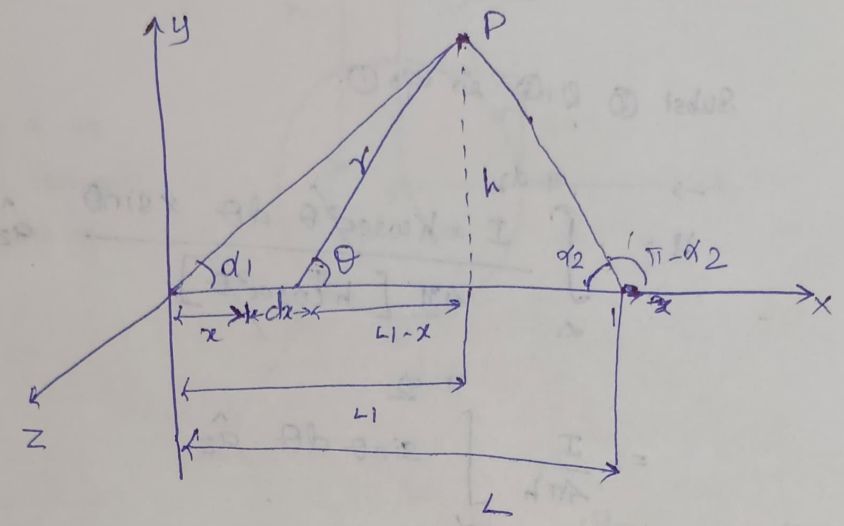
Estimation of Magnetic Field intensity due to straight and circular conductors:

1. H due to finite and infinite length current element.



Consider a straight conductor of length 'L' with current I flowing through it. Consider a point P at height 'h'.

Current element $I\,dx$.

Magnetic field intensity at P due to current element $I\,dx$ is,

$$\vec{dH} = \frac{I\,dx \sin\theta}{4\pi r^2}\,\hat{a_z}$$

Magnetic field intensity due to entire conductor

$$\vec{H} = \int \frac{I\,dx\,\sin\theta}{4\pi r^2}\,\hat{a_z} \rightarrow (1)$$

from figure,

$$\tan\theta = \frac{h}{L_1-x}$$

$$L_1-x = h\cot\theta$$

differentiating, x wrt θ,

$$-dx = h[-\operatorname{cosec}^2\theta\,d\theta)$$

$$dx = h\,\mathrm{cosec}^2\theta\, d\theta \qquad \rightarrow (2)$$

$$\sin\theta = \frac{h}{r}$$

$$r = \frac{h}{\sin\theta} = h\,\mathrm{cosec}\,\theta \qquad \rightarrow (3)$$

Subst (2) & (3) in eqn (1)

$$\vec{H} = \int_{\alpha_1}^{\pi-\alpha_2} \frac{I \times \cancel{h}\,\cancel{\mathrm{cosec}^2\theta}\, d\theta \times \sin\theta}{4\pi\,[h^{\cancel{2}}\,\cancel{\mathrm{cosec}^2\theta}]}\,\hat{a}_z$$

$$= \frac{I}{4\pi h}\int_{\alpha_1}^{\pi-\alpha_2} \sin\theta\, d\theta\,\hat{a}_z$$

$$= \frac{I}{4\pi h}\left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}\hat{a}_z$$

$$= -\frac{I}{4\pi h}\left(\cos(\pi-\alpha_2) - \cos\alpha_1\right)\hat{a}_z$$

$$= -\frac{I}{4\pi h}\left(-\cos\alpha_2 - \cos\alpha_1\right)\hat{a}_z$$

$$\vec{H} = \frac{I}{4\pi h}\left(\cos\alpha_1 + \cos\alpha_2\right)\hat{a}_z$$

For infinitely long conductor, $\alpha_1 = \alpha_2 = 0$

$$\vec{H} = \frac{I}{4\pi h}(2)\,\hat{a}_z$$

$$\vec{H} = \frac{I}{2\pi h}\,\hat{a}_z$$