SNS COLLEGE OF TECHNOLOGY

19MCB204 – SOLID MECHANICS

UNIT II - BEAMS -SHEAR FORCE, BENDING MOMENT AND THEORY OF BENDING

Types of beams: Supports and Loads

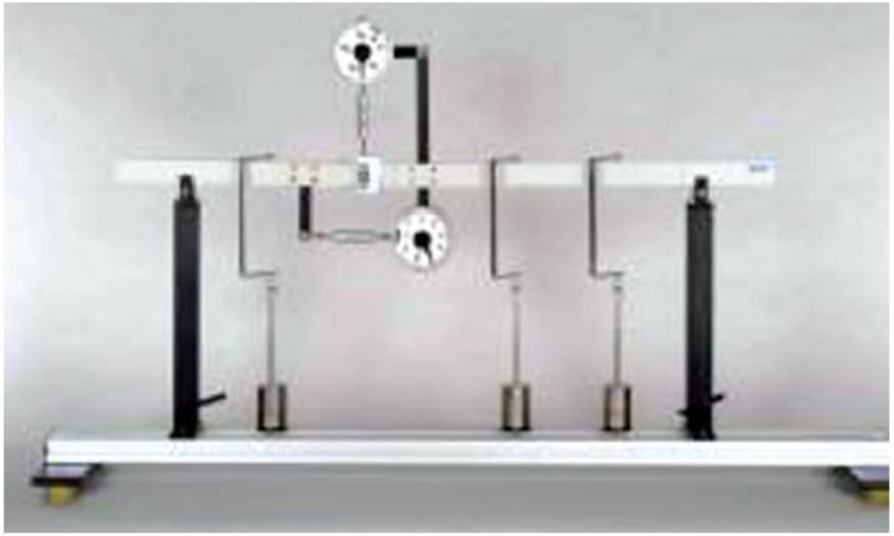
The scientific way of studying the deflection or any other effect is to draw and analyse the shear force or bending moment diagrams of a beam. In general, the beams are classified as under:

1. Cantilever beam,
2. Simply supported beam,
3. Overhanging beam,
4. Rigidly fixed or built-in-beam and
5. Continuous beam.

13.2. Types of Loading

A beam may be subjected to either or in combination of the following types of loads:

1. Concentrated or point load,
2. Uniformly distributed load and
3. Uniformly varying load.



Shearing force

13.3. Shear Force

The shear force (briefly written as S.F.) at the cross-section of a beam may be defined as the unbalanced vertical force to the right or left of the section.

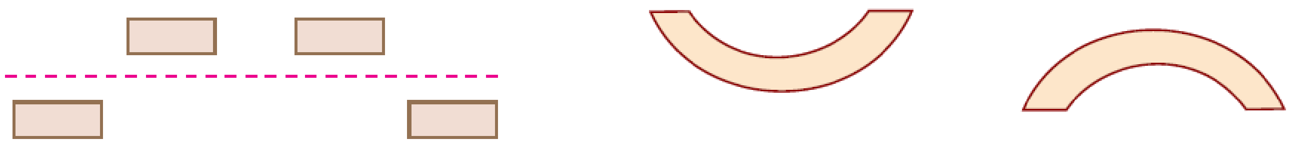
13.4. Bending Moment

The bending moment (briefly written as B.M.) at the cross-section of a beam may be defined as the algebraic sum of the moments of the forces, to the right or left of the section.

Note. While calculating the shear force or bending moment at a section, the end reactions must also be considered alongwith other external loads.

13.5. Sign Conventions

We find different sign conventions in different books, regarding shear force and bending moment at a section. But in this book the following sign conventions will be used, which are widely followed and internationally recognised.



(*a*) Positive S.F. (*b*) Negative S.F. (*c*) Positive B.M. (*d*) Negative B.M.

