SNS COLLEGE OF TECHNOLOGY

19MCB204 – SOLID MECHANICS

UNIT II - BEAMS -SHEAR FORCE, BENDING MOMENT AND THEORY OF BENDING

Types of beams: Supports and Loads

EXAMPLE 13.10. *A simply supported beam AB, 6 m long is loaded as shown in Fig. 13.21.*

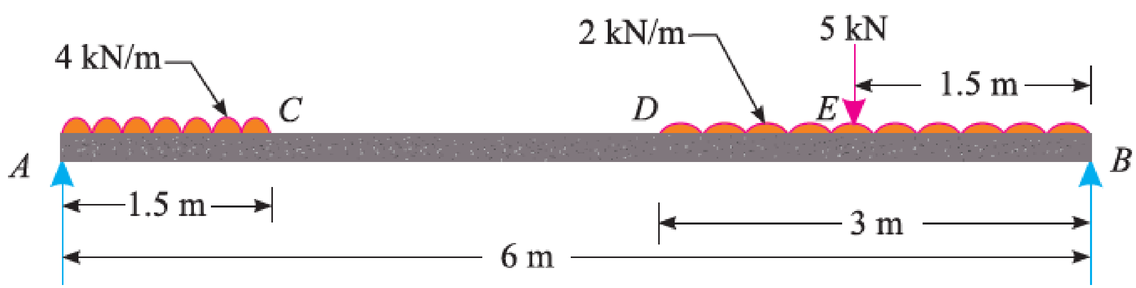


Fig. 13.21

Construct the shear force and bending moment diagrams for the beam and find the position and value of maximum bending moment.

SOLUTION. Given : Span (l) = 6 m ; Point load at E (W) = 5 kN ; Uniformly distributed load between A and C (w_1) = 4 kN/m and uniformly distributed load between D and B = 2 kN/m.

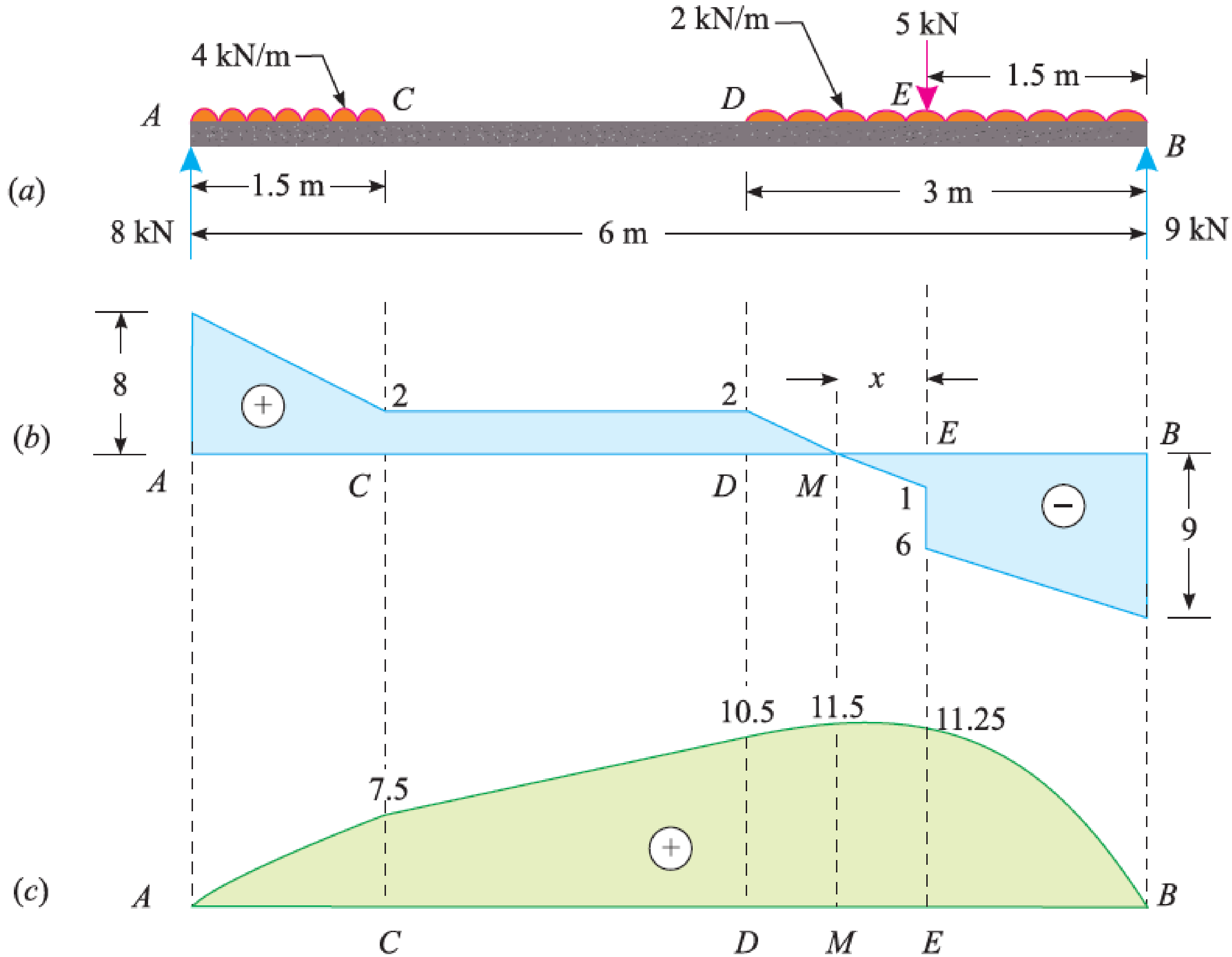
First of all, let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 6 = (4 \times 1.5 \times 0.75) + (2 \times 3 \times 4.5) + (5 \times 4.5) = 54$$

