1. Solve $[D^2 - 5D + 6]y = 0$

Solution :-

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$[D^2 - 5D + 6]y = 0$$

Replace :

$D^2 = m^2 \quad D = m \quad y = 1$

Auxillary Equation (A.E)

$$m^2 - 5m + 6 = 0$$

$$\begin{array}{c} 6 \\ -3 \quad 2 \\ -5 \end{array}$$

$$(m-3)(m-2) = 0$$

$\boxed{m = 3}$ $\boxed{m = 2}$

∴ Therefore values are different therfore,

$$C.F = Ae^{m_1x} + Be^{m_2x}$$

$$\boxed{C.F = Ae^{3x} + Be^{2x}}$$

---

2. Solve : $[D^2 + D + 1]y = 0$

Solution :-

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1y = 0$$

$$[D^2 + D + 1]y = 0$$

Replace :-

$D^2 = m^2 \quad D = m \quad y = 1$

Auxillary Equation (A.E)

$$m^2 + m + 1 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1 \quad b = 1 \quad c = 1$

$$m = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{i^2 3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$= -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$= (\alpha \pm i\beta)$$

$\boxed{\alpha = -\frac{1}{2}}$; $\boxed{\beta = \sqrt{3}/2}$

Values are imagenary therefore.

$$C.F = e^{\alpha x}\left[A \cos \beta x + B \sin \beta x\right]$$

$$\boxed{C.F = e^{-1/2 x}\left[A \cos \sqrt{3}/2\, x + B \sin \sqrt{3}/2\, x\right]}$$ ~~.

3. Solve : $[D^2+4]y=0$

Solution:-

$$\frac{d^2y}{dx^2}+4y=0$$

$$[D^2+4]y=0$$

Replace :-

$$D^2=m^2 \qquad y=1.$$

Auxillary Equation (A.E)

$$m^2+4=0$$

$$m^2=-4$$

$$m=\pm\sqrt{-4}$$

$$m=\pm\sqrt{i^2 4}$$

$$m=\pm 2i$$

$$m=0\pm 2i$$

$$\alpha=0 \qquad \beta=2i$$

$$(\alpha\pm\beta i)$$

$$C.F=e^{\alpha x}[A\cos\beta x+B\sin\beta x]$$

$$C.F=e^{0x}[A\cos 2x+B\sin 2x]$$

$e^0=0$

$$\boxed{C.F=[A\cos 2x+B\sin 2x}$$

4. Solve : $[D^2-4]y=0$

Solution:-

$$\frac{d^2y}{dx}-4y=0$$

$$[D^2-4]y=0$$

Replace :-

$$D^2=m^2 \qquad y=1$$

Auxillary Equation (A.E)

$$m^2-4=0$$

$$m^2=4$$

$$m=\pm\sqrt{4}$$

$$m=\pm 2$$

$$\boxed{m=2} \quad \boxed{m=-2}$$

Values are different therefore

$$C.F=Ae^{m_1x}+Be^{m_2x}$$

$$\boxed{C.F=Ae^{2x}+Be^{-2x}}$$

Solve $[D^2 + 7D + 12]y = 0$

Solution :-

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 12y = 0$$

$$[D^2 + 7D + 12]y = 0$$

Replace :-

$D^2 = m^2 \quad D = m \quad y = 1.$

Auxiliary Equation (A.E)

$$m^2 + 7m + 12 = 0$$

12
4 3
7

$$(m+4)(m+3) = 0$$

$$\boxed{m = -4} \quad \boxed{m = -3}$$

Values are different therefore,

$$C.F = Ae^{m_1 x} + Be^{m_2 x}$$

$$\boxed{C.F = Ae^{-4x} + Be^{-3x}}$$

---

6. $[D^2 + 4D + 4]y = 0$

Solution :-

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = 0$$

$$[D^2 + 4D + 4]y = 0$$

Replace :-

$D^2 = m^2 \quad D = m \quad y = 1.$

Auxiliary Equation (A.E)

4
2 2
4

$$m^2 + 4m + 4 = 0$$

$$(m+2)(m+2) = 0$$

$$\boxed{m = -2} \quad \boxed{m = -2}$$

values are Equal ∴ therefore,

$$C.F = e^{mx}[A+Bx]$$

$$C.F = e^{-2x}[A+Bx]$$

Particular Intergaral. (Trignamatric include function)

$$P.I = \frac{1}{f(D)} \; R.H.S.$$

Failure:-

$$P.I = \frac{x}{f'(D)} \; R.H.S$$

Again failure:-

$$P.I = \frac{x^2}{f''(D)} \; R.H.S.$$

Types:-

$$R.H.S = e^{ax}$$

Rule : Replace $D=a$

1. Solve : $(D^2-6D+5)y = e^{3x}$ (or) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = e^{3x}$

Sollution:-

$$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 5y = e^{3x} \Rightarrow (D^2-6D+5)y = e^{3x}$$

Auxillary Equation:-

$D^2 = m^2$ $D = m$ $y = 1.$

$m^2 - 6m + 5 = 0$

5: -1, -5

$(m-1)(m-5) = 0$

$\boxed{m=1}$ $\boxed{m=5}$

values are different therefore

$$C.F = Ae^{m_1x} + Be^{m_2x}$$

$$\boxed{C.F = Ae^{1x} + Be^{5x}}$$

$$P.I = \frac{1}{f(D)} \cdot RHS$$

$$P.I = \frac{1}{D^2 - 6D + 5} \cdot e^{3x}$$

Replace $D = a = 3$

$$P.I = \frac{1}{(3)^2 - 6(3) + 5} \cdot e^{3x}$$

$$= \frac{1}{9 - 18 + 5} \cdot e^{3x}$$

$$\boxed{P.I = \frac{1}{-4} \cdot e^{3x}}$$

$$y = C.F + P.I$$

$$= \left[Ae^{1x} + Be^{5x}\right] + \left[\frac{1}{-4} \cdot e^{3x}\right]$$

---

2) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = e^{2x} + 4$ (or) $[D^2 + 4D + 8]y = e^{2x} + 4$