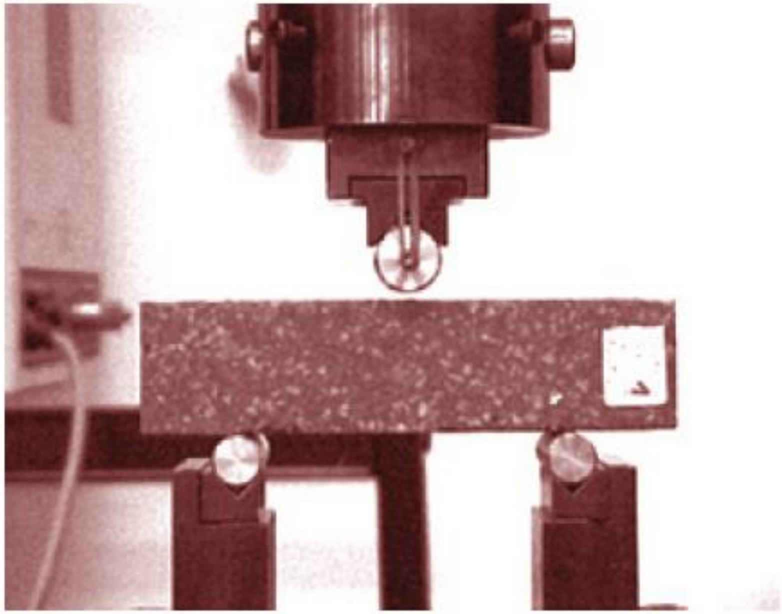
Fig. 13.1

1. ***Shear Force.*** We know that as the shear force is the unbalanced vertical force, therefore it tends to slide one portion of the beam, upwards or downwards with respect to the other. The shear force is said to be positive, at a section, when the left hand portion tends to slide downwards or the right hand portion tends to slide upwards shown in Fig. 13.1 (*a*). Or in other words, all the downward forces to the left of the section cause positive shear and those acting upwards cause negative shear as shown in Fig. 13.1 (*a*).

Similarly, the shear force, is said to be negative at a section when the left hand portion tends to slide upwards or the right hand portion tends to slide downwards as shown in Fig. 13.1 (*b*). Or in other words, all the upward forces to the left of the section cause negative shear and those acting downwards cause positive shear as shown in Fig. 13.1 (*b*).

2. ***Bending Moment.*** At sections, where the bending moment, is such that it tends to bend the beam at that point to a curvature having concavity at the top, as shown in Fig. 13.1 (*c*) is taken as positive. On the other hand, where the bending moment is such that it tends to bend the beam at that point to a curvature having convexity at the top, as shown in Fig. 13.1 (*d*) is taken as negative. The positive bending moment is often called sagging moment and negative as hogging moment.

A little consideration will show that the bending moment is said to be positive, at a section, when it is acting in an anticlockwise direction to the right and negative when acting in a clockwise direction. On the other hand, the bending moment is said to be negative when it is acting in a clockwise direction to the left and positive when it is acting in an anticlockwise direction.



Bending test of resin concrete

NOTE. While calculating bending moment or shear force, at a section the beam will be assumed to be weightless.

13.6. Shear Force and Bending Moment Diagrams

The shear force and bending moment can be calculated numerically at any particular section. But sometimes, we are interested to know the manner, in which these values vary, along the length of the beam. This can be done by plotting the shear force or the bending moment as ordinate and the position of the cross as abscissa. These diagrams are very useful, as they give a clear picture of the distribution of shear force and bending moment all along the beam.

Note. While drawing the shear force or bending moment diagrams, all the positive values are plotted above the base line and negative values below it.

13.7. Relation between Loading, Shear Force and Bending Moment

The following relations between loading, shear force and bending moment at a point or between any two sections of a beam are important from the subject point of view:

1. If there is a point load at a section on the beam, then the shear force suddenly changes (*i.e.*, the shear force line is vertical). But the bending moment remains the same.
2. If there is no load between two points, then the shear force does not change (*i.e.*, shear force line is horizontal). But the bending moment changes linearly (*i.e.*, bending moment line is an inclined straight line).
3. If there is a uniformly distributed load between two points, then the shear force changes linearly (*i.e.*, shear force line is an inclined straight line). But the bending moment changes according to the parabolic law. (*i.e.*, bending moment line will be a parabola).
4. If there is a uniformly varying load between two points then the shear force changes according to the parabolic law (*i.e.*, shear force line will be a parabola). But the bending moment changes according to the cubic law.

