SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT211 – ELECTROMAGNETIC FIELDS

II YEAR/ IV SEMESTER

UNIT 2 – DIELECTRICS & STEADY ELECTRIC CURRENT

TOPIC 4 – CAPACITANCE: ISOLATED SPHERE, SPHERICAL SHELLS & ENERGY AND ENERGY DENSITY IN A CAPACITOR

CAPACITANCE – SPHERICAL CAPACITOR

Consider a spherical capacitor formed of two concentric spherical conducting shells of radius a and b. The capacitor is shown in the Fig. 5.18.

The radius of outer sphere is 'b' while that of inner sphere is 'a'. Thus b > a. The region between the two spheres is filled with a dielectric of permittivity ε. The inner sphere is given a positive charge + Q while for the outer sphere it is – Q.

Considering Gaussian surface as a sphere of radius r, it can be obtained that $\bar{E}$ is in radial direction and given by,

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r \text{ V/m}$$

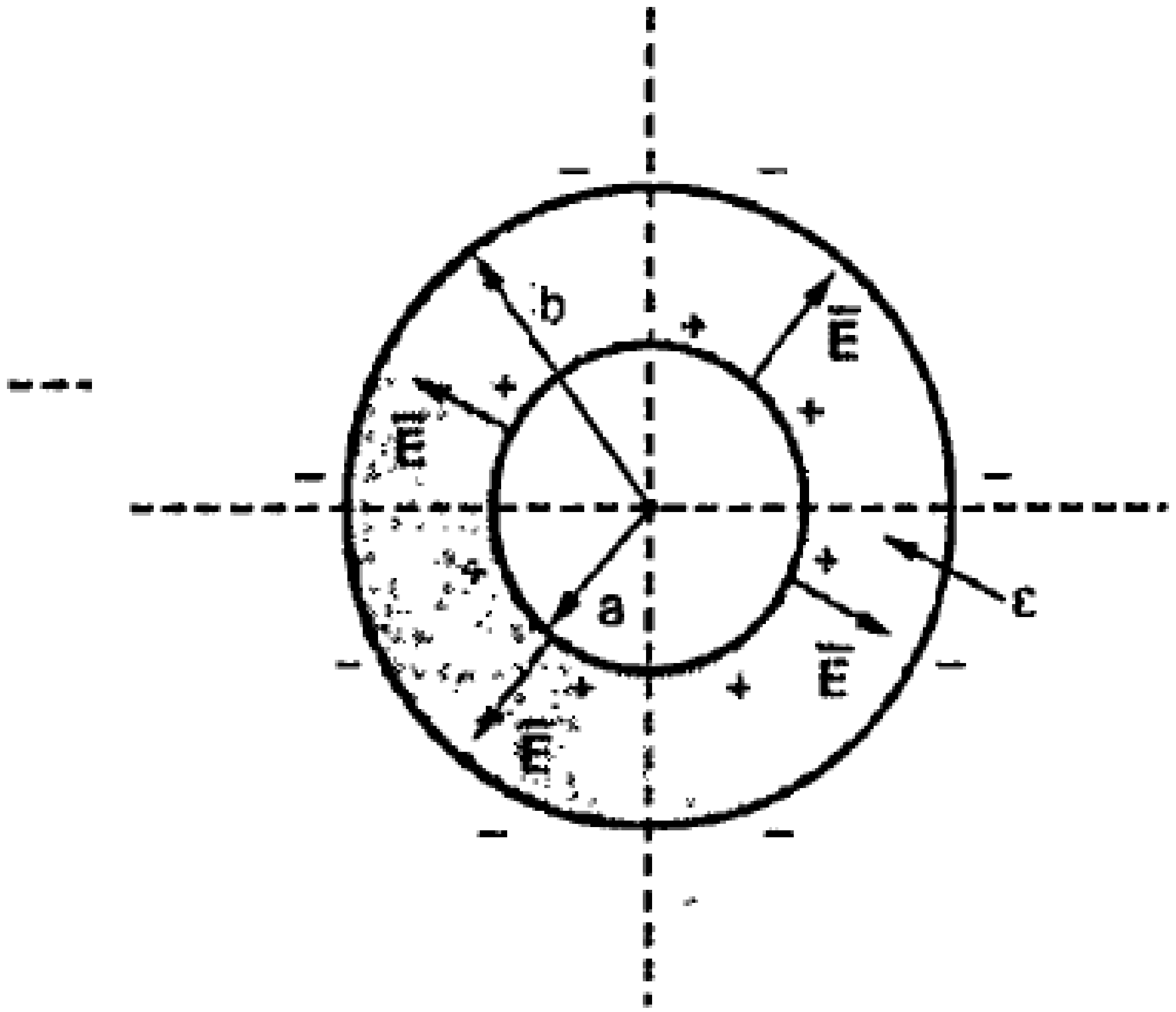

Fig. 5.18 Spherical capacitor

CAPACITANCE – SPHERICAL CAPACITOR

The potential difference is work done in moving unit positive charge against the direction of $\bar{E}$ i.e. from r = b to r = a.

$$\therefore \quad V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r \cdot d\bar{L} \quad \dots (2)$$

CAPACITANCE – SPHERICAL CAPACITOR

Now $d\bar{L} = dr\,\bar{a}_r$ ... In radial direction

$$\therefore \quad V = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r \bullet dr\,\bar{a}_r = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{r=b}^{r=a} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]_{r=b}^{r=a}$$

