PRP

UNIT I

1. A random variable $\boldsymbol{X}$ has the following probability distribution.

X	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find

(1) The value of $\boldsymbol{k}$

(2) Evaluate P(X<6),P(0<X<5)

(3) The smallest value of a for which $P(X \le a) > \frac{1}{2}$.

(4) The Cumulative distribution function.

2. A random variable X has the following probability function

X	0	1	2	3	4	5	6	7	8
P(x)	A	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) Determine the value of 'a'

(ii) Find P(X<3), P(X≥ 3), P(0<X<5)

(iii) Find the distribution function of X.

3. A random variable X has the following probability distribution

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

Find (1) The value of K (2) Evaluate P(X< 2) and P(-2<X<2) (3) Find the Cumulative distribution of X (4) Find the mean of X.

4. If the Random variable X takes the value 1,2,3,4 such that 2P(X=1) =3P(X=2)=P(X=3)=5P(X=4). Find the probability distribution.

5. A continuous R.V X has the p.d.f $f(x) = 3x^2,\ 0 \le x \le 1$. Find the value of a, such that $P(X \le a) = P(X > a)$. Find the value b such that P(X > b) = 0.05.

6. A continuous R.V. $\boldsymbol{X}$ has the p.d.f. $f(x) = \begin{cases} \frac{k}{1+x^2} & -\infty < x < \infty \\ 0 & otherwise \end{cases}$

. Find

(1) The value of $\boldsymbol{k}$

(2) Distribution function of $\boldsymbol{X}$

(3) $\boldsymbol{P(X \geq 0)}$

7. The probability function of an infinite discrete distribution is given by $P(X = j) = \frac{1}{2^x}$ $(x = 1,2,3, \ldots)$

(1) Mean and variance of $\boldsymbol{X}$

(2) $M.G.F$

(3) $P(X$ is even)

8. A random variable has the pdf $(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & otherwise \end{cases}$. Obtain the mgf and first four moments about the origin. Also find the mean and variance.

9. Find the M.G.F of the random variable with the probability law $P(X = x) = q^{x-1}p,\ x = 1,2,3, \ldots.$ Find the mean and variance.

10. A continuous Random variable X has the distribution function $F(x) = \begin{cases} 0 & x \leq 1 \\ k(x-1)^4 & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$.

(1) Find K (2) p.d.f f(x) (3) P(X< 2).

11. The diameter of an electric cable say X, is assumed to be a continuous Random variable with P.d.f

$$f(x) = 6x(1-x) \ ,0 \le x \le 1$$

(i) Check that the above is a P.d.f

(ii) Determine a and b such that $P(X < b) = P(X > b)$

(iii) Find the distribution function of X

(iv) Find $P(X \le \frac{1}{2} / \frac{1}{3} < X < \frac{2}{3})$

12. If the probability density of X is given by

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find its rth moment. Hence evaluate $E[(2x+1)^2]$

13. If the cumulative distribution function of X is given by $F(x) = \begin{cases} 1 - \frac{4}{x^2}, x > 2 \\ 0 \quad , x \le 2 \end{cases}$

Find (i) P(X<3) (ii) P(4<X<5) (iii) $P(X \ge 3)$.

14. Experience has shown that walking in a certain park, the time X(in mins) , between seeing two people smoking has a density function of the form $f(x) = \begin{cases} \lambda x e^{-x} & x > 0 \\ 0 & elsewhere \end{cases}$

(a) Calculate the value of λ

(b) Find the distribution function of X

(c) What is the probability that a person who has just seen a person smoking will see another person smoking in 2 to 5 minutes? In atleast 7 minutes?

15. The density function of a random variable X is given by $f(x) = \begin{cases} kx(2-x)^2 & 0 < x < 2 \\ 0 & otherwise \end{cases}$ find (i) k (ii) Mean and variance of the distribution.

16. Find the M.G.F for the distribution $f(x) = \begin{cases} \frac{x}{4} \; e^{-\frac{x}{2}} & x > 0 \\ 0 & otherwise \end{cases}$ find (i) M.G.F (ii) First Four moments obout the origin

17. A random variable has the p.d.f given by

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find (a) The moment generating function
(b) First four moments about the origin.

18.Describe Binomial distribution B(n,p) and obtain the moment generating function. Hence compute (1) the first four moments and (2) the recursion relation for the central moments.

19. Derive the MGF of Poisson distribution and hence or otherwise deduce its mean and variance.

20. Find the nth moment about mean of normal distribution.

21. 4 coins were tossed simultaneously. What is the probability of getting (i) 2 heads (ii) atleast 2 heads (iii) at most 2 heads.

22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of 2 successes.

