Coimbatore-35
An Autonomous Institution

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RTMENT OF INFORMATION TECHNOLOGY

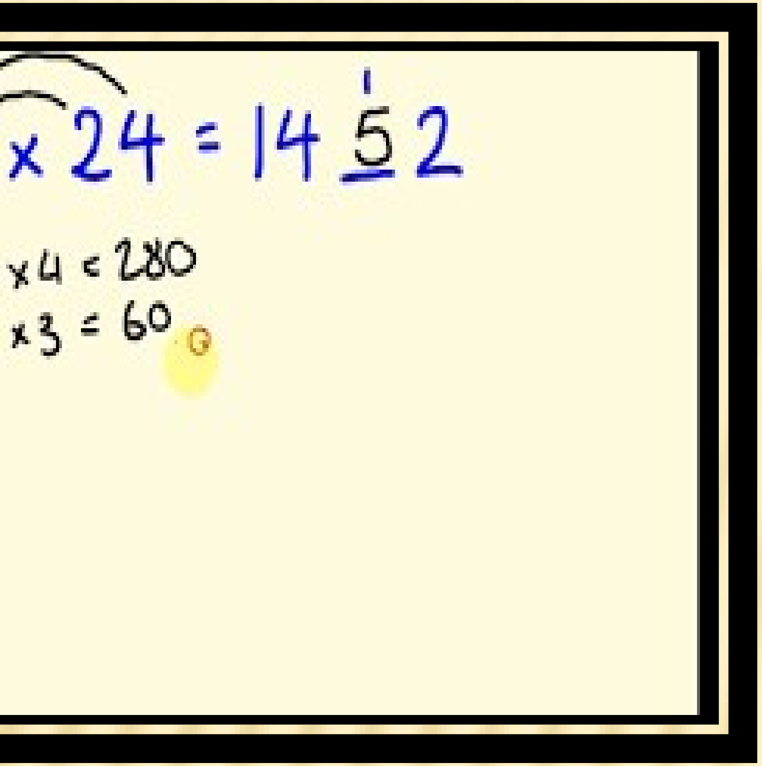
TB201 – Design and Analysis of Algorithms

