Fig. 9.10 shows a boundary between two isotropic homogeneous linear materials with permeabilities $\mu_1$ and $\mu_2$. The boundary condition on the normal components is determined by allowing the surface to cut a small cylindrical gaussian surface. Applying Gauss's law for the magnetic field from Sec. 8.5,

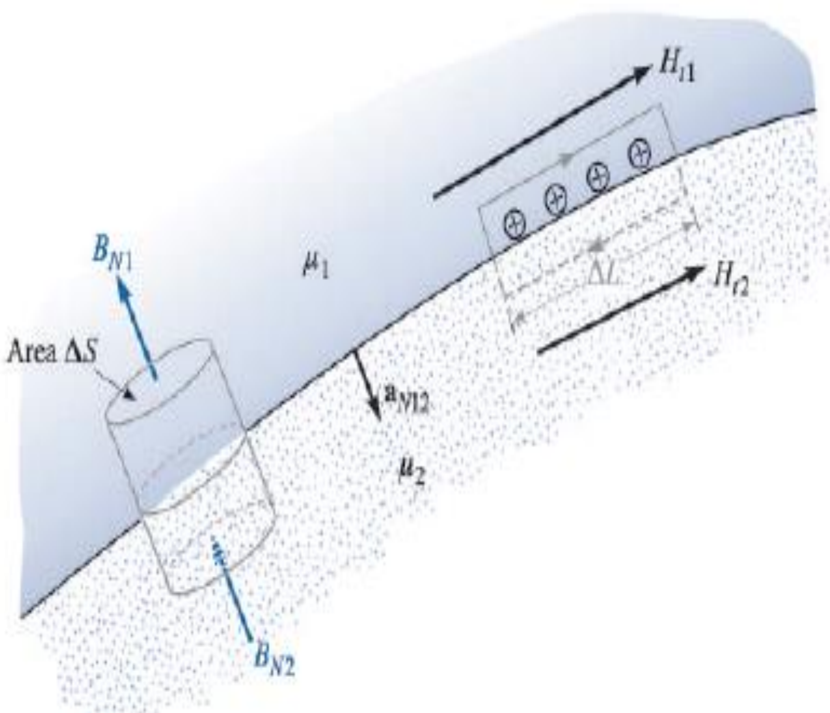
$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

we find that

$$B_{N1}\Delta S - B_{N2}\Delta S = 0$$

or

$$\boxed{B_{N2} = B_{N1}} \qquad (32)$$



Thus

$$H_{N2} = \frac{\mu_1}{\mu_2} H_{N1} \tag{33}$$

The normal component of **B** is continuous, but the normal component of **H** is discontinuous by the ratio $\mu_1/\mu_2$.

The relationship between the normal components of **M**, of course, is fixed once the relationship between the normal components of **H** is known. For linear magnetic materials, the result is written simply as

$$M_{N2} = \chi_{m2} \frac{\mu_1}{\mu_2} H_{N1} = \frac{\chi_{m2}\mu_1}{\chi_{m1}\mu_2} M_{N1} \tag{34}$$

Next, Ampère's circuital law

$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

is applied about a small closed path in a plane normal to the boundary surface, as shown to the right in Fig. 9.10. Taking a clockwise trip around the path, we find that

$$H_{t1}\Delta L - H_{t2}\Delta L = K\Delta L$$

where we assume that the boundary may carry a surface current **K** whose component normal to the plane of the closed path is $K$. Thus

$$\boxed{H_{t1} - H_{t2} = K} \tag{35}$$

The directions are specified more exactly by using the cross product to identify the tangential components,

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{N12} = \mathbf{K}$$

where $\mathbf{a}_{N12}$ is the unit normal at the boundary directed from region 1 to region 2. An equivalent formulation in terms of the vector tangential components may be more convenient for **H**:

$$\mathbf{H}_{t1} - \mathbf{H}_{t2} = \mathbf{a}_{N12} \times \mathbf{K}$$

For tangential **B**, we have

$$\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K \qquad (36)$$

The boundary condition on the tangential component of the magnetization for linear materials is therefore

$$M_{t2} = \frac{\chi_{m2}}{\chi_{m1}} M_{t1} - \chi_{m2} K \qquad (37)$$

The last three boundary conditions on the tangential components are much simpler, of course, if the surface current density is zero. This is a free current density, and it must be zero if neither material is a conductor.