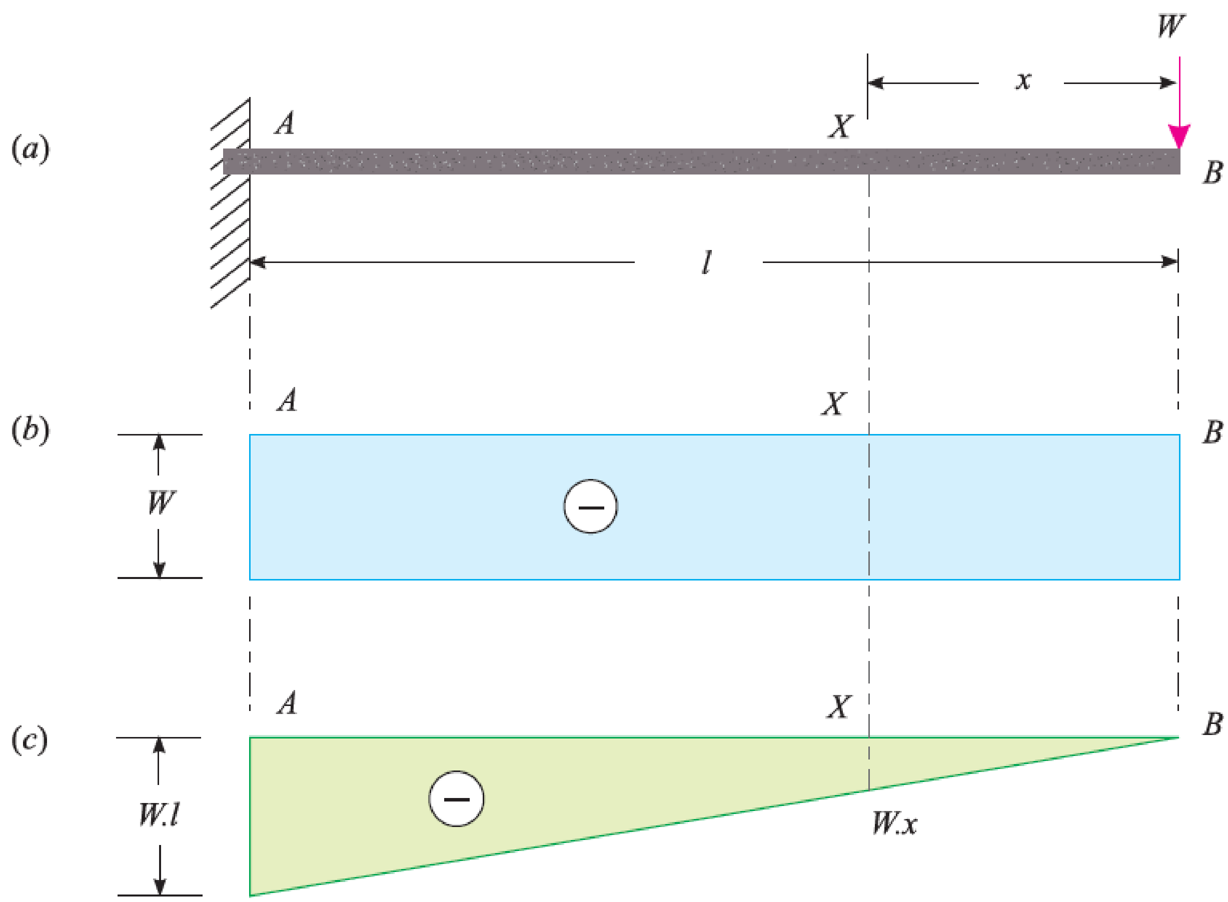


Fig. 13.2. Cantilever with a point load

13.9. Cantilever with a Uniformly Distributed Load

Consider a cantilever AB of length l and carrying a uniformly distributed load of w per unit length, over the entire length of the cantilever as shown in Fig. 13.4 (*a*).

We know that shear force at any section X, at a distance x from B,

$$F_x = -w \cdot x \qquad \text{... (Minus sign due to right downwards)}$$

Thus we see that shear force is zero at B (where $x = 0$) and increases by a straight line law to $-wl$ at A as shown in Fig. 13.4 (*b*).