23.If 10% of the screws produced by an automatic machine are defective, find the probability that out of 20 screws selected at random, there are (i) exactly 2 defective (ii) Atmost 3 defective (iii) Atleast 2 defectives

23.In a large consignment of electric bulbs 10% are defective. A random sample of 20 is taken for inspection. Find the probability that (i) All are good bulbs, (ii) Atmost there are 3 defective bulbs (iii) Exactly there are three defective bulbs.

24. A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective what is the probability that a box will fail to meet the guaranteed quality?($e^{-2} = 0.13534$)

25. If X is a Poisson variate $P(X = 2) = 9P(X = 4) + 90P(X = 6)$, find (i) mean of X (ii) variance of X.

26. Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.8.
   (i) What is the probability that the target would be hit on 6th attempt
   (ii) What is the probability that it takes him less than 5 shots
   (iii) What is the probability that it takes him an even number of shots

27. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$.
   What is the probability that the repair time
   (a) exceeds 2 hours
   (b) exceeds 5 hours

28. A car hire firm has 2 cars which it hires out day by day. The number of demands for a car in each day is distributed with mean 1.5. Calculate the preposition of days in which
   I) Neither car is used
   II) Some demand is refused.

29. The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and S.D. 5. If 3 students are selected at random from this group, what is the probability that atleast one of them would have scored above 75? (Given the area between z=0 and z=2 under the standard normal curve is 0.4772) .

30. The weekly wages of 1000 workmen are normally distributed around a mean of Rs.70 with a S.D. of Rs.5. Estimate the number of workers whose weekly wages will be (i) between Rs.69 and Rs.72 (ii) less than Rs.69 (iii) more than Rs.72.

Unit II

1. From the following distribution of (X,Y) find. (i) $P(X \le 1)$ (ii) $P(Y \le 3)$ (iii) $P(X \le 1, Y \le 3)$ (iv)$P\left(X \le \frac{1}{Y} \le 3\right)$ (v) $P\left(Y \le \frac{3}{X} \le 1\right)$ (vi) $P(X + Y \le 4)$.

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

2. The joint probability function (X,Y) is given by $P(x, y) = k(2x + 3y) \quad x = 0,1,2; \; y = 1,2,3$
   (i) Find the marginal distributions.
   (ii) Find the probability distributions of (X+Y)
   (iii) Find all conditional probability distributions.

3. The joint p.d.f of the random variable (X,Y) is given by

$$f(x, y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2, \; 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Find

(i) Marginal density function s of X and Y

(ii) Conditional density of X given Y

(iii) $P\left(\frac{1}{4} < X < \frac{\frac{1}{2}}{Y} = \frac{1}{3}\right)$

4. The joint p.d.f of the two dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{8xy}{9} & : 1 \le x \le y \le 2 \\ 0 & : otherwise \end{cases}$$

Find

(i) Marginal densities of X and Y

(ii) The conditional density functions $f(x/y)$ $and\ f(y/x)$.

5. If the joint p.d.f of a two dimensional random variable (X,Y) is given by $f(x,y) =$

$$\begin{cases} x^2 + \frac{xy}{3}: & 0 < x < 1; 0 < y < 2 \\ 0: & otherwise \end{cases}$$

Find (i) $P(X > {}^{1}/_{2})$ (ii) $P(Y > 1)$ (iii) $P(Y < X)$

(iii) $P\left(Y < \frac{1}{2} \Big/ X < \frac{1}{2}\right)$ (v) $P(X + Y \ge 1)$

(vi) find the conditional density functions.

(vii) Check whether the conditional density functions are valid.

6. The joint p.d.f of the random variable (X,Y) is given by $f(x,y) = kxye^{-(x^2+y^2)}$ $x > 0, y > 0$

(i) Find k (ii) Prove that X and Y are independent.

7. Given $f_{XY}(x,y) = \begin{cases} cx(x-y) & ,0 < x < 2, -x < y < x \\ 0 & otherwise \end{cases}$
   (i) Evaluate c
   (ii) Find $f_X(x)$
   (iii) $f_{\frac{Y}{X}}(y/x)$
   (iv) $f_Y(y)$.
8. Two random variables X and Y have the following joint probability density functions
   $$f(x,y) = \begin{cases} 2-x-y: 0 \le x \le 1, 0 \le y \le 1 \\ 0 \quad : otherwise \end{cases}$$
   (i) Find the marginal density functions of X and Y
   (ii) Conditional density function
   (iii) Var X and Var Y
   (iv) Correlation coefficient between X and Y.
9. Given the joint p.d.f of X and Y is
   $$f(x,y) = \begin{cases} 8xy: 0 < x < y < 1 \\ 0 \quad : otherwise \end{cases}$$
   Find the marginal and conditional p.d.f's X and Y.Are X and Y independent?
10. Let (X,Y) be the two dimensional random variable described by the joint p.d.f
   $$f(x,y) = \begin{cases} 8xy: 0 \le x \le 1, 0 \le y \le x \\ 0 \quad : otherwise \end{cases}$$
   Find the Cov(X,Y).
11. The joint p.d.f of the random variable (X,Y) is $f(x,y) = 3(x+y): 0 \le x \le 1, 0 \le y \le 1, x+y \le 1$. Find Cov(X,Y).
12. If X and Y are uncorrelated random variables with variances 16 and 9, find the correlation co-efficient between x+y and x-y.