$$R_B = 54/6 = 9 \text{ kN}$$

and

$$R_A = (4 \times 1.5) + (2 \times 3) + 5 - 9 = 8 \text{ kN}$$



4 kN/m
2 kN/m
5 kN
1.5 m
A
C
D
E
B
(a)
1.5 m
3 m
8 kN
6 m
9 kN
8
2
2
x
(b)
A
C
D
M
E
B
1
6
9
10.5
11.5
11.25
7.5
(c)
A
B
C
D
M
E

Shear force diagram

The shear force diagram is shown in Fig. 13.22 (*b*) and the values are tabulated here:

$$F_A = +R_A = +8 \text{ kN}$$
$$F_C = 8 - (4 \times 1.5) = 2 \text{ kN}$$
$$F_D = 2 \text{ kN}$$
$$F_E = 2 - (2 \times 1.5) - 5 = -6 \text{ kN}$$
$$F_B = -6 - (2 \times 1.5) = -9 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.22 (c) and the values are tabulated here:

$$M_A = 0$$
$$M_C = (8 \times 1.5) - (4 \times 1.5 \times 0.75) = 7.5 \text{ kN-m}$$
$$M_D = (8 \times 3) - (4 \times 1.5 \times 2.25) = 10.5 \text{ kN-m}$$
$$M_E = (9 \times 1.5) - (2 \times 1.5 \times 0.75) = 11.25 \text{ kN-m}$$
$$M_B = 0$$

We know that maximum bending moment will occur at M, where the shear force changes sign. Let x be the distance between E and M. From the geometry of the figure between D and E, we find that

$$\frac{x}{1} = \frac{1.5 - x}{2} \quad \text{or} \quad 2x = 1.5 - x$$
$$3x = 1.5 \quad \text{or} \quad x = 1.5/3 = 0.5 \text{ m}$$
$$\therefore \quad M_M = 9(1.5 + 0.5) - (2 \times 2 \times 1) - (5 \times 0.5) = 11.5 \text{ kN-m}$$

EXAMPLE 13.13. *An overhanging beam ABC is loaded as shown in Fig. 13.30.*

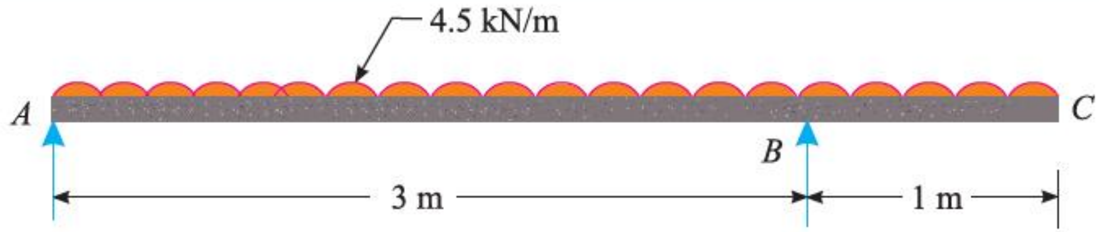


Fig. 13.30

Draw the shear force and bending moment diagrams and find the point of contraflexure, if any.

SOLUTION. Given : Span (l) = 4 m ; Uniformly distributed load (w) = 4.5 kN/m and overhanging length (c) = 1 m.

First of all, let us find out the reactions R_A and R_B. Taking moment about A and equating the same,

$$R_B \times 3 = (4.5 \times 4) \times 2 = 36$$

$$\therefore \quad R_B = 36/3 = 12 \text{ kN}$$

$$\text{and} \quad R_A = (4.5 \times 4) - 12 = 6 \text{ kN}$$

$$F_A = +R_A = +6 \text{ kN}$$
$$F_B = +6 - (4.5 \times 3) + 12 = 4.5 \text{ kN}$$
$$F_C = +4.5 - (4.5 \times 1) = 0$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.31 (*c*) and the values are tabulated here:

$$M_A = 0$$
$$M_B = -\left(4.5 \times 1 \times \frac{1}{2}\right) = -2.25 \text{ kN-m}$$
$$M_C = 0$$