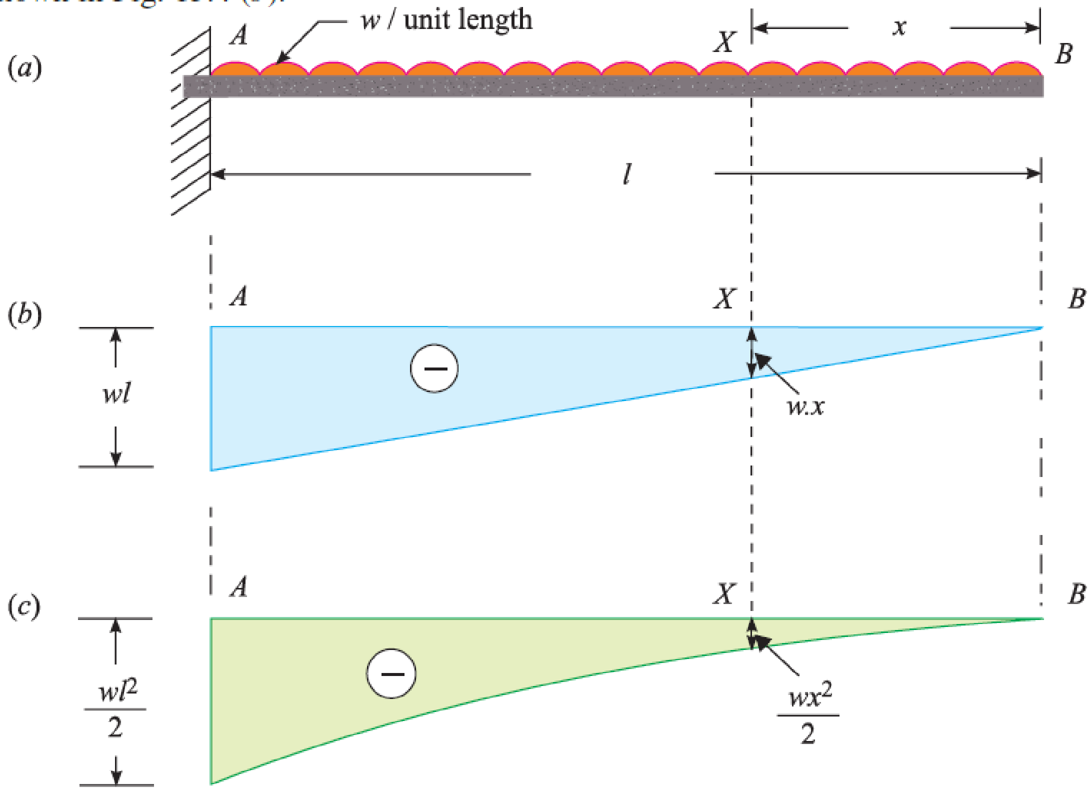


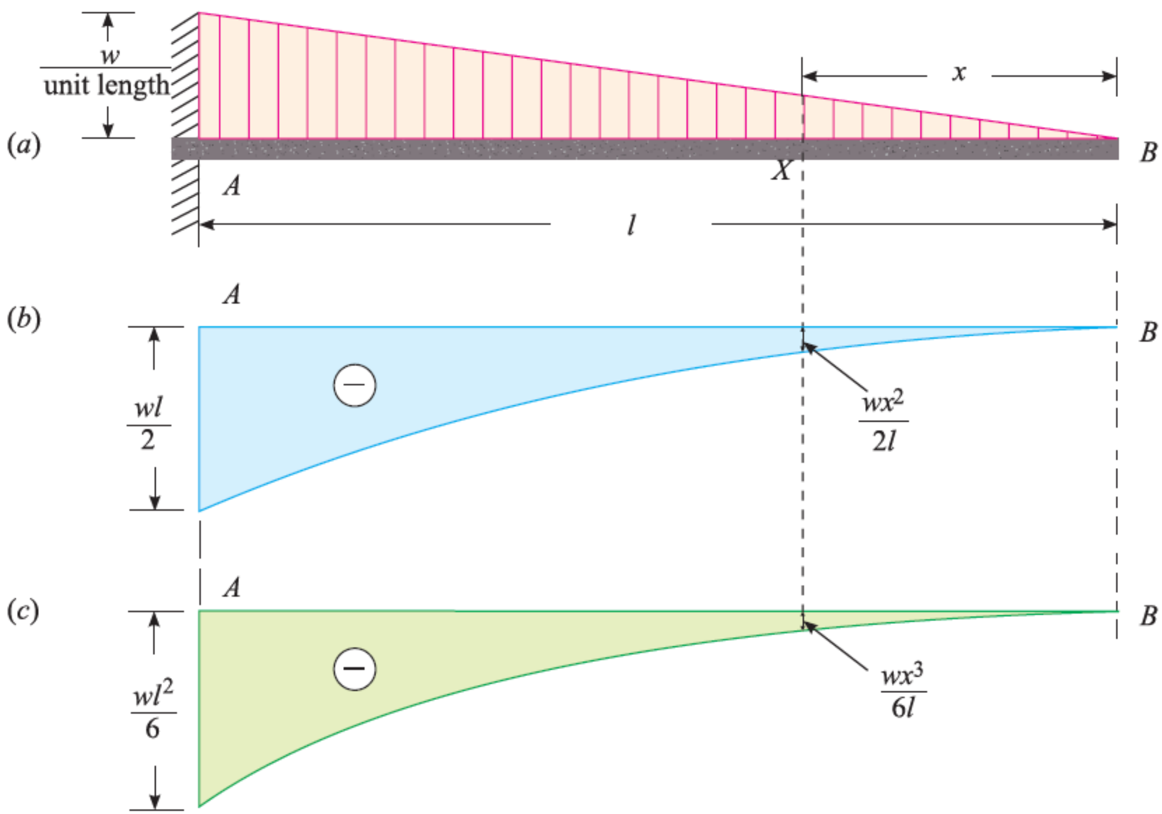
Fig. 13.4. Cantilever with a uniformly distributed load

