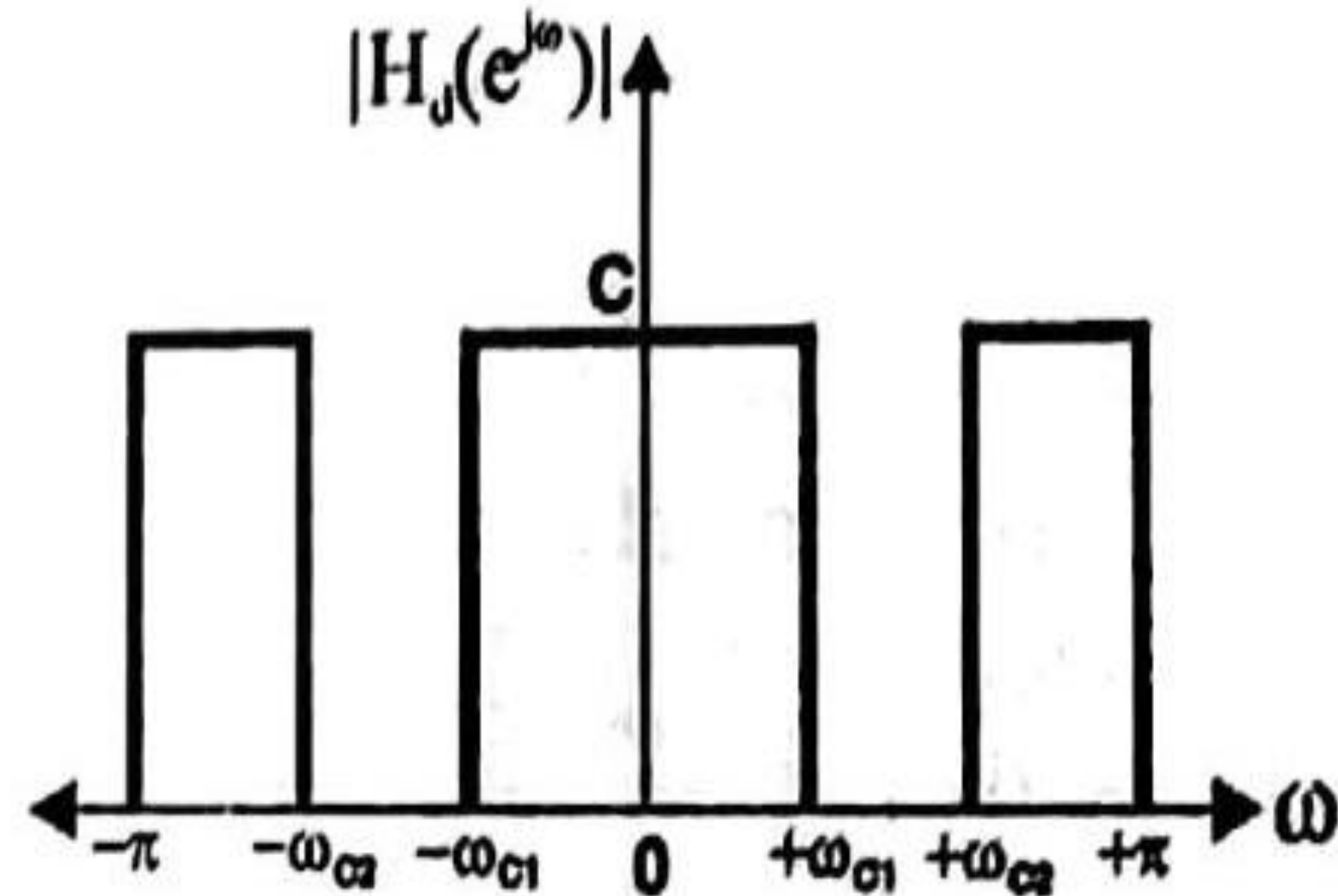
