SNS COLLEGE OF TECHNOLOGY

19MCB204 – SOLIDMECHANICS

UNIT- IV DEFLECTION OF BEAMS AND BUCKLING OF COLUMNS

Macaulay Method-Probems

Macaulay Method-Probems

A steel girder of uniform section, 14 meters long is simply supported at its ends. It carries concentrated loads of 90 KN and 60 KN at two points 3 meters and 4.5 meters from the two ends respectively. Calculate: (i) The deflection of the girder at the points under the two loads.(ii)The maximum deflection. Take:I = 64 x 10-4 m4 and E = 210 x 10-6 KN/m2.

Solution :-

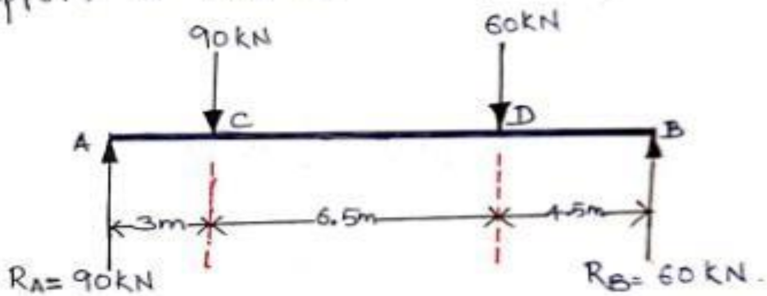
Span of steel girder,

$l = 14.0\,m$.

Moment of Inertia, $I = 64 \times 10^{-4}\,m^4$.

Young's Modulus, $E = 210 \times 10^6\,kN/m^2$

Let R_A and R_B be the reactions at support A and B respectively.



Taking moments about A, we get.

$$R_B \times 14 = 90 \times 3 + 60 \times 9.5 = 840.$$

$$R_B = 60\,kN.$$

$$R_A + R_B = 90 + 60 = 150\,kN$$

$$R_A = 150 - 60 = 90\,kN$$

$$R_A = 90\,kN.$$

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Macaulay Method-Probems

Consider any section XX at a distance x from end A, following Macaulay's method, the bending moment is given by,

$$M_x = EI\frac{d^2y}{dx^2} = 90x \,\Big|\, -90(x-3) \,\Big|\, -60(x-9.5) \rightarrow (1)$$

Integrating, we get.

$$EI\frac{dy}{dx} = 45x^2 + C_1 \,\Big|\, -45(x-3)^2 - 30(x-9.5)^2 \rightarrow (2)$$

Integrating again, we get.

$$EIy = 15x^3 + C_1x + C_2 \,\Big|\, -15(x-3)^3 - 10(x-9.5)^3 \rightarrow (3)$$

When, $x = 0,\ y = 0$.

$\therefore\ C_2 = 0$.

When, $x = 14m,\ y = 0$.

$$0 = 15\times(14)^3 + C_1\times 14 - 15(14-3)^3 - 10(14-9.5)^3$$

$$= 41160 + 14C_1 - 19965 - 911.25 = 14C_1 - 20283.75$$

$$C_1 = -1448.84$$

~~Macaulay Method-Probems~~

Hence the deflection at any section is given by.

$$EIy = 15x^3 - 1448.84x1 - 15(x-3)^3 - 10(x-9.5)^3$$

$\longrightarrow$ Deflection Equation.

i) y_C and y_D :-

Deflection at C, y_c.

Putting $x = 3m$ in the deflection equation, we get.

$$EIy_c = 15 \times 3^3 - 1448.84 \times 3 = 405 - 4346.52$$

$$EIy_c = -3941.52.$$

$$y_c = -\frac{3941.52}{EI} = \frac{3941.52}{210 \times 10^6 \times 64 \times 10^{-4}}$$

$$y_c = -0.00291 m = -2.93mm.$$

(or)

Downward deflection of C.

$$y_c = -2.93mm$$

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Deflection at D, y_D: Putting $x = 9.5$ mm in the deflection equation, we get.

$$EIy_D = 15\times 9.5^3 - 1448.84\times 9.5 - 15(9.5-3)^3$$

$$\Rightarrow 12860.6 - 13764 - 4119.4 \Rightarrow -5022.8$$

$$y_D = \frac{-5022.8}{EI}$$

$$= \frac{-5022.8}{210\times 10^6\times 64\times 10^{-4}}$$

$$y_D = -0.00373\text{ m (or)} -3.73\text{mm}$$

Downward deflection of D = 3.73mm.

ii) Maximum deflection, y_{max}:-

Let us assume that the deflection will be maximum at a section between C and D. Equating the slope at the section to zero, we get.

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$$EI\frac{dy}{dx} = 45x^2 - 1448.84 - 45(x-3)^2 = 0.$$

Or,

$$45x^2 - 1448.84 - 45(x^2 - 6x + 9) = 0.$$

Or,

$$45x^2 - 1448.84 - 45x^2 + 270x - 405 = 0.$$

Or,

$$270x = +1853.84.$$

$$\boxed{x = 6.87m}$$

Putting this value of x in the deflection equation, we get.

$$EIy_{max} = 15 \times 6.87^3 - 1448.84 \times 6.87 - 15(6.87-3)^3.$$

$$= 4863.6 - 9953.5 - 869.4 = -5959.3.$$

$$y_{max} = -\frac{5959.3}{EI}$$

$$= -\frac{5959.3}{210 \times 10^6 \times 64 \times 10^{-4}}$$

$$y_{max} = 0.0043m \text{ (or) } -4.43mm$$

Downward deflection, $y_{max} = 4.43mm.$