13. Marks obtained by 10 students in Mathematics(x) and statistics(y) are given below

x:	60	34	40	50	45	40	22	43	42	64
y:	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find y when x=55.

14. In a correlation analysis the equations of the two regression lines are $3x + 12y = 9; and\ 3y + 9x = 46$. Find (i) The value of the correlation coefficient (ii) Mean value of X and Y.

15. Find the correlation coefficient and the equation of the regression lines for the following values of X and Y.

X	1	2	3	4	5
Y	2	5	3	8	7

16. Find the most likely price in City A corresponding to the price of Rs.70 at City B from the following:

	City B	City A
Average Price	65	67
S.D. of Price	2.5	3.5

Correlation coefficient is 0.8.

17. The joint p.d.f of the random variable (X,Y) is given as

$$f(x,y) = \begin{cases} e^{-(x+y)} & ; x > 0, y > 0 \\ 0 & ;\ otherwise \end{cases}$$

Find the distribution of $\frac{1}{2}(X - Y)$.

18. The independent random variables X and Y follow exponential distribution with parameter $\lambda = 1$. Find the p.d.f of $U = X - Y$.

19. Let X and Y are normally distributed independent random variables with mean 0 and variance σ^2. Find the p.d.f's of $R = \sqrt{x^2 + y^2}\ and\ \theta = tan^{-1}\left(\frac{y}{x}\right)$.

20. The joint p.d.f of a two dimensional random variable (X,Y) is given by $f(x,y) = x + y, 0 \leq x, y \leq 1$. Find the p.d.f of U=XY.
21. If X and Y are independent random variables , with p.d.f $f(x) = e^{-x}\ , x \geq 0 \quad : f(y) = e^{-y}, y \geq 0$. Show that $U = \frac{X}{X+Y}\ and\ V = X + Y$ are independent.

Unit III

SSS and WSS

1. The probability distribution of the process $\{X(t)\}$ is given by $P(X(t)=n)=\begin{cases}\frac{(at)^{n-1}}{(1+at)^{n-1}}, n=1,2,\ldots\\ \frac{at}{1+at} \quad ,n=0\end{cases}$
   Show that {X(t)}is not stationary.
2. Verify whether the random process $X(t)=Acos(w_0t+\theta)$where A and $w_0$are constants , $\theta$is a uniformly distributed random variable $(0,2\pi)$ is wide sense stationary.
3. Show that the random process $X(t)=Acos\lambda t+Bsin\lambda t$, where λ is a constant, A and B are uncorrelated random variables with 0 mean and equal variance, is a WSS.
4. Consider a random process {X(t)} defined by $X(t)=Acost+Bsint$ when A and B are independent random variables each of which assumes the values -2 and 1 with probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Show that {X(t)}is wide sense stationary and not strict sense stationary.
5. Define random telegraph process. Prove that it is stationary in the wide sense.
6. If {X(t)} is a Gaussian process with $\mu\{X(t)\}=10$ and $C(t_1,t_2)=16e^{-|t_1-t_2|}$, find the probability that (i) $X(10)\leq 8$ and (ii) $|X(10)-X(6)|\leq 4$
7. Suppose that $X(t)$ is a Gaussian process with $\mu_X=2, R_{XX}(\tau)=5e^{-0.2|\tau|}$, Find the probability that $X(4)\leq 1$.
8. The one step T.P.M of a Markov chain (X_n;n=0,1,2,…) having state space $S=(1,2,3)$ is $\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution $\pi_0=(0.7,0.2,0.1)$. Find
   1. $P(X_2=3/X_0=1)$

2. $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$
3. $P(X_2 = 3)$

10. The t.p.m of a Markov chain with three states 0,1,2 is $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = \frac{1}{3}$, $i = 0,1,2$. Find $(i) P[X_2] = 2$ $(ii) P[X_3 = 1 , X_2 = 2, X_1 = 1, X_0 = 2]$ (iii) $P[X_2 = 1, X_0 = 0]$

Poisson Process

11. If the process $\{X(t); t \geq 0\}$ is a Poisson process with parameter λ, obtain $P[X(t) = n]$. Is the process first order stationary?