$$\therefore \quad V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right] V \qquad \text{... (3)}$$

Now

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]}$$

$$\therefore \quad \boxed{C = \frac{4\pi\varepsilon}{\left[\frac{1}{a}-\frac{1}{b}\right]}\ F} \qquad \text{... (4)}$$

CAPACITANCE – SINGLE ISOLATED SPHERE

Consider a single isolated sphere of radius 'a', given a charge + Q. It forms a capacitance with an outer plate which is infinitely large hence b = ∞.

The capacitance of such a single isolated spherical conductor can be obtained by substituting b = ∞ in the equation (4).

$$\therefore \qquad C = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{\infty}\right]} \qquad \text{but } \frac{1}{\infty} = 0$$

$$\therefore \qquad \boxed{C = 4\pi\varepsilon a \text{ F}} \qquad \dots (5)$$

This is the case of a spherical conductor at a large distance from other conductors. Practically this fact is important in obtaining the stray capacitance of an isolated body.

CAPACITANCE – SINGLE ISOLATED SPHERE COATED WITH DIELECTRIC

Consider a single isolated sphere coated with a dielectric having permittivity ε_1, upto radius r_1. The radius of inner sphere is 'a' as shown in the Fig. 5.19. It is placed in a free space so outside sphere $\varepsilon = \varepsilon_0$. It carries a charge + Q.

So for $a < r < r_1, \varepsilon = \varepsilon_1$

and for $r > r_1, \varepsilon = \varepsilon_0$.

