SOLUTIONS OF EQUATIONS AND EIGEN VALUE PROBLEMS

PART-A

1. The condition for convergence of Newton's Raphson method is (a)

a) $\left|f(x).f''(x)\right| < \left|f'(x)\right|^2$ b) $\left|f(x).f''(x)\right| < \left|f'''(x)\right|^2$

c) $\left|f'(x).f''(x)\right| < \left|f'(x)\right|^2$ d) None of the above

2. The order of convergence for Newton's Raphson method is

a)3 **b) 2** c)5 d)6 (b)

3. Iteration formula for Newton's Raphson method is

a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ **b)** $x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$

c) $x_{n+1} = x_n - \frac{f''(x_n)}{f'(x_n)}$ **d)None of the above** (a)

4. Newton –Raphson method is also getting of the equations

a)True b)False c) Both d) None of the above (b)

5. In Gaussian elimination method ,the co-efficient matrix is transformed to ___________ form

a)Upper Triangular b)Lower Triangular (a)

c) Both d)) None of the above

6. Gauss elimination method is

a)Direct b) Indirect c) Iterative d)Both(a) and (b) (a)

7. Gauss Jacobi and Gauss Seidal method is (c)

a)Direct b) Indirect c) Iterative d)Both a) and b)

8. Gauss Seidal method is better than Gauss Jacobi method

a)True b) False c) Both (a) and (b) d) None of the above (a)

9. Gauss Seidal method Converges _________a)Quickly b) Slowly c) not Converge (a)
d) None of the above

10 The rate of convergence of Gauss seidal method is roughly _______ times that of Gauss Jacobi method (a)

a)Two b) Three c) Four d) Five

11. The co-efficient matrix of Gauss Seidal method must be

a)Diagonally dominant b) Upper triangular c) Lower triangular d)None of the above (a)

12. Newton Raphson method is also known as (c)

a)Secant b) Cosecant c)Tangent d) None of the above

13. Gauss Jordan method involves more computation than in

a) Gaussian method b) Iteration method c) Triangular method d) Back substitution method (a)

14. Newton Raphson method is Convergent

a)Linearly b) Quadratic ally c) Cubically d)Biquadratically (b)

15. Newton's Method is useful in cases where the graph of the function when it crosses the x axis is nearly **horizontal** (b)

a) True b) False c) None of the above

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16. Newton's Method is useful in cases where the graph of the function when it crosses the x axis is nearly **vertical** (a)

a)True b) False c) None of the above

17. The iterative formula for finding the reciprocal of N is

a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ **b)** $x_{n+1} = x_n(2 - Nx_n)$ **c)** $x_{n+1} = x_n - \frac{f''(x_n)}{f'(x_n)}$

d) $x_{n+1} = x_n(3 - Nx_n)$

(b)

18. In Newton Raphson method the error at any stage is proportional to the _______ of the error in the previous stage (a)

a) Square b) Cube c)multiple d) None of the above

19. In general the Iteration method is a self correcting method, since the round of error is

a)Smaller b) Larger c) bigger d) None of the above (a)

20. As soon as a new value for a variable is found by iteration, it is used immediately in the following equation. This method is called (a)

a)Gauss- Seidal b)Jacobi's c)Gauss- Jordan d)Relaxation

21. The power method will work satisfactorily only if A has a ___________ eigen value

a) Dominant b) No dominant c) Upper d) None of the above (a)

22. It the eigen values of A are -4,3,1 the the dominant eigen value of A is

a) 4 b)1 c)-4 d) 3 (c)

PART-B

1. Write the iterative formula of Newton-Raphson method.

Solution:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

2. Newton Raphson method is also known as

Solution:

Method of Iteration (or)

Newton's Iteration Method.

3. Derive Newton's algorithm for finding the p^{th} root of a number N.

Solution:

If $x = N^{\frac{1}{p}}$

Then $X^p = N$ $\Rightarrow X^p - N = 0$ **is the equation.**

$$\therefore f(x) = x^p - N$$
$$f'(x) = px^{p-1}$$

By Newton's Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^p - N}{p x_n^{p-1}}$$

$$= \frac{p x_n^p - x_n^p + N}{p x_n^{p-1}}$$

$$= \frac{(p-1)x_n^p + N}{p x_n^{p-1}}$$

4. Show that the Newton's Raphson formula to find $\sqrt{a}$ can be expressed in the form

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{a}{x_n}\right], n = 0,1,2....$$

Solution:

If $x=\sqrt{a}$

Then $x^2 - a = 0$ **is the equation.**

$$\therefore f(x) = x^2 - a$$

$$f'(x) = 2x$$

By Newton's Raphson rule for n^{th} **iterate,**

$$
\begin{aligned}
x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_{n})} \\
&= x_n - \frac{x_n^2 - a}{2x_n} \\
&= \frac{2x_n^2 - x_n^2 + a}{2x_n} \\
&= \frac{x_n^2 + a}{2x_n} \\
&= \frac{1}{2}\left[x_n + \frac{a}{x_n}\right], n = 0,1,2...
\end{aligned}
$$

5. What are the merits of Newton's method of iteration?

<u>Solution:</u>

Newton's method is successfully used to improve the result odtained by other methods. It is applicable to the solution of equations involving algebrical functions as well as transcendental functions.