Solution:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = e^{2x} + 4$$

$$[D^2 + 4D + 8]y = e^{2x} + 4$$

Auxiliary equation :-

$D^2 = m^2$ $D = m$ $y = 1.$

$$m^2 + 4m + 8 = 0$$

$a = 1 \quad b = 4 \quad c = 8$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 32}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$$= \frac{-4 \pm \sqrt{i^2 16}}{2}$$

$$= \frac{-4 \pm i \cdot 4}{2}$$

$$= -\frac{\cancel{4}^{2}}{\cancel{2}} \pm \frac{i\cancel{4}^{2}}{\cancel{2}_{1}}$$

$$= (\alpha \pm i\beta)$$

$\boxed{\alpha = -2} \quad \boxed{\beta = 2}$

$$C.F = e^{\alpha x}\left[A \cos\beta x + B \sin\beta x\right]$$

$$\boxed{C.F = e^{-2x}\left[A \cos(2)x + B \sin 2x\right]}$$

$$P.I_1 = \frac{1}{f(D)} \cdot RHS. \qquad f(D) = D^2 + 4D + 8$$

$$= \frac{1}{(D^2 + 4D + 8)} \cdot e^{2x}$$

$D = a = 2$

$$= \frac{1}{2^2 + 4(2) + 8} \cdot e^{2x}$$

$$= \frac{1}{4 + 8 + 8} \cdot e^{2x}$$

$$= \frac{1}{4+16} \cdot e^{2x}$$

$$\boxed{P.I_1 = \frac{1}{20} e^{2x}}$$

$$P.I_2 = \frac{1}{f(D)} \cdot RHS$$

$$= \frac{1}{D^2+4D+8} \cdot 4e^{0x}$$

$$D = a = 0$$

$$= \frac{1}{0^2+4(0)+8} \cdot 4e^{0x}$$

$$= \frac{1}{\cancel{8}\,2} \cdot \cancel{4}\,e^{0x}$$

$$\boxed{P.I_2 = \frac{1}{2}}$$

$$y = C.F + P.I$$

$$\boxed{y = e^{-2x}[A\cos 2x + B\sin 2x] + \left[\frac{1}{20}e^{2x} + \frac{1}{2}\right]}$$

4-2-23

1. Solve : $(D^2 - 2D + 1)y = 5e^{-x}$

Solution :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 1y = 5e^{-x}$$

$$[D^2 - 2D + 1]y = 5e^{-x}$$

$D^2 = m^2 \quad D = m \quad y = 1$

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$m = 1 \quad m = 1$

Values are equal therfore

$$C.F = e^{mx}[A + Bx]$$

$$\boxed{C.F = e^{1x}[A + Bx]}$$

$$P.I = \frac{1}{f(D)} \cdot RHS$$

$$f(D) = D^2 - 2D + 1$$

$$P.I = \frac{1}{D^2 - 2D + 1} \cdot 5e^{-x}$$

Replace :-

$$D = a = +1$$

$$P.I = \frac{1}{(+1)^2 - 2(+1) + 1} \cdot 5e^{-x}$$

$$= \frac{1}{1 - 2 + 1} \cdot 5e^{-x}$$

$$P.I = \frac{1}{0} \cdot 5e^{-x} \quad \text{(failure)}$$

differciate with respect to f(d)

$$P.I = \frac{x}{f'(D)} \, RHS$$

$$= \frac{x}{2D - 2} \, 5e^{x}$$

Replace $D = a = 1$

$$P.I = \frac{x}{2(1) - 2} \, 5e^{x}$$

$$= \frac{x}{2 - 2} \, 5e^{x}$$

$$= \frac{x}{0} \, 5e^{x} \quad \text{(again failure)}$$

Again differenciate with respect to $f'(D)$

$$P.I = \frac{x^2}{f''(D)} \, RHS$$

$$P.I = \frac{x^2}{2} 5e^x$$

$$\boxed{P.I = \frac{5}{2} x^2 e^x}$$

solution :-

$y = C.F + P.I$

$$\boxed{y = e^{1x}\left[A + Bx\right] + \frac{5}{2} x^2 e^x}$$

solve : $[D^2 - 6D + 9]y = 3e^{3x} + 5e^{-2x}$

solution :-

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 3e^{3x} + 5e^{-2x}$$

$$[D^2 - 6D + 9]y = 3e^{3x} + 5e^{-2x}$$

$D^2 = m^2 \quad D = m \quad y = 1$

$$m^2 - 6m + 9 = 0$$

$$(m-3)(m-3) = 0$$

$\boxed{m = 3}$ $\boxed{m = 3}$

9
-3 | -3
-6

values are Equal therefore,

$$C.F = e^{mx}[A + Bx]$$

$$\boxed{C.F = e^{3x}[A + Bx]}$$

$$P.I_1 = \frac{1}{f(D)} \text{ RHS}$$

$$f(D) = [D^2 - 6D + 9]$$

$$P.I = \frac{1}{D^2 - 6D + 9} \cdot 3e^{3x}$$

Replace : $D = a = 3$

$$P.I = \frac{1}{3^2 - 6(3) + 9} \; 3e^{3x}$$

$$= \frac{1}{9 - 18 + 9} \cdot 3e^{3x}$$

$$= \frac{1}{0} \; 3e^{3x}$$ (failure)

