SNS COLLEGE OF TECHNOLOGY

Coimbatore-37.
An Autonomous Institution

COURSE NAME : 19ITB201 & DESIGN AND ANALYSIS OF ALGORITHMS

II YEAR/ IV SEMESTER

UNIT – 3 DYNAMIC PROGRAMMING AND GREEDY TECHNIQUE
Topic:

Dynamic Programming: Computing a Binomial Coefficient

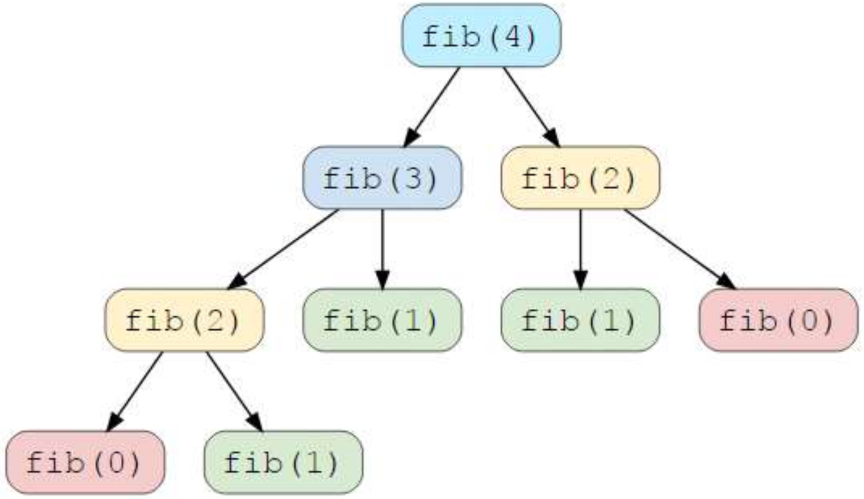
Dr.B.Vinodhini
Associate Professor
Department of Computer Science and Engineering

Dynamic Programming: Computing a Binomial Coefficient

- Dynamic Programming
  - ***Computing a Binomial Coefficient***
  - Warshall's algorithm
  - Floyd's algorithm
  - Optimal Binary Search Trees
  - Knapsack Problem and Memory functions

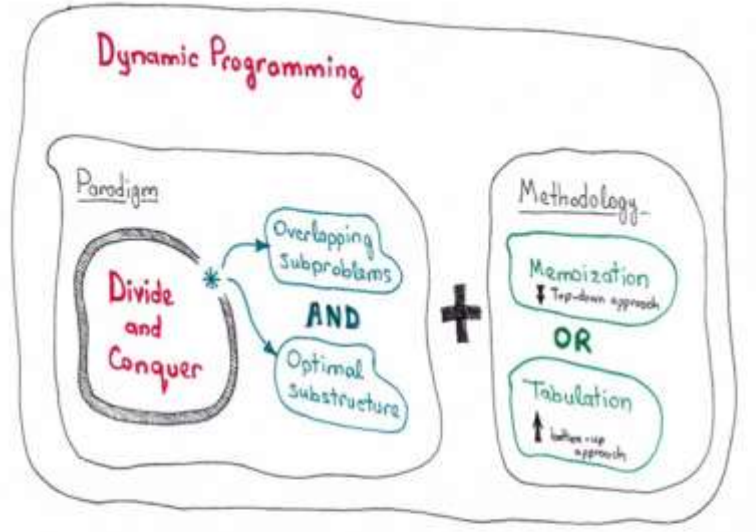
Dynamic Programming: Computing a Binomial Coefficient

- Dynamic programming – pblm →similar sub problems →reuse the solution
- **Characteristics**
  - Overlapping sub problems – solving same sub problems
  - Optimal substructure property – optimal solution can be built from sub problem
  - Example : Fibor



Dynamic Programming: Computing a Binomial Coefficient

- **Methodology**
  - Top-down with memoization
    - Storing the result of already solved sub-problem is called memoization
  - Bottom-up with Tabulation
    - Sub-problems (bottom – up)