6. State the order of convergence and convergence condition for Newton's Raphson method.

<u>Solution:</u>

Order of convergence is 2.[ie quadratic]

Condition for convergence is $|f(x)f''(x)| < |f'(x)|^2$

7. What is the condition for applying the fixed point iteration to find the real root?

Solution:

Let x=r be a root of x=g(x). Let I be an interval combining the point x=r.

The condition for fixed point iteration method is $|g'(x)|<1$ for all x in I, the sequence of approximation $x_0, x_1, \ldots, x_n$ will converge to the root r.

8. Check whether the fixed point method is applicable to the equation $x^3 - 2x + 5 = 0$

Solution:

$$f(x) = x^3 - 2x + 5 = 0$$

$$\begin{aligned} & f(0) = -5 = -ve \\ \text{Given } & f(1) = -6 = -ve \\ & f(2) = -1 = -ve \\ & f(3) = 16 = +ve \end{aligned}$$

The root lies between 2 & 3.

The given equation f(x) =0 is written as x=g(x)

$$x^3 = 2x + 5$$

$$x = (2x+5)^{\frac{1}{3}} = g(x)$$

$$\Rightarrow g'(x) = \frac{1}{3}(2x+5)^{-\frac{2}{3}}.2$$

$$= \frac{2}{3}\frac{1}{(2x+5)^{\frac{2}{3}}}$$

$$|g'(x)| = \left|\frac{2}{3}\frac{1}{(2x+5)^{\frac{2}{3}}}\right| < 1$$

We can apply fixed point iteration method..

9. What is the order of convergence for fixed point iteration?

Solution:

The convergence is linear and the order of convergence is 2.

10.Write the two method to solve simultaneous linear algebraic equations:

Solution:

By Direct Method:

1)Gauss Elimination Method and

2) Gauss Jordan Method

By Indirect (or) Iterative Method:

1)Gauss Jacobi Method and

2) Gauss Seidal Method

11.compare Gauss elimination & Gauss Jordan method.

Gauss Elimination Method	Gauss Jordan Method
1)Coefficient matrix is transformed into upper triangular matrix.	Coefficient matrix is transformed into upper diagonal matrix
2)Direct method,need the back substitution method to obtain the soltution	Indirect method,no need of back substitution method

12.Write a sufficient condition for Gauss-seidal & Jacobi method to converge.

Solution:

Let the linear equation be,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b3y + c_3z = d_3$$

Then sufficient condition is

$$|a_1| > |b_1| + |c_1|$$
$$|b_2| > |a_2| + |c_2|$$
$$|c_3| > |a_3| + |b_3|$$

ie The coefficient of matrix should be diagonally dominant.

13.State the iterative formula for Gauss Jacobi method.

Solution:

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$$

with the initial condition x=y=z=0.

14. State the iterative formula for Gauss Seidal method.

Solution:

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$
$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$
$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

with the initial condition y=z=0.

15.When will iteration method succeed?

Solution:

Iteration method may succeed,the equation of system must contain one large coefficient and it should be along the leading diagonal of the matrix of the coefficient.

16.Whether the given system of equation is solvable using iterative method.

Solution:

$$x + 3y + 52z = 173.61$$
$$41x - 2y + 3z = 65.46$$
$$x - 27y + 2z = 71.31$$

As the sufficient condition is not satisfied by the system of equations[ie the coefficient matrix is not diagonally dominant],we write the equation as,

$$41x - 2y + 3z = 65.46$$
$$x - 27y + 2z = 71.31$$
$$x + 3y + 52z = 173.61$$

Now the diagonal elements are dominant in the coefficient matrix is solvable using iterative method.

17.Compare Gauss Jacobi and Gauss seidal methods.

Solution:

Gauss Jacobi method	Gauss Seidal method
1)Convergence rate is slow	The rate of convergence of Gauss Seidal method is roughly twice that of Gauss Jacobi.
2)Iterative method	Iterative method
3)Condition for convergence is the coefficient matrix is diagonally dominant	Condition for convergence is the coefficient matrix is diagonally dominant

18. Why Gauss-Seidal method is a better method than Jacobi's iterative method.

Solution:

Since the current value of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration, the convergence in Gauss Seidal method will be more rapid than in Gauss Jacobi method.