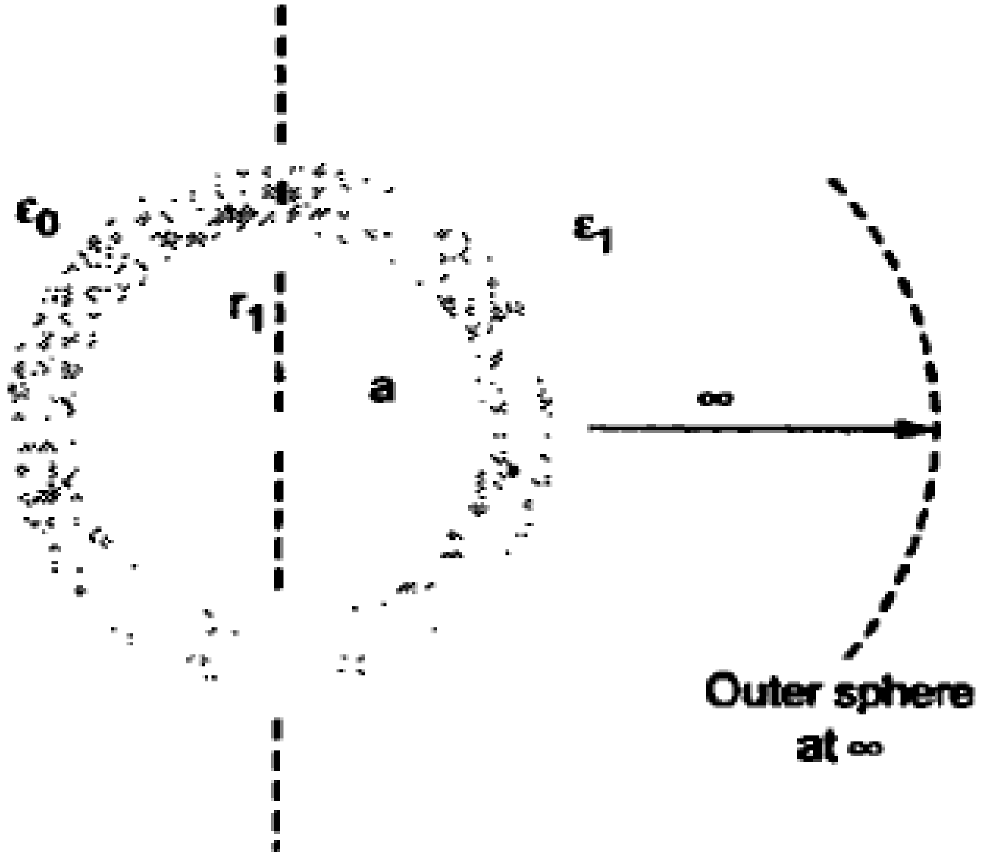

Fig. 5.19

CAPACITANCE – SINGLE ISOLATED SPHERE COATED WITH DIELECTRIC

The potential difference is work done in bringing unit positive charge from outer sphere r = ∞ to inner sphere r = a against $\bar{E}$. This is to be splitted in two as,

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{r=\infty}^{r=a} \bar{E} \cdot d\bar{L}$$

$$\therefore \quad V = -\int_{r=\infty}^{r=r_1} \bar{E} \cdot d\bar{L} - \int_{r=r_1}^{r=a} \bar{E} \cdot d\bar{L} \quad \dots (6)$$

Now for $a < r < r_1$,

$$\bar{E}_1 = \frac{Q}{4\pi\varepsilon_1 r^2} \bar{a}_r \quad \dots (7)$$

Now for $r_1 < r < \infty$,

$$\bar{E}_2 = \frac{Q}{4\pi\varepsilon_0 r^2} \bar{a}_r \quad \dots (8)$$

while $d\bar{L} = dr\,\bar{a}_r$ as $\bar{E}_1$ and E_2 are in radial direction.

CAPACITANCE – SINGLE ISOLATED SPHERE COATED WITH DIELECTRIC

$$C = \frac{4\pi}{\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right)+\frac{1}{\varepsilon_0 r_1}\right]} F$$

ENNERGY STORED IN A CAPACITOR

It is seen that capacitor can store the energy. Let us find the expression for the energy stored in a capacitor.

Consider a parallel plate capacitor as shown in the Fig. 5.25. It is supplied with the voltage V.

Let $\bar{a}_N$ is the direction normal to the plates.