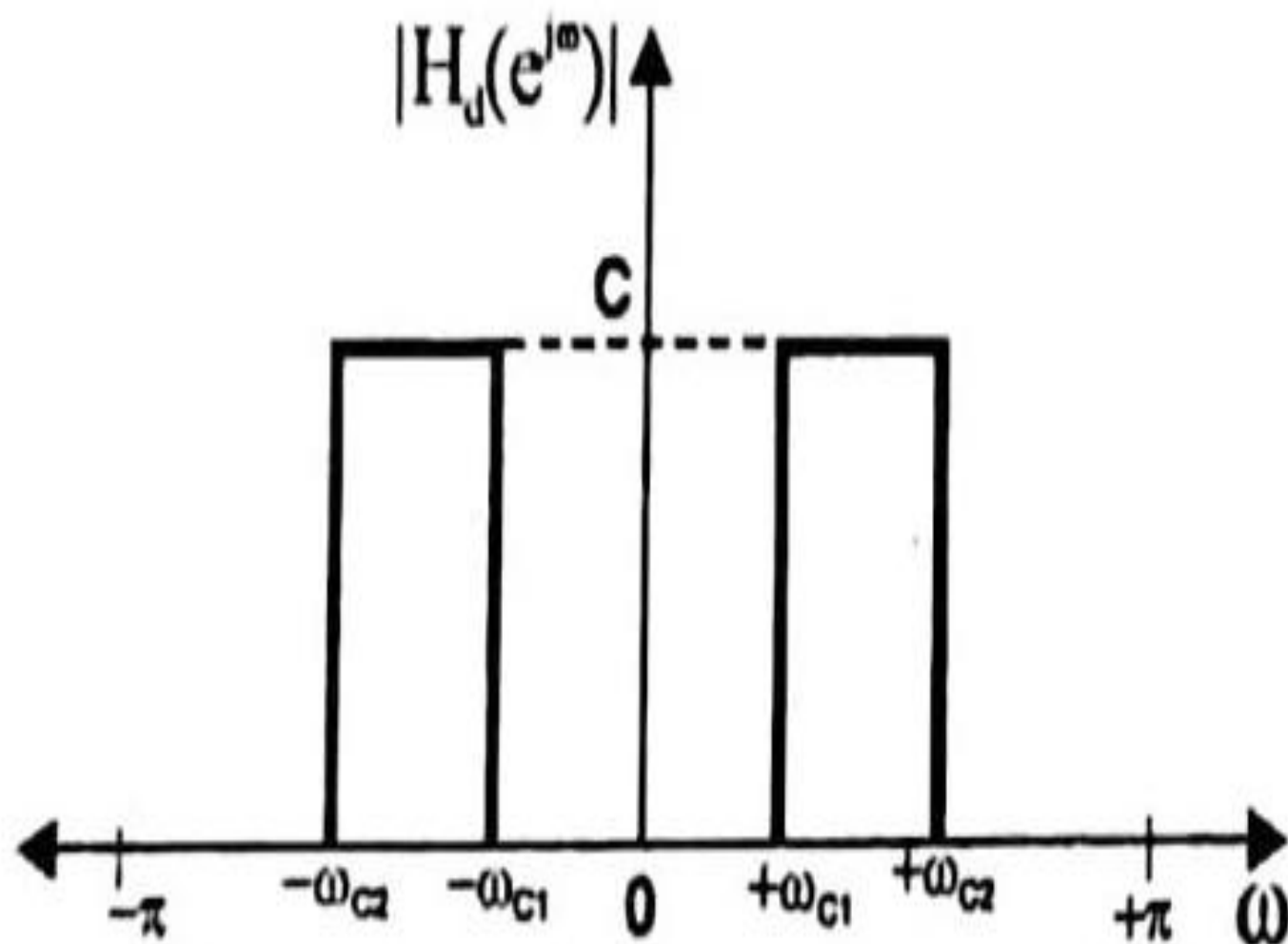