19. State the merits and demerits of Elimination and Iterative methods for solving a system of equations.

Solution:

Elimination method involves a certain amount of fixed computation and they are exact solutions.

Iterative method is those in which the solution is got by successive approximations and they are approximate solutions.

20. Find the inverse of the coefficient matrix by Gauss Jordan elimination method $5x-2y=10; 3x+4y=12.$

Solution:

The coefficient matrix is

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$$

$$[A, I] = \begin{bmatrix} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2/5 & 1/5 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix} \qquad R_1 \to R_1/5$$

$$= \begin{bmatrix} 1 & -2/5 & 1/5 & 0 \\ 0 & 26/5 & -3/5 & 1 \end{bmatrix} \qquad R_2 \to R_2 - 3R_1$$

$$= \begin{bmatrix} 1 & -2/5 & 1/5 & 0 \\ 0 & 1 & -3/26 & 5/26 \end{bmatrix} \qquad R_2 \to R_2 * 5/26$$

$$= \begin{bmatrix} 1 & 0 & 2/13 & 1/13 \\ 0 & 1 & -3/26 & 5/26 \end{bmatrix} \qquad R_1 \to R_1 + (2/5)R_2$$

$$\therefore A^{-1} = \begin{bmatrix} 2/13 & 1/13 \\ -3/26 & 5/26 \end{bmatrix}$$

$$= \frac{1}{26}\begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$$

21. Find the power method, the largest Eigen value of $\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ correct to 2 decimal places, choose $[1,1]^T$ as the initial eigen vector.

Solution:

Let

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 0.8 \end{bmatrix} = 5X_2$$

$$AX_2 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 4.8 \\ 3.4 \end{bmatrix} = 4.8\begin{bmatrix} 1 \\ 0.71 \end{bmatrix} = 4.8X_3$$

$$AX_3 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.71 \end{bmatrix} = \begin{bmatrix} 4.71 \\ 3.13 \end{bmatrix} = 4.71\begin{bmatrix} 1 \\ 0.67 \end{bmatrix} = 4.71X_4$$

$$AX_4 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 4.67 \\ 3.01 \end{bmatrix} = 4.67\begin{bmatrix} 1 \\ 0.65 \end{bmatrix} = 4.67X_5$$

$$AX_5 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 4.65 \\ 2.95 \end{bmatrix} = 4.65 \begin{bmatrix} 1 \\ 0.63 \end{bmatrix} = 4.65X_6$$

$$AX_6 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.63 \end{bmatrix} = \begin{bmatrix} 4.63 \\ 2.89 \end{bmatrix} = 4.63 \begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = 4.63X_7$$

$$AX_7 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = \begin{bmatrix} 4.62 \\ 2.86 \end{bmatrix} = 4.62 \begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = 4.62X_8$$

Eigen value=4.62 &corresponding Eigen vector=$\begin{bmatrix} 1 \\ 0.62 \end{bmatrix}$.

22.Determine the largeset Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ correct to two decimal places using power method.

Solution:

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2X_2$$

$$AX_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2X_3$$

Largest Eigen value=2 and Corresponding Eigen vector=$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

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PART-C

Using Newton's iterative method find the root between 0 and 1 of $x^3 = 6x - 4$ **correct to two decimal places.**

2. Find the real positive root of **3x –cosx – 1 = 0** by Newton's method correct to6 decimal places
3. Find a root of $x\log_{10} x - 1.2 = 0$ by Newton's method correct to 3 decimal places
4. Find a root of $x\log_{10} x - 12.34 = 0$ start with $x_0 = 10$ by Newton's method correct to 3 decimal places
5. Obtain Newton's Iterative formula for finding $\sqrt{N}$ **where N is a** positive real number. Hence evaluate $\sqrt{142}$
6. Find the iterative formula for finding the value of $\frac{1}{N}$ where N is a real number, using Newton – Raphson method . Hence evaluate $\frac{1}{26}$ correct to 4 decimal places.
7. Solve the system of equations by (i) Gauss elimination method (ii) Gauss- Jordan method
   10x + y + z = 12
   2x + 10y + z = 13
   x + y + z = 7
8. Solve the system of equations by (i) Gauss- Jacobi method (ii) Gauss- Seidal method
   27x +6 y - z = 85
   x + y + 54z = 110
   6x + 15 y +2 z = 72
9. Using Gauss- Jordan method, Find the Inverse of the matrix $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

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10. Determine the Largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{5} \end{bmatrix}$