$$\therefore \quad \bar{E} = \frac{V}{d}\,\bar{a}_N \qquad \ldots (1)$$

The energy stored is given by,

$$W_E = \frac{1}{2}\int_{vol} \bar{D}\cdot\bar{E}\,dv$$

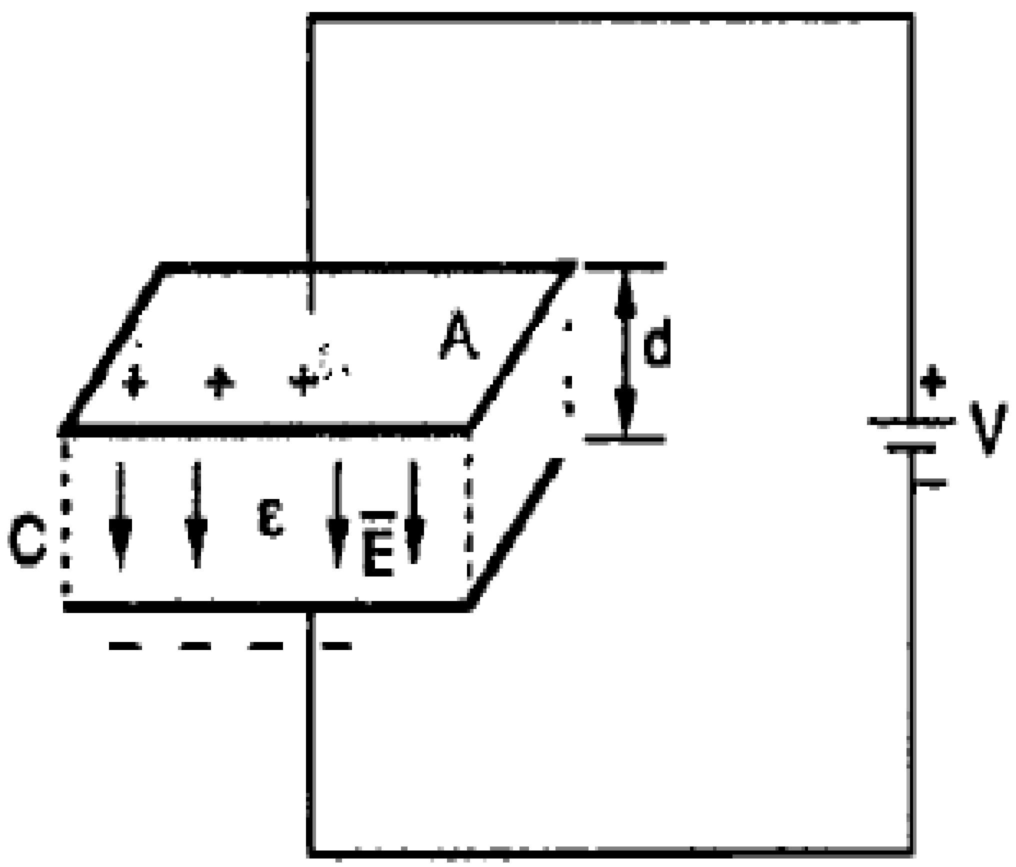

Fig. 5.25 Parallel plate capacitor

ENNERGY STORED IN A CAPACITOR

$$= \frac{1}{2}\int_{vol} \varepsilon \overline{E}\cdot\overline{E}\, dv \qquad \text{but} \quad \overline{E}\cdot\overline{E} = \left|\overline{E}\right|^2$$

$$= \frac{1}{2}\int_{vol} \varepsilon \left|\overline{E}\right|^2 dv \qquad \text{but} \quad \left|\overline{E}\right| = \frac{V}{d}$$

$$= \frac{1}{2}\varepsilon \frac{V^2}{d^2}\int_{vol} dv \qquad \text{but} \quad \int_{vol} dv = \text{Volume} = A \times d$$

$$= \frac{1}{2}\varepsilon \frac{V^2 A d}{d^2} = \frac{1}{2}\frac{\varepsilon A}{d}V^2$$

.

$$\boxed{W_E = \frac{1}{2}CV^2 \ J}$$

ENNERGY DENSITY

As seen in earlier chapter, energy density is the energy stored per unit volume as volume tends to zero.

$$\therefore \quad W_E = \frac{1}{2}\varepsilon \int_{Vol} |\bar{E}|^2 \, dv$$

$$\therefore \quad W_E = \frac{1}{2}\varepsilon |\bar{E}|^2 \ \text{J/m}^3 = \text{Energy density}$$

Using $|\bar{D}| = \varepsilon |\bar{E}|$ we can write,

$$\boxed{W_E = \frac{1}{2}\frac{|\bar{D}|^2}{\varepsilon} = \frac{1}{2}|\bar{D}||\bar{E}| \ \text{J/m}^3}$$

ENNERGY DENSITY

As seen in earlier chapter, energy density is the energy stored per unit volume as volume tends to zero.

$$\therefore \qquad W_E = \frac{1}{2}\varepsilon \int_{Vol} \left|\bar{E}\right|^2 dv$$

$$\therefore \qquad W_E = \frac{1}{2}\varepsilon \left|\bar{E}\right|^2 \ J/m^3 = \text{Energy density}$$

Using $\left|\bar{D}\right| = \varepsilon\left|\bar{E}\right|$ we can write,

$$W_E = \frac{1}{2}\frac{\left|\bar{D}\right|^2}{\varepsilon} = \frac{1}{2}\left|\bar{D}\right|\left|\bar{E}\right| \ J/m^3$$

REFERENCES

- John.D.Kraus , “ Electromagnetics “,5th Edition , Tata McGraw Hill, 2010
- W. H.Hayt & J A Buck: “Engineering Electromagnetics” Tata McGraw-Hill, 7th Edition 2007

THANK YOU