$$P.I = \frac{x}{f'(D)} \; RHS$$

$$= \frac{x}{2D - 6} \; 3e^{3x}$$

$$= \frac{x}{2(3) - 6} \; 3e^{3x}$$

$$= \frac{x}{6 - 6} \; 3e^{3x}$$

$$= \frac{x}{0} \; 3e^{3x}$$

$$P.I = \frac{x^2}{f''(D)} \; RHS$$

$$\boxed{P.I = \frac{x^2}{2D} \; 3e^{3x}}$$

$$P.I_2 = \frac{1}{f(D)} \; RHS$$

$$= \frac{1}{D^2 - 6D + 9} \; 5e^{-2x}$$

Replace $= D = a = -2$

$$P.I = \frac{1}{(-2)^2 - 6(-2) + 9} \; 5e^{-2x}$$

$$= \frac{1}{4 + 12 + 9}$$

$$\boxed{P.I_2 = \frac{1}{25} \; 5e^{-2x}}$$

$y = C.F + P.I$

$y = e^{3x}[A+Bx] + \left[\frac{x^2}{2} 3e^{3x} + \frac{1}{25} 5e^{-2x}\right]$ (Ans)

Type - II. Trignomentry Function

R.H.S = sin ax (or) cos ax

Rule :-

1. Replace $D^2 = -(a)^2$ (More important formula)

1. Solve : $[D^2 - 3D + 2]y = \sin 5x$

Solution :

Complementry function

$$\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = \sin 5x$$

$$[D^2 - 3D + 2]y = \sin 5x$$

$D^2 = m^2 \quad D = m \quad y = 1$

$$m^2 - 3m + 2 = 0$$

$$(m-1)(m-2) = 0$$

$$m = 1 \quad m = 2$$

$$C.F = Ae^{m_1 x} + Be^{m_2 x}$$

$$\boxed{C.F = Ae^{1x} + Be^{2x}}$$

$$P.I = \frac{1}{f(D)} RHS$$

$$P.I = \frac{1}{D^2 - 3D + 2} \sin 5x$$

Replace : $D^2 = -(a)^2$

$D^2 = -(5)^2$

$D^2 = -(25) \Rightarrow D^2 = -25$

$$P.I = \frac{1}{-25-3D+2} \sin 5x$$

$$= \frac{1}{-23-3D} \sin 5x$$

$$= \frac{1}{-3D-23} \sin 5x$$

$$= \frac{1}{-23-3D} \times \frac{-23+3D}{-23+3D} \sin 5x$$

$(a-b)$ $(a+b) = a^2-b^2$

$$= \frac{1(-23+3D)}{(-23)^2-(3D)^2} \sin 5x$$

$$= \frac{-23+3D}{529-9D^2} \sin 5x$$

$$= \frac{-23 \sin 5x + 3D \sin 5x}{529-9(-25)}$$

$$P.I = \frac{-23 \sin 5x + 3(\cos 5x \times 5)}{754}$$

$y = C.F + P.I$

$$y = \left[Ae^{1x} + Be^{2x}\right] + \left[\frac{-23 \sin 5x + 3(\cos 5x \times 5)}{754}\right]$$

Home work Sums:-

$(D^2 + 4D + 4)y = 11e^{-2x}$

Solution:-

$D^2 + 4D + 4y = 11e^{-2x}$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 11e^{-2x}$$

$D^2 = m^2$ $D = m$ $y = 1.$

$$m^2 + 4m + 4 = 0$$

$$(m+2)(m+2) = 0$$

$$\boxed{m = -2} \quad \boxed{m = -2}$$

values are Equal therefore,

$$C.F = e^{mx}[A + Bx]$$

$$\boxed{C.F = e^{-2x}[A + Bx]}$$

$$P.I = \frac{1}{f(D)} RHS$$

$$f(D) = D^2 + 4D + 4$$

$$P.I = \frac{1}{D^2 + 4D + 4} 11e^{-2x}$$

$$D = a = -2$$

$$P.I = \frac{1}{(-2)^2 + 4(-2) + 4} 11e^{-2x}$$

$$= \frac{1}{4 - 8 + 4} 11e^{-2x}$$

$$= \frac{1}{0} 11e^{-2x} \quad \text{(failure)}$$

$$P.I = \frac{x}{f'(D)} RHS$$

$$= \frac{x}{2D + 4} 11e^{-2x}$$

$$= \frac{x}{2(-2) + 4} 11e^{-2x}$$

$$= \frac{x}{-4 + 4} 11e^{-2x}$$

$$P.I = \frac{x}{0} 11e^{-2x} \quad \text{(Again failure)}$$

$$P.I = \frac{x^2}{f''(D)} RHS$$

$$\boxed{P.I = \frac{x^2}{2} 11e^{-2x}}$$

$y = C.F + P.I$

$$y = \left[e^{-2x}\left[A+Bx\right] + \frac{x^2}{2}\, 11e^{-2x}\right]$$

2) $$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 14e^{-3x}$$

Solution :