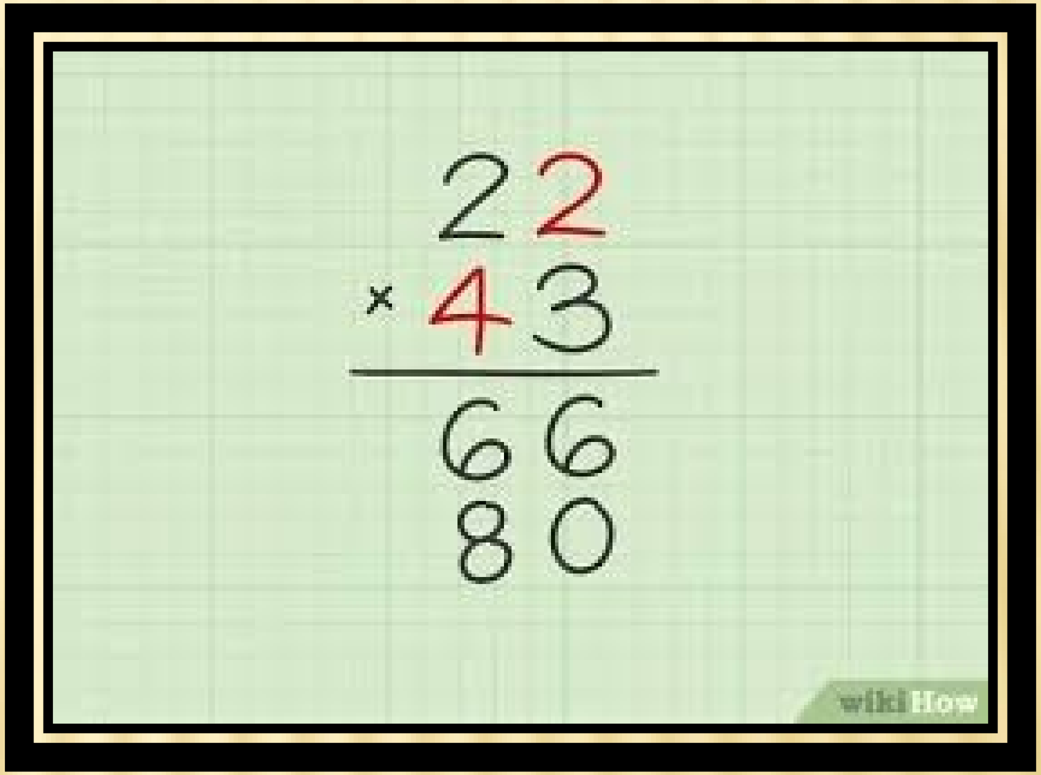
II YEAR IV SEM

UNIT 2 – Brute Force and Divide and Conquer

– Divide and Conquer-Multiplication of large Integers

ltiplying Two digit integers





ltiplying Four digit integers

	T	U	
	4	3	
	1	2	
	8	6	(43 x 2)
4	3	0	(43 x 1)
5	1	6	

```
       1 2 7 2
     x 1 3 6 4
   -----------
       5 0 8 8
     7 6 3 2 0
   3 8 1 6 0 0
+ 1 2 7 2 0 0 0
   -----------
 1 7 3 5 0 0 8
```

er the problem of multiplying two n-digit inte

ated by arrays of their digits such as $a = a_1 a_2$..

de-school algorithm:

$a_n \; b_1 \quad b_2 \ldots \quad b_n$

$d_{12} \ldots d_{1n}$

$d_{22} \ldots d_{2n}$

$\ldots \; \ldots$

$d_{n2} \ldots d_{nn}$

EXAMPLE

2-digit integers $a = 23$ and $b = 14$ can be

ted as follows:

$^1 + 3.10^0$ and $14 = 1.10^1 + 4.10^0$.

s multiply them:

$(2.10^1 + 3.10^0) * (1.10^1 + 4.10^0)$

$0^2 + (2 * 4 + 3 * 1)10^1 \quad + \; (3 * 4)10^0$

ula uses four digit multiplications (i.e., n^2).

erm can be computed by

1 = (2 + 3) * (1 + 4) - 2 * 1 - 3 * 4.

any multiplications? 1 or 3

-digit integers $a = a_1a_0$ and $b = b_1b_0$, th
can be computed by

$c_2 10^2 + c_1 10^1 + c_0$,

is the product of their first digits,

b_0 is the product of their second digits,

+ a_0) * (b_1 + b_0) - (c_2 + c_0) is the product

the a's digits and the sum of the b's digit

of c_2 and c_0.

eral, for two n-digit integers a and b where n
ve even number. We denote the first half of th
by a_1 and the second half by a_0; for b, the notat
and b_0, respectively. In these notations, a =
s that $a = a_1 10^{n/2} + a_0$, and $b = b_1 b_0$ implies tha
$^2 + b_0$.

ple: $a = 21|35$, $b = 40|14$

ITHM

$= (a_1 10^{n/2} + a_0) * (b_1 10^{n/2} + b_0)$

$_1)10^n + (a_1 * b_0 + a_0 * b_1)10^{n/2} + (a_0 * b_0)$

$c_1 10^{n/2} + c_0,$ where

b_1 is the product of their first halves,

e-and-Conquer Algorithm

$* b_0$ is the product of their second halves,

$_1 + a_0) * (b_1 + b_0) - (c_2 + c_0)$ is the product of the su

halves and the sum of the b's halves minus the su

c_0.

nstead of using 4 multiplications to

e $a * b$, we just need to compute 3

lications (i.e., $a_1 * b_1$, $(a_0 * b_0)$, and $(a_1 + a_0)$

b_0)).

e and conquer Algorithm

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we have recursive algorithm for computing prod
egers. The recursion is stopped when n becomes o

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e and Conquer Algorithm

tiplication of n-digit numbers requires three mult
git numbers, the recurrence for the nu
tions $M(n)$ will be

$(n/2)$ for $n > 1$, $M(1) = 1$.

e and conquer Algorithm

g it by backward substitutions for n =

$= 3M(2^{k-1}) = 3[3M(2^{k-2})] = 3^2M(2^{k-2})$

$3^iM(2^{k-i}) = \ldots = 3^kM(2^{k-k}) = 3^k$.

$k = \log_2 n$,

$$M(n) = 3^{\log_2 n} = n^{\log_2 3} \approx n^{1.585} < n^2.$$

e running time of multiplication of large integers
nquer?

er multiplication cant solved by using brute force.


Thank You