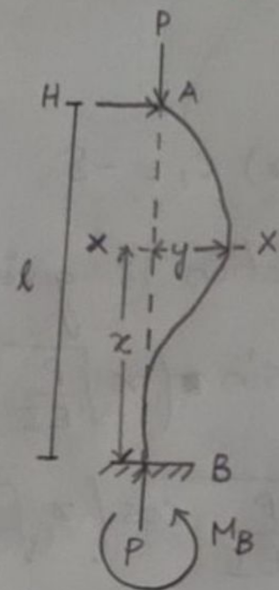
Considering, the first practical Value

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

Case III : When one end of the Column is fixed & other end pinned or Hinged.



The fig shows a column AB of Length l, whose Upper end A is hinged. While its Lower End is fixed.

Let P be the Crippling Load, studying the nature of bending, We realize that there will be a restraint moment M_B, at the Lower fixed end. The existence of restraint moment therefore justifies the need of horizontal force also at the top end A without which no bending moment can occur at B.

Hence, the hinge at A must exert a horizontal force H at A.

Consider any Section XX at a distance x from the lower fixed end B,

The bending moment at the Section is given by,

$$EI.\frac{d^2y}{dx^2} = -Py + H(l-x)$$

$$\therefore EI.\frac{d^2y}{dx^2} + Py = H(l-x)$$

The Solution to the above differential Eqn,

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x.\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l-x).$$

Where, C_1 & C_2 are Constants of Integration.

The Slope at any Section is given by,

$$\frac{dy}{dx} = -C_1.\sqrt{\frac{P}{EI}}\sin\left(x.\sqrt{\frac{P}{EI}}\right) + C_2\sqrt{\frac{P}{EI}}.\cos\left(x.\sqrt{\frac{P}{EI}}\right) - \frac{H}{P}$$

At B, the deflection is Zero.

At $x = 0$, $y = 0$.

$$0 = C_1 + \frac{H}{P}l.$$

$$\therefore C_1 = -\frac{H}{P} \cdot l.$$

At B, the slope is Zero.

At $x=0$, $\frac{dy}{dx} = 0$

$$0 = C_2\sqrt{\frac{P}{EI}} - \frac{H}{P} \text{ (or) } C_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}$$

At A, the deflection is Zero,

At $x = l$, $y = 0$

$$0 = -\frac{H}{P} l \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(l \cdot \sqrt{\frac{P}{EI}}\right)$$

Simplifying, We get

$$\tan\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \left(l \cdot \sqrt{\frac{P}{EI}}\right)$$

The Solution to this eqn is

$$l\sqrt{\frac{P}{EI}} = 4.5 \text{ radians.}$$

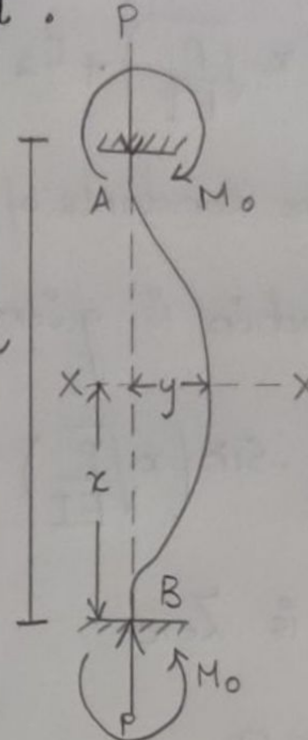
$$\frac{l^2 P}{EI} = (4.5)^2 = 20.25$$

$$P = \frac{20.25\, EI}{l^2}$$

Approximately $20.25 = 2\pi^2$

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

Case IV : When both ends of the Column are Fixed.



This fig shows a Column AB of Length l, whose both the ends A & B are fixed. Obviously there will be a restraint moment say M_o at each end. Let, P be the Crippling Load.

Considering any section XX distant x from the Lower end B. The Bending Moment at the Section XX, given by,

$$EI \cdot \frac{d^2y}{dx^2} = M_o - Py$$

$$EI \cdot \frac{d^2y}{dx^2} + Py = M_o$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_o}{EI}$$

The Soln: to the above differential eqn:

$$y = C_1 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

Where, C_1 & C_2 are Constants of Integration.

The slope at any Section is given by,

$$\frac{dy}{dx} = -C_1 \cdot \sqrt{\frac{P}{EI}} \cdot \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right)$$

At B, the deflection is Zero

At, $x = 0$, $y = 0$

$$0 = C_1 + \frac{M_0}{P} \text{ (or) } \boxed{C_1 = -\frac{M_0}{P}}$$

At B, the slope is Zero

At, $x = 0$, $\frac{dy}{dx} = 0$

$$0 = C_2 \sqrt{\frac{P}{EI}} \text{ (or) } \boxed{C_2 = 0}$$

At A, the deflection is Zero,

At, $x = l$, $y = 0$

$$0 = \frac{-M_0}{P} \cos\left(l \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

$$\frac{M_0}{P}\left[1 - \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right)\right] = 0$$

$$\cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = 1$$

$$l \times \sqrt{\frac{P}{EI}} = 0, 2\pi, 4\pi, 6\pi$$

Considering first practical Value,

$$l\sqrt{\frac{P}{EI}} = 2\pi$$

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$