13.10. Cantilever with a Gradually Varying Load

Consider a cantilever *AB* of length *l*, carrying a gradually varying load from zero at the free end to w per unit length at the fixed end, as shown in Fig. 13.7 (*a*).

We know that, the shear force at any section *X*, at a distance *x* from the free end *B*,

$$F_X = -\left(\frac{wx}{l}\cdot\frac{x}{2}\right) = -\frac{wx^2}{2l} \qquad \text{...}(i) \quad \text{(Minus sign due to right downward)}$$



13.11. Simply Supported Beam with a Point Load at its Mid-point

Consider a *simply supported beam *AB* of span *l* and carrying a point load *W* at its mid-point *C* as shown in Fig. 13.12 (*a*). Since the load is at the mid-point of the beam, therefore the reaction at the support *A*,

$$R_A = R_B = 0.5\ W$$

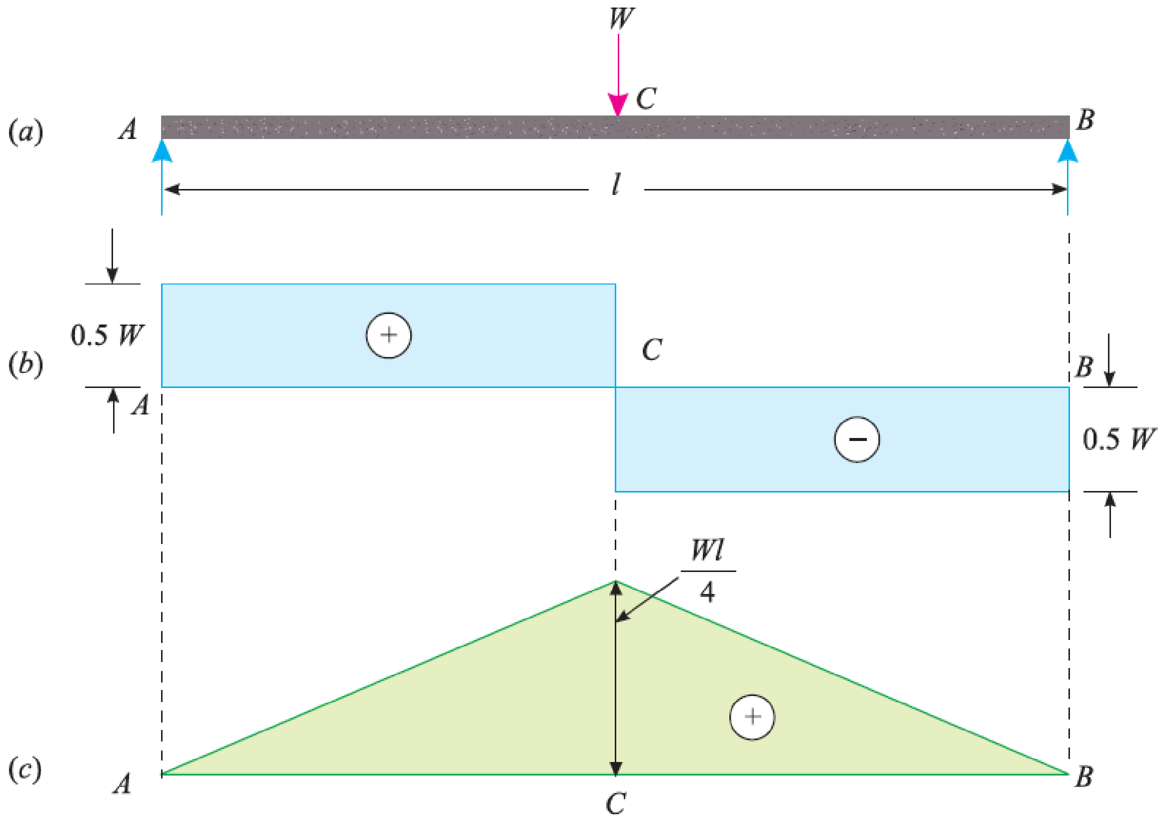


Fig. 13.12. Simply supported beam with a point load

13.12. Simply Supported Beam with a Uniformly Distributed Load

Consider a simply supported beam *AB* of length *l* and carrying a uniformly distributed load of *w* per unit length as shown in Fig. 13.15. Since the load is uniformly distributed over the entire length of the beam, therefore the reactions at the supports *A*,