$$\left[D^2+7D+12\right]y = 14e^{-3x}$$

$D^2 = m^2 \quad D = m \quad y = 1.$

$$m^2 + 7m + 12 = 0$$

12

4 | 3

7

$$(m+4)(m+3) = 0$$

$\boxed{m=-4}$ $\boxed{m=-3}$

values are different therefore,

$$C.F = Ae^{m_1x} + Be^{m_2x}$$

$$\boxed{C.F = Ae^{-4x} + Be^{-3x}}$$

$$P.I_1 = \frac{1}{f(D)} RHS$$

$$f(D) = \left[D^2+7D+12\right]$$

$$= \frac{1}{D^2+7D+12}\, 14e^{-3x}$$

Replace:-

$$D = a = -3$$

$$= \frac{1}{(-3)^2+7(-3)+12}\, 14e^{-3x}$$

$$= \frac{1}{9-21+12}\, 14e^{-3x}$$

$$= \frac{1}{21-21}\, 14e^{-3x}$$

$$P.I_1 = \frac{1}{0}\, 14e^{-3x} \text{ (failure)}$$

$$P.I_2 = \frac{x}{f'(D)} \text{ RHS}$$

$$= \frac{x}{2D+7} \; 14e^{-3x}$$

$$= \frac{x}{2(-3)+7} \; 14e^{-3x}$$

$$= \frac{x}{-6+7} \; 14e^{-3x}$$

$$P.I_2 = \frac{x}{1} \; 14e^{-3x}$$

$$y = C.F + P.I$$

$$\boxed{y = \left[Ae^{-4x} + Be^{-3x} + \frac{x}{1} \; 14e^{-3x}\right]}$$

3. $[D^2 + 6D + 5]y = e^{3x} - 5e^{-x}$

solution :

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = e^{3x} - 5e^{-x}$$

$$[D^2 + 6D + 5]y = e^{3x} - 5e^{-x}$$

$$D^2 = m^2 \quad D = m \quad y = 1.$$

$$m^2 + 6m + 5 = 0$$

$$(m+1)(m+5) = 0$$

5
1 5
6

$$\boxed{m = -1} \; \boxed{m = -5}$$

Values are different therefore

$$C.F = Ae^{m_1x} + Be^{m_2x}$$

$$\boxed{C.F = Ae^{-1x} + Be^{-5x}}$$

$$P.I_1 = \frac{1}{f(D)} \text{ RHS}$$

$$f(D) = D^2 + 6D + 5$$

Replace :

$$D = a = 3$$

$$P.I_1 = \frac{1}{D^2+6D+5} e^{3x}$$

$$= \frac{1}{(3)^2+6(3)+5} e^{3x}$$

$$= \frac{1}{9+18+5} e^{3x}$$

$$= \frac{1}{17+5} e^{3x}$$

$$\boxed{P.I_1 = \frac{1}{22} e^{3x}}$$

$$P.I_2 = \frac{1}{f(D)} R.HS$$

$$= \frac{1}{D^2+6D+5} 5e^{-x}$$

Replace:

$$D = a = -1.$$

$$= \frac{1}{(-1)^2+6(-1)+5} 5e^{-x}$$

$$= \frac{1}{1-6+5} 5e^{-x}$$

$$= \frac{1}{6-6} 5e^{-x}$$

$$= \frac{1}{0} 5e^{-x} \text{ (failure)}$$

$$P.I = \frac{x}{f'(D)} RHS$$

$$= \frac{x}{2D+6} 5e^{-x}$$

$$= \frac{x}{2(-1)+6} 5e^{-x}$$

$$= \frac{x}{-2+6} 5e^{-x}$$

$$\boxed{P.I_2 = \frac{x}{4} 5e^{-x}}$$

$y =$ C.F + P.I

$$y = \left[Ae^{-1x} + Be^{-5x}\right] + \left[\frac{1}{22}e^{3x} - \frac{x}{4}5e^{-x}\right]_{//} \quad (\text{Ans})$$

Solve:

$$[D^2 - D + 1]y = e^{2x} - 5$$

Solution:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1y = e^{2x} - 5$$

$$[D^2 - D + 1]y = e^{2x} - 5$$

$$D^2 = m^2 \quad D = m \quad y = 1.$$

$$m^2 - m + 1 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1 \quad b = -1 \quad c = 1.$$

$$m = \frac{(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{i^2(-3)}}{2}$$

$$= \frac{-1 \pm \sqrt{i^2 3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$= \frac{-1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$= (\alpha \pm i\beta) \qquad \alpha = -1/2 \; ; \; \beta = \sqrt{3}/2$$

Values are imaginary therefore,

$$C.F = e^{\alpha x}\left[A\cos\beta x + B\sin\beta x\right]$$

$$C.F = e^{-1/2 x}\left[A\cos\sqrt{3}/2\, x + B\sin\sqrt{3}/2\, x\right]$$

$$P.I_1 = \frac{1}{f(D)}\ RHS$$

$$f(D) = D^2 - D + 1$$

$$= \frac{1}{D^2 - D + 1}\, e^{2x}$$

Replace:

$$D = a = 2$$

$$= \frac{1}{(2)^2 - 2 + 1}\, e^{2x}$$

$$= \frac{1}{4 - 2 + 1}\, e^{2x}$$

$$= \frac{1}{2 + 1}\, e^{2x}$$

$$\boxed{P.I_1 = \frac{1}{3}\, e^{2x}}$$

$$P.I_2 = \frac{1}{f(D)}\ RHS$$

$$= \frac{1}{D^2 - D + 1}\ -5e^{0x}$$

Replace $= D = a = 0$

$$= \frac{1}{(0)^2 - (0) + 1}\ -5e^{0x}$$

$$\boxed{P.I_2 = \frac{1}{1}\ -5e^{0x}}$$

$$y = C.F + P.I$$

$$y = \left[e^{-1/2x}\left[A\cos\sqrt{3}/2\, x + B\sin\sqrt{3}/2\, x\right] + \left[1/3\, e^{2x} - \frac{1}{1}\, 5e^{0x}\right]\right]$$

(Ans)