12. State the postulates of a Poisson process and derive the probability distribution. Also prove that the sum of two independent Poisson processes is a Poisson process.

13. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 minute and 2 minute and (3) 4 minute or less

14. Assume that the number of messages input to a communication channel in an interval of duration t seconds, is a Poisson process with mean $\lambda = 0.3$. Compute

(1) The probability that exactly 3 messages will arrive during 10 second interval.

(2) The probability that the number of message arrivals in an interval of duration 5 seconds is between 3 and 7.

15. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min.
(1) exactly 4 customers arrive and
(2) more than 4 customers arrive

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Unit IV

Auto correlation

1. Find the autocorrelation function of the random process $X(t) = A\sin(\omega t + \theta)\,, where$ A and ω are constants and θ is uniformly distributed in $(0,2\pi)$.

2. X(t) and Y(t) are zero mean and stochastically independent random process having auto correlation function $R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = cos2\pi\tau$ respectively.
   (i) Find the A.C.F of $W(t) = X(t) + Y(t)$
   (ii) Find the A.C.F of $Z(t) = X(t) - Y(t)$
   (iii) Find the cross correlation function of $W(t)\ and\ Z(t)$.
3. State and prove Wiener -Khinchine theorem.

Relationship between $R_{XX}(\tau) and\ S_{XX}(\omega)$

4. Define spectral density of a stationary random process $X(t)$. Prove that for a real random process X(t), the power spectral density is an even function.
5. Find the power spectral density of the random process whose auto correlation function is $R(\tau) = \begin{cases} 1-|\tau| & for\ \ |\tau| \le 1 \\ 0, & elsewhere \end{cases}$.
6. Find the power spectral density function whose auto correlation is given by $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$.
7. A random process {X(t)} is given by $X(t) = Acospt + Bsinpt$, where A and B are independent random variables such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$. Find the power spectral density of the process.
8. The auto correlation function of a random process is given by $R(\tau) = \begin{cases} \lambda^2 & |\tau| > \varepsilon \\ \lambda^2 + \frac{\lambda}{\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right) & |\tau| \le \varepsilon \end{cases}$. Find the power spectral density of the process.

9. The auto correlation function of a WSS process with autocorrelation function $R(\tau) = \alpha^2 e^{-2\lambda\sqrt{|\tau|}}$, determine the power spectral density of the process.

10. Determine the power spectral density of a WSS process X(t) which has an auto correlation $R_{XX}(\tau) = \begin{cases} \left[1 - \frac{|\tau|}{T}\right], -T \le t \le T \\ 0 \quad otherwise \end{cases}$. Show that the process is mean ergodic.

11. The power spectral density function of a zero mean WSS process {X(t)} is given by $S(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & otherwise \end{cases}$. Find $R(\tau)$. Show that X(t) and $X(t + \frac{\pi}{\omega_0})$ are uncorrelated.

12. Find the auto correlation function of the process $S_{XX}(\omega) = \begin{cases} 1 + \omega^2 & |\omega| < 1 \\ 0 & otherwise \end{cases}$

13. Find the cross correlation {X(t)} and {Y(t)} whose cross power spectrum is $S_{XY}(\omega) = \begin{cases} p + \frac{iq\omega}{B};\ -B < \omega < B \\ 0 \quad otherwise \end{cases}$

14. Find the cross correlation {X(t)} and {Y(t)} whose cross power spectrum is $S_{XY}(\omega) = \begin{cases} a + jb\omega;\ |\omega| < 1 \\ 0 \quad otherwise \end{cases}$.

15. If the cross correlation of two processes {X(t)} and {Y(t)} is $R_{XY}(t, t+\tau) = \frac{AB}{2}[sin\omega_0\tau + cos\omega_0(2t+\tau)]$ where A,B,ω_0 are constants. Find the cross power spectrum.

16. Prove that, if the input to a time – invariant, stable linear system is a WSS process, then the output will also be a WSS process.
17. Prove that (i) $R_{XY}((\tau) = R_{XX}(\tau) * h(\tau)$ (ii) $R_{YY}((\tau) = R_{XY}(\tau) * h(-\tau)$ (iii) $S_{XY}(\omega) = S_{XX}(\omega)H(\omega)$ (iv) $S_{YY}(\omega) = S_{XY}(\omega)H^*(\omega)$ (v) $S_{YY}(\omega) = S_{XX}(\omega)|H(\omega)|^2$
18. X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary Random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ and $R_{YY}(\tau)$, if the power transfer function is $H(\omega) = \frac{R}{R+iL\omega}$.