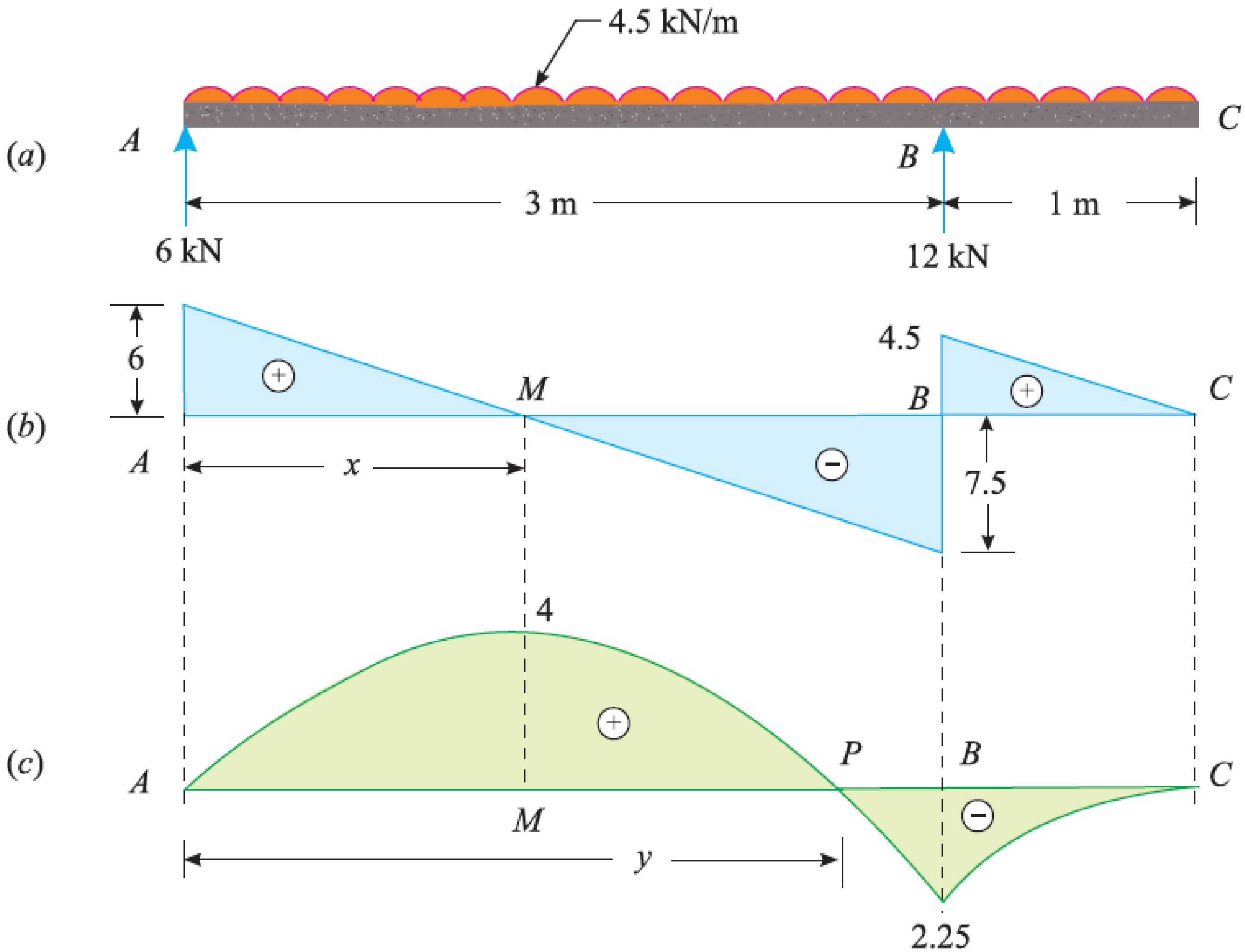
We know that maximum bending moment will occur at M, where the shear force changes sign. Let x be the distance between A and M. From the geometry of the figure between A and B, we find that

$$\frac{x}{6} = \frac{3-x}{7.5} \quad \text{or} \quad 7.5\,x = 18 - 6\,x$$
$$13.5\,x = 18 \quad \text{or} \quad x = 18/13.5 = 1.33 \text{ m}$$
$$\therefore \quad M_M = (6 \times 1.33) - 4.5 \times 1.33 \times \frac{1.33}{2} = 4 \text{ kN-m}$$

Point of contraflexure

Let P be the point of contraflexure at a distance y from the support A. We know that bending moment at P.

$$M_P = 6 \times y - 4.5 \times y \times \frac{y}{2} = 0$$
$$2.25\,y^2 - 6\,y = 0 \quad \text{or} \quad 2.25\,y = 6$$
$$\therefore \quad y = 6/2.25 = 2.67 \text{ m}$$ **Ans.**


4.5 kN/m
(a)
A
B
C
3 m
1 m
6 kN
12 kN
(b)
6
4.5
M
B
C
A
x
7.5
(c)
4
P
B
C
A
M
y
2.25