6. Solve : $(D^2+4D+2)y = \sin 3x$

Solution :-

$$(D^2+4D+2)y = \sin 3x$$

$$D^2 = m^2 \quad D = m \quad y = 1.$$

$$m^2 + 4m + 2 = 0$$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$a = 1 \quad b = 4 \quad c = 2$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-8}}{2}$$

$$= \frac{-4 \pm \sqrt{8}}{2}$$

$$= \frac{-4}{2} \pm \frac{\sqrt{8}}{2}$$

$$= -2 \pm \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$$

$$= -2 \pm \sqrt{2}$$

$$\alpha = -2+\sqrt{2} \qquad \beta = -2-\sqrt{2}$$

Values are different, therefore

$$C.F = Ae^{m_1 x} + Be^{m_2 x}$$

$$C.F = Ae^{-2+\sqrt{2}} + Be^{-2-\sqrt{2}}$$

$$P.I = \frac{1}{f(D)} \cdot RHS$$

$$f(D) = D^2+4D+2$$

$$= \frac{1}{D^2+4D+2} \sin 3x$$

Replace : ~~D = a = 3~~

$$D^2 = -(a)^2$$

$$D^2 = -(3)^2$$

$$D^2 = -(9)$$

$$D^2 = -9$$

$$P.I = \frac{1}{(-9)^2 + 4(D) + 2} \sin 3x$$

$$= \frac{1}{81 + 4D + 2} \sin 3x$$

$$= \frac{1}{83 + 4D} \sin 3x$$

$$= \frac{1}{83+4D} \sin 3x$$

$$= \frac{1}{83+4D} \times \frac{83-4D}{83-4D} \sin 3x$$

$$= \frac{1(83-4D)}{(83)^2 - (4D)^2} \sin 3x$$

$$= \frac{(83-4D) \sin 3x}{6889 - 4(-9)}$$

$$= \frac{83 \sin 3x - 4D \sin 3x}{6889 + 36}$$

$$= \frac{83 \sin 3x - 4(\cos 3x \cdot 3)}{6925}$$

$$P.I = \frac{83 \sin 3x - 36 \cos 3x}{6925}$$

$$y = C.F + P.I$$

$$y = \left[Ae^{-2+\sqrt{2}} + Be^{-2-\sqrt{2}}\right] + \left[\frac{83 \sin 3x - 36 \cos 3x}{6925}\right]$$

6. $[D^2 - 7D + 12]y = e^{5x} + \cos 2x.$

Solution:-

$D^2 = m^2 \quad D = m \quad y = 1.$

$m^2 - 7m + 12 = 0$

$(m-3)(m-4) = 0$

$\boxed{m = 3} \quad \boxed{m = 4}$

$C.F = Ae^{m_1 x} + Be^{m_2 x}$

$C.F = Ae^{3x} + Be^{4x}$

$$P.I = \frac{1}{f(D)} RHS$$

$$= \frac{1}{D^2 - 7D + 12} e^{5x} + \cos 2x$$

$$P.I_1 = \frac{1}{D^2 - 7D + 12} e^{5x}$$

Replace : $D = a = 5$

$$PI_1 = \frac{1}{(5)^2 - 7D + 12} e^{5x}$$

$$= \frac{1}{25 - 7(5) + 12} e^{5x}$$

$$PI_1 = \frac{1}{\cancel{25} - 35 + 12} e^{5x}$$

$$= \frac{1}{37 - 35} e^{5x}$$

$$\boxed{PI_1 = \frac{1}{2} e^{5x}}$$

$$PI_2 = \frac{1}{D^2 - 7D + 12} \cos 2x$$

Replace : $D^2 = -(a)^2$

$= -(2)^2$

$D^2 = -4$

$$PI_2 = \frac{1}{-4 - 7D + 12} \cos 2x$$

$$= \frac{1}{8 - 7D} \cos 2x$$

$$= \frac{1}{8 - 7D} \times \frac{8 + 7D}{8 + 7D} \cos 2x$$

$$= \frac{1(8 + 7D)}{(8)^2 - (7D)^2} \cos 2x$$

$$= \frac{(8 + 7D) \cos 2x}{64 - 49D^2}$$

$$= \frac{8\cos 2x + 7D\cos 2x}{64 - 49(-4)^2}$$

$$= \frac{8\cos 2x + 7D\cos 2x}{64 + 49(16)}$$

$$= \frac{8\cos 2x + 7D\cos 2x}{860}$$

$$= \frac{8\cos 2x + 7(-\sin 2x \cdot 2)}{860}$$

$$\boxed{P.I_2 = \frac{8\cos 2x - 14\sin 2x}{860}}$$

$y = C.F + PI_1 + PI_2$

$$y = \left[Ae^{3x} + Be^{4x}\right] + \left[\frac{1}{2}e^{5x} + \frac{8\cos 2x - 14\sin 2x}{860}\right]$$