Difference between Divide and conquer and Dynamic Programming

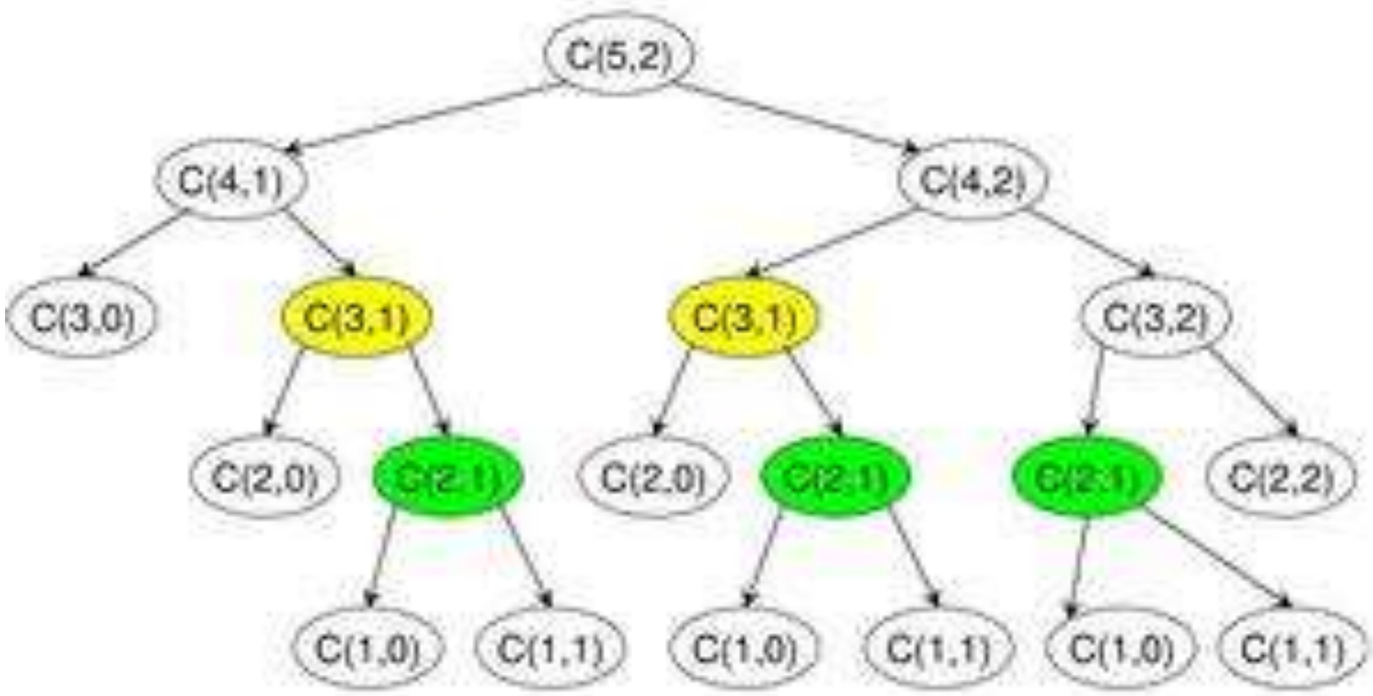
Divide and conquer	**Dynamic Programming**
Sub problems are not dependent on each other	Sub problems are dependent on each other
Doesn’t store the solution of sub-problem	Stores the solution of sub problem

Computing a Binomial Coefficient

- Binomial coefficient – computation of no of ways r items that can be chosen from n elements $C(^{n}_{r})$
- C(n, k) = n! / (n-k)! * k!
- C(n, k) = C (n-1, k-1) + C (n-1, k) , n>k, k>0
- C (n,0) = 1, C (n,n) = 1
- ***Example:***
- 1^{st} formula : C (4,2) → 4! /(2!) * 2! → 24 / 4 →6
- 2^{nd} formula : C(4,2) → C(3,1) + C(3,2) → ….. → 6
- C (4,2) → how many two combinations of elements can be picked from set of 4 elements
- Example: possibilities of 1,2,3,4 → (1,2) (1,3) (1,4) (2,3) (2,4) (3,4)

Computing a Binomial Coefficient

- Example : C (5,2)
- C(n, k) = C (n-1, k-1) + C (n-1, k) , n>k, k>0
- C (n,0) = 1, C (n,n) = 1



Computing a Binomial Coefficient - Tabulation

	0	1	2	3	4	5	...	$(k-1)$	k
0	1								
1	1	1							
2	1	2	1						
3	1	3	3	1					
4	1	4	6	4	1				
5									
⋮									
k	1								1
⋮									
$(n-1)$	1							$C(n-1, k-1)$	$C(n-1,k)$
n	1								$C(n,k)$

Computing a Binomial Coefficient - Algorithm

Algorithm *Binomial*(n, k)
 for $i \leftarrow 0$ **to** n **do** // fill out the table row wise
  for $i = 0$ **to** min(i, k) **do**
   if j==0 or j==i **then** $C[i, j] \leftarrow 1$ // IC
   else $C[i, j] \leftarrow C[i-1, j-1] + C[i-1, j]$ // recursive relation
 return $C[n, k]$

Computing a Binomial Coefficient - Analysis

- Cost of the algorithm – table
- Sum – 2 parts (upper and lower triangle)
- $A(n, k)$ = sum for upper triangle + sum for the lower rectangle

$$A(n,k) = \sum_{i=1}^{k} \sum_{j=1}^{i-1} 1 + \sum_{i=k+1}^{n} \sum_{j=1}^{k} 1$$

$$\Rightarrow \sum_{i=1}^{k} ((i-1) - \cancel{1} + \cancel{1}) + \sum_{i=k+1}^{n} (k - \cancel{1} + \cancel{1})$$

$$\Rightarrow \sum_{i=1}^{k} (i-1) + \sum_{i=k+1}^{n} k$$

$$\Rightarrow \left[\sum_{i=1}^{k} i - \sum_{i=1}^{k} 1 \right] + k \sum_{i=k+1}^{n} 1$$

$$\Rightarrow \frac{k(k+1)}{2} - (k - \cancel{1} + \cancel{1}) + k\left[n - (k+1) + 1\right]$$

Computing a Binomial Coefficient - Analysis

$$\Rightarrow \frac{k^2+k}{2} - k + k\left[n - k - \cancel{1} + \cancel{1}\right]$$

$$\Rightarrow \frac{k^2 + k - 2k + 2(nk - k^2)}{2}$$

$$\Rightarrow \frac{k^2 - k + 2nk - 2k^2}{2}$$

$$\Rightarrow \frac{-k^2 - k + 2nk}{2}$$

$$\approx nk$$

$$\boxed{O(nk)}$$

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References

1. Anany Levitin, “Introduction to the Design and Analysis of Algorithms”, Pearson Education, 3rd Edition, 2012
2. Ellis Horowitz, SartajSahni and SanguthevarRajasekaran, “Fundamentals of Computer Algorithms”, Galgotia Publications, 2nd edition, 2003