MAGNITUDE RESPONSE



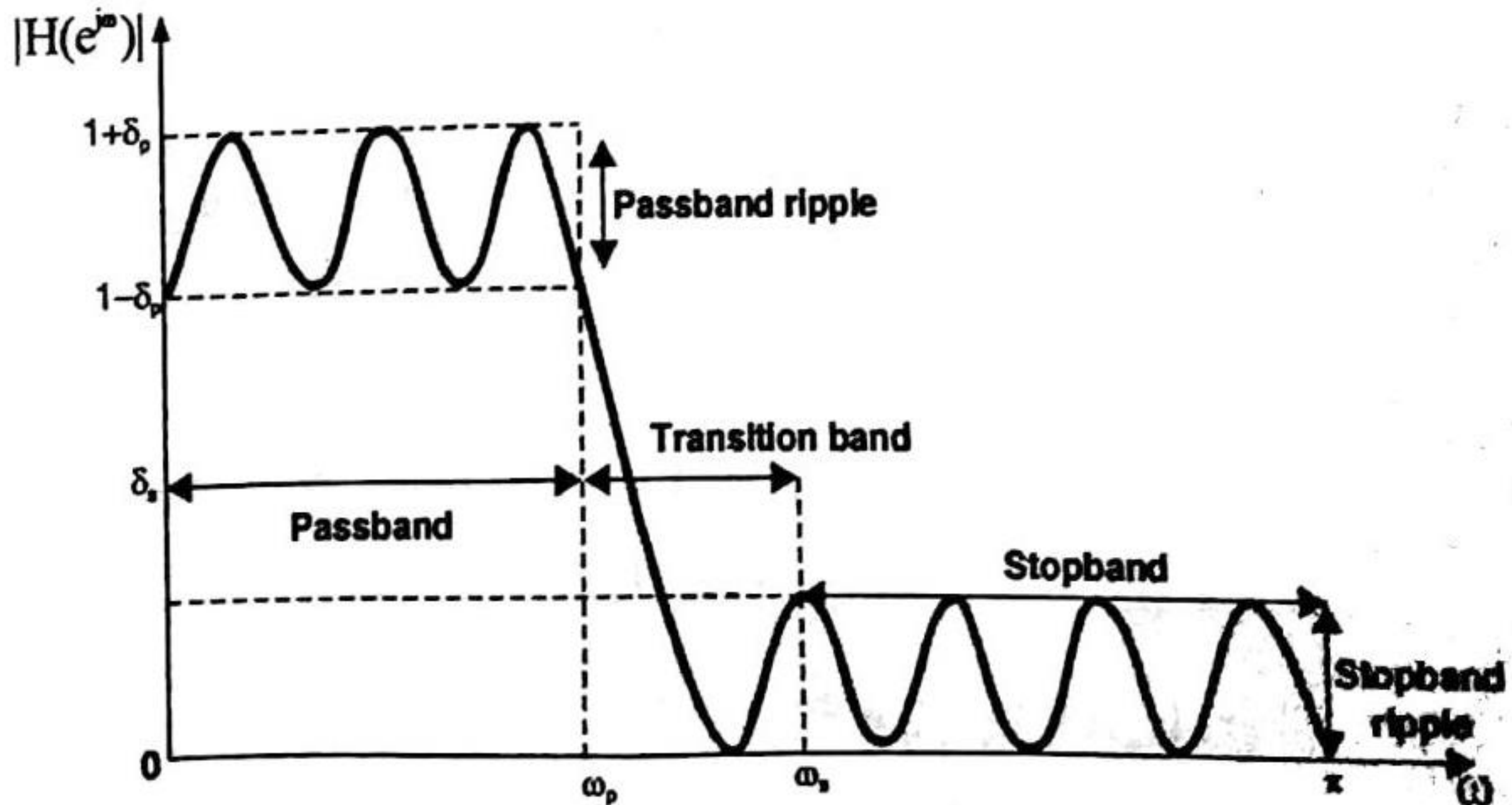
IDEAL BANDPASS
FILTER

IDEAL BANDSTOP
FILTER





MAGNITUDE RESPONSE OF A PRACTICAL LOWPASS FILTER





MAGNITUDE RESPONSE OF A PRACTICAL LOWPASS FILTER



- The transition of the frequency response from pass band to stop band defines the transition band or transition region of the filter
- The pass band edge frequency ω_p defines the edge of the pass band, while the stop band edge frequency ω_s denotes the beginning of the stop band
- δ_p - Pass band ripple
- δ_s - Stop band ripple
- ω_p - Pass band edge frequency
- ω_s - Stop band edge frequency



CHARACTERISTICS OF FIR FILTERS WITH LINEAR PHASE



- Let $h(n)$ be the causal finite duration sequence defined over the interval $0 \leq n \leq N-1$ and the samples of $h(n)$ be real
- The Fourier transform of $h(n)$ is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

Which is periodic in frequency with period 2π

$$\therefore H(e^{j\omega}) = H(e^{j\omega + 2\pi m}); \text{ for } m = 0, \pm 1, \pm 2, \dots$$



CHARACTERISTICS OF FIR FILTERS WITH LINEAR PHASE



- Since $H(e^{j\omega})$ is complex it can be expressed as **Amplitude function, Magnitude function and Phase function**

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| e^{j\angle H(e^{j\omega})} = A(\omega) e^{j\theta(\omega)}$$

$$\text{where, } A(\omega) = \pm |H(e^{j\omega})| = \text{Amplitude function}$$

$$\theta(\omega) = \angle H(e^{j\omega}) = \text{Phase function}$$

$$|H(e^{j\omega})| = \text{Magnitude function}$$

- When $h(n)$ is real, the magnitude function is a symmetric function and the phase function is an asymmetric function

$$\therefore |H(e^{j\omega})| = |H(-e^{j\omega})|$$

$$|\theta(\omega)| = -|\theta(-\omega)|$$



ASSESSMENT



1. Define FIR Systems.
2. Mention the advantages and disadvantages of FIR Filters.
3. Based on frequency response the filters are classified into four basic types. They are -----, -----, ----- and -----
4. What are the steps involved in designing FIR Filter?
5. In order to examine the linear and nonlinear phase characteristics, two delay functions are ----- and -----
6. The Fourier transform of $h(n)$ is -----



THANK YOU