$$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 3e^{3x} + 5\sin 2x$$

Solution :-

$[D^2 - 6D + 9]y = 3e^{3x} + 5\sin 2x$

$D^2 = m^2 \quad D = m \quad y = 1.$

$m^2 - 6m + 9 = 0$

9

$-3 \quad -3$

$(m-3)(m-3) = 0$

$\boxed{m = 3}$ $\boxed{m = 3}$

Values are equal

$C.F = e^{mx}[A + Bx]$

$$\boxed{CF = e^{3x}[A + Bx]}$$

$$P.I_1 = \frac{1}{f(D)} \text{ RHS}$$

$$f(D) = D^2 - 6D + 9$$

$$= \frac{1}{D^2 - 6D + 9} 3e^{3x}$$

Replace : $D = a = 3$

$$= \frac{1}{(3)^2 - 6(3) + 9} 3e^{3x}$$

$$= \frac{1}{9 - 18 + 9} 3e^{3x}$$

$$= \frac{1}{0} 3e^{3x} \text{ (failure)}$$

$$P.I_1 = \frac{x}{f'(D)} \text{ RHS}$$

$$= \frac{x}{2D - 6} 3e^{3x}$$

$$= \frac{x}{2(3) - 6} 3e^{3x}$$

$$= \frac{x}{6 - 6} 3e^{3x}$$

$$= \frac{x}{0} 3e^{3x} \text{ (failure)}$$

$$P.I_1 = \frac{x^2}{f''(D)} \text{ RHS.}$$

$$= \frac{x^2}{2D} 3e^{3x}$$

$$= \frac{x^2}{2(3)} 3e^{3x}$$

$$\boxed{P.I_1 = \frac{x^2}{6} 3e^{3x}}$$

$$P.I_2 = \frac{1}{f(D)} \text{ RHS}$$

$$f(D) = D^2 - 6D + 9$$

$$= \frac{1}{D^2 - 6D + 9} 5\sin 2x$$

Replace : $D^2 = -(a)^2$

$$D^2 = -(2)^2$$

$$D^2 = -4$$

$$P.I = \frac{1}{(-4)^2 - 6D + 9} \, 5\sin 2x$$

$$= \frac{1}{16 - 6D + 9} \, 5\sin 2x$$

$$= \frac{1}{25 - 6D} \, 5\sin 2x$$

$$= \frac{1}{25 - 6D} \times \frac{25 + 6D}{25 + 6D} \, 5\sin 2x$$

$$= \frac{1(25 + 6D)}{(25)^2 - (6D^2)} \, 5\sin 2x$$

$$= \frac{25 + 6D}{625 - 6(D)^2} \, 5\sin 2x$$

$$= \frac{25 + 6D\,(5\sin 2x)}{625 - 6(-4)}$$

$$= \frac{(5\sin 2x)(25 + 6D)}{625 + 24}$$

$$= \frac{125\sin 2x + 6D.(5\sin 2x)}{649}$$

$$= \frac{125\sin 2x + 30D\sin 2x}{649}$$

$$= \frac{125\sin 2x + 30(\cos 2x . 2)}{649}$$

$$P.I_2 = \frac{125\sin 2x + 60\cos 2x}{649}$$

$$y = C.F + P.I_1 + PI_2$$

$$= e^{3x}[A + Bx] + \left[\frac{x^2}{6} 3e^{3x} + \frac{125\sin 2x + 60\cos 2x}{649}\right.$$

Type : 3

first $[1 \pm x]$ is 1 [illegible] type 3 & power -1 [illegible]

Formula:-

$$[1+x]^{-1} = 1 - x + x^2 - x^3 + \dots$$

$$[1-x]^{-1} = 1 + x + x^2 + x^3 + \dots$$

1) $[D^2 - 3D + 2]y = x^3$

$D^2 = m^2 \quad D = m \quad y = 1$

$m^2 - 3m + 2 = 0$

$(m-1)(m-2) = 0$

$m = 1 \quad m = 2$

$C.F = Ae^{1x} + Be^{2x}$

$C.F = Ae^{x} + Be^{2x}$

$$P.I = \frac{1}{f(D)} \text{ RHS.}$$

$$D = [2 - 3D + D^2]$$

$$= \frac{1}{2 - 3D + D^2} x^3$$

$$= \frac{1}{2\left[\frac{\cancel{2}}{\cancel{2}} - \frac{3D}{2} + \frac{D^2}{2}\right]} x^3$$

$$= \frac{1}{2}\left[\frac{1}{1 - \frac{3D}{2} + \frac{D^2}{2}}\right] x^3$$

$$= \frac{1}{2}\left[1 - \frac{3D}{2} + \frac{D^2}{2}\right]^{-1} x^3$$

$$= \frac{1}{2}\left[1 - \left(-\frac{3D}{2} + \frac{D^2}{2}\right) + \left(-\frac{3D}{2} + \frac{D^2}{2}\right)^2 - \left(\frac{-3D}{2} + \frac{D^2}{2}\right)^3\right]$$

$$= \frac{1}{2}\left[1 + \frac{3D}{2} - \frac{D^2}{2} + \frac{9D^2}{4} - \frac{6D^3}{4} + \frac{27D^3}{8}\right] x^3$$

$$= \cancel{\frac{1}{2}\left[1 + \frac{3D}{2} + \frac{7D^2}{4} + \frac{15D^3}{8}\right] x^3}$$

$$= \frac{-D^2}{2} + \frac{9D^2}{4}$$

$$= \frac{-2D^2 + 9D^2}{4}$$

$$= \frac{-2D^2 + 9D^2}{4}$$

$$= \frac{7D^2}{4}$$

$$= \frac{-6D^3}{4} + \frac{27D^3}{8}$$

$$= \frac{-12D^3 + 27D^3}{8}$$

$$= \frac{15D^2}{8}$$

$$= \frac{1}{2}\left[1 + \frac{3D}{2} + \frac{7D^2}{4} + \frac{15D^3}{8}\right] x^3$$ u.

24/02/23

CAUCHY'S LINEAR EQUATION. (variable co-efficient in higher order)

Rule :-

1. Replace R.H.S $\boxed{x = e^z}$

2. $\log x = \log e^z$ ↓ apply the log.