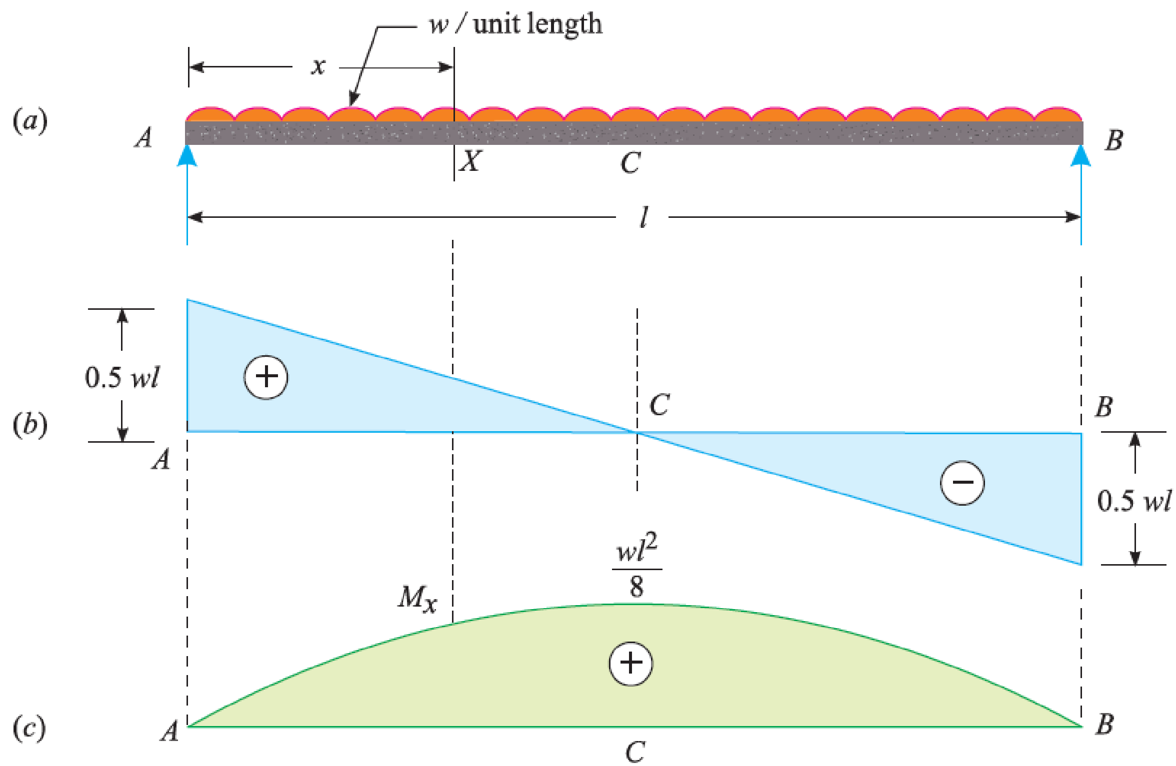


Fig. 13.15. Simply supported beam with a uniformly distributed load

13.15. Overhanging Beam

It is a simply supported beam which overhangs (*i.e.*, extends in the form of a cantilever) from its support.

For the purposes of shear force and bending moment diagrams, the overhanging beam is analysed as a combination of a simply supported beam and a cantilever. An overhanging beam may overhang on one side only or on both sides of the supports.

13.16. Point of Contraflexure

We have already discussed in the previous article that an overhanging beam is analysed as a combination of simply supported beam and a cantilever. In the previous examples, we have seen that the bending moment in a cantilever is negative, whereas that in a simply supported beam is positive. It is thus obvious that in an overhanging beam, there will be a point, where the bending moment will change sign from negative to positive or *vice versa*. Such a point, where the bending moment changes sign, is known as a point of contraflexure.