$\log x = z \log e$

$\boxed{\log x = z}$

3. $xD = \theta$ LHS =

4. $x^2 D^2 = \theta(\theta - 1)$

θ values is $\theta = d/dz$

Linear Equation type $(ax + b)$ model

Solve : $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$

Solution :

Given $\left[x^2D^2 + 4xD + 2\right]y = x^2 + \frac{1}{x^2}$

Replace $x = e^z$; $xD = \theta$

$$\left[\theta(\theta-1) + 4\theta + 2\right]y = (e^z)^2 + \frac{1}{(e^z)^2}$$

$$\left[\theta^2 - \theta + 4\theta + 2\right]y = e^{2z} + \frac{1}{e^{2z}}$$

$$\left[\theta^2 + 3\theta + 2\right]y = e^{2z} + e^{-2z}$$

$$\frac{d^2y}{dz^2} + 3\frac{dy}{dz} + 2y = e^{2z} + e^{-2z}$$

Complementary function (C.F)

Replace $\theta^2 = m^2$; $\theta = m$; $y = 1$

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$\boxed{m = -1}$ $\boxed{m = -2}$

$$C.F = Ae^{m_1 z} + Be^{m_2 z}$$

$$\boxed{C.F = Ae^{-1z} + Be^{-2z}}$$

$P.I = \frac{1}{f(\theta)}$ RHS $\qquad \theta = \theta^2 + 3\theta + 2$

$$= \frac{1}{\theta^2 + 3\theta + 2}\left(e^{2z} + e^{-2z}\right)$$

~~P.I₁ Repla~~

$$P.I_1 = \frac{1}{\theta^2 + 3\theta + 2} e^{2z}$$

Replace : $\theta = a = 2$

$$P.I_1 = \frac{1}{(2)^2 + 3(2) + 2} e^{2z}$$

$$= \frac{1}{4+6+2} e^{2z}$$

$$= \frac{1}{12} e^{2z}$$

$$P.I_2 = \frac{1}{\theta^2 + 3\theta + 2} e^{-2z}$$

Replace $\theta = a = -2$

$$= \frac{1}{(-2)^2 + 3(-2) + 2} e^{-2z}$$

$$= \frac{1}{4 - 6 + 2} e^{-2z}$$

$$= \frac{1}{-2+2} e^{-2z}$$

$$= \frac{1}{0} e^{-2z} \text{ (failure)}$$

$$P.I_2 = \frac{z}{f'(\theta)} e^{-2z}$$

$$= \frac{z}{2\theta + 3} e^{-2z}$$

$$= \frac{z}{2(-2)+3} e^{-2z}$$

$$= \frac{z}{-4+3} e^{-2z}$$

$$P.I_2 = \frac{z}{-1} e^{-2z} \Rightarrow \frac{-z}{1} e^{-2z}$$

$$y = A(e^z)^{-1} + B(e^z)^{-2}$$

$$= \frac{1}{12}(e^z)^2 - z(e^z)^{-2}$$

$= \cancel{\frac{1}{12}}$ $y = C.F + P.I$

$$y = Ax^{-1} + Bx^{-2} + \frac{1}{12}x^2 - zx^{-2}$$

$$\boxed{y = Ax^{-1} + Bx^{-2} + \frac{1}{12}x^2 - \log x \, x^{-2}}$$

$$[x^2D^2 + 4xD + 2]y = \sin(2\log x)$$

Sollution:-

$$[\theta(\theta-1) + 4\theta + 2]y = \sin(2z)$$

$$[\theta^2 - \theta + 4\theta + 2]y = \sin 2z$$

$$[\theta^2 + 3\theta + 2]y = \sin 2z$$

Auxillary Equation is:

$$\theta^2 = m^2 \ ; \ \theta = m \ ; \ y = 1$$

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$$\boxed{m = -1}, \ \boxed{m = -2}$$

$$C.F = Ae^{m_1 z} + Be^{m_2 z}$$

$$\boxed{C.F = Ae^{-1z} + Be^{-2z}}$$

$$P.I = \frac{1}{f(\theta)} R.H.S$$

$$= \frac{1}{\theta^2 + 3\theta + 2} \sin 2z$$

Replace $\theta^2 = -(a)^2$

$$= -(2)^2$$

$$\theta^2 = -4$$

$$P.I = \frac{1}{-4 + 3\theta + 2} \sin 2z$$

$$= \frac{1}{3\theta - 2} \sin 2z$$

$$= \frac{1}{-2+3\theta} \sin 2z$$

$$= \frac{1}{-2+3\theta} \times \frac{-2-3\theta}{-2-3\theta} \sin 2z$$

$$= \frac{-2-3\theta}{(-2)^2 + (-3\theta)^2} \sin 2z$$

$$= \frac{-2-3\theta}{4+9\theta^2} \sin 2z$$

$$= \frac{-2-3\theta\,(\sin 2z)}{4+9\theta^2}$$

$$= \frac{-2-3(2)\sin 2z}{4+9(2)^2}$$

$$= \frac{-2-6\sin 2z}{4+9(4)}$$

$$= \frac{-2-6\sin 2z}{4+36}$$

$$= \frac{\cancel{-}2\sin 2z - (-6\sin 2z)}{\cancel{-}32}$$

$$= \frac{2\sin 2z + 6\sin 2z}{32}$$

$$P.I = \frac{2\sin 2z + 6\cos 2z}{32}$$

$$y = C.F + P.I$$

$$y = Ae^{-1z} + Be^{-2z} + \frac{2\sin 2z + 6\cos 2z}{32}$$

$$= Ax^{-1} + Bx^{-2} + \frac{2\sin 2z + 6\cos 2z}{32}.$$

3) solve : $\left[x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2\right]y = \sin(\log x)$

Sollution :-

$$[x^2D^2 + 4xD + 2]y = \sin(\log x)$$

$$[\theta(\theta-1) + 4\theta + 2]y = \sin(\log x)$$

$$[\theta(\theta-1) + 4\theta + 2]y = \sin(z)$$

$$[\theta^2 - \theta + 4\theta + 2]y = \sin z$$

$$[\theta^2 + 3\theta + 2]y = \sin z$$

Auxillary equation is

$\theta^2 = m^2$; $\theta = m$; $y = 1$.

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$$\boxed{m=-1} \quad \boxed{m=-2}$$

$$C.F = Ae^{m_1 z} + Be^{m_2 z}$$

$$\boxed{C.F = Ae^{-1z} + Be^{-2z}}$$

$$P.I = \frac{1}{f(\theta)} R.H.S \qquad \theta = \theta^2 + 3\theta + 2$$

$$= \frac{1}{\theta^2 + 3\theta + 2} \sin z$$

Replace $\theta^2 = -(a)^2$

$$= -(1)^2$$

$$\theta^2 = -1$$

$$P.I = \frac{1}{(-1) + 3\theta + 2} \sin z$$

$$= \frac{1}{-1 + 3\theta + 2} \sin z$$

$$= \frac{1}{1 + 3\theta} \sin z$$

$$= \frac{1}{1+3\theta} \times \frac{1-3\theta}{1-3\theta} (\sin z)$$

$$= \frac{1-3\theta}{(1)^2-(3\theta)^2} (\sin z)$$

$$= \frac{1-3\theta}{1-9\theta^2} (\sin z)$$

$$= \frac{1-3\theta}{1-9(-1)} \sin z$$

$$= \frac{1-3\theta}{1+9} \sin z$$

$$P.I = \frac{\sin z - 3\theta \sin z}{10}$$

$$y = C.F + P.I$$

$$y = Ae^{-1z} + Be^{-2z} + \frac{\sin z - 3\theta \sin z}{10}$$

$$y = Ax^{-1} + Bx^{-2} + \frac{\sin z - 3\theta \sin z}{10}$$

2/3/23

Legendre's Linear Equation.

$$(ax+b)^2 \frac{d^2y}{dx^2} + (ax+b)\frac{dy}{dx} + y = R$$

This is also called as Legendre Linear Equation

Rules:- Replace

LHS : $(ax+b)\frac{dy}{dx}$ (or) $D = a\theta$

$(ax+b)^2 \frac{d^2y}{dx^2}$ (or) $D^2 = a^2\theta(\theta-1)$

RHS Replace:-

give log we can use z in RHS

$ax+b = e^{z}$

log value is 1 so e^{z}

$\log(ax+b) = z$

$\log(ax+b) = z$

$\cos(\log(ax+b)) = z$

1 Solve:

marks (6 marks)

$$(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$$

Sollution :-

Given $\left[(2x+3)^2 D^2 - 2(2x+3)D - 12\right] y = 6x$

Replace LHS:-

$(\because a\theta)$

$(2x+3)D = 2\theta$

$(\because a^2\theta(\theta-1))$

$(2x+3)^2 D^2 = 2^2\theta(\theta-1)$

$= 4\theta(\theta-1)$

Replace RHS:-

$2x+3 = e^{z}$

$2x = e^{z}-3$

$x = \frac{e^{z}-3}{2}$

$\left[(2x+3)^2 D^2 - 2(2x+3)D - 12\right] y = 6x$

$\left[4\theta(\theta-1) - 2(2\theta) - 12\right] y = \overset{3}{\cancel{6}}\left[\frac{e^{z}-3}{\cancel{2}}\right]$ (x)

$\left[4\theta^2 - 4\theta - 4\theta - 12\right] y = 3\left[e^{z}-3\right]$

$\left[4\theta^2 - 8\theta - 12\right] y = 3e^{z} - 9$

complementary function.

$D^2 = m^2 \quad D = m \quad y = 1.$

$4m^2 - 8m - 12 = 0$

$m^2 - 2m - 3 = 0$

$(m+1)(m-3) = 0$

$m = -1 \quad m = 3$

$C.F = Ae^{m_1 z} + Be^{m_2 z}$

$$\boxed{C.F = Ae^{-1z} + Be^{3z}}$$

$P.I = \frac{1}{f(D)} RHS$

$P.I = \frac{1}{4D^2 - 8D - 12} [3e^z - 9]$

$P.I_1 = \frac{1}{4D^2 - 8D - 12} 3e^z$

Replace $D = a = 1$

$= \frac{1}{4-8-12} 3e^z$

$$\boxed{P.I_1 = \frac{1}{16} 3e^z}$$

$P.I_2 = \frac{1}{f(D)} RHS$

$= \frac{1}{4D^2 - 8D - 12} (-9 \cdot e^{0z})$

put $D = a = 0$

$P.I_2 = \frac{1}{-\cancel{12}\,4} - \cancel{9}^{3} e^{0z}$

$= \frac{3}{4} e^{0z}$

$= 3/4$

$y = C.F + P.I$

$$\boxed{y = Ae^{-1z} + Be^{3z} + \left(\frac{1}{-16}\, 3e^{z}\right) + \frac{3}{4}}$$

Transform the. Legendre's linear Equations.

1) $\left[(3x+2)^2 D^2 + 3(3x+2)D - 36\right]y = 4x$

Transform the linear differential equation

2m 2) $(2x+7)^2 \frac{d^2y}{dx^2} - 6(2x+7)\frac{dy}{dx} + 8y = 8x$ into constant coefficient

3) $\left[(x+1)^2 D^2 + (x+1)D + 1\right]y = 4\left[\cos \log (x+1)\right]$