MAXIMA AND MINIMA

WORKING RULE TO FIND MAXIMUM OR MINIMUM VALUES [EXTREMUM VALUES] OF $f(x, y)$.

★ Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

★ Set $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

Solve it Simultaneously

The Solution point of these equations are called stationary points.

★ Find the values of $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$, $t = \frac{\partial^2 f}{\partial y^2}$ at these points.

★ (a) If $rt-s^2>0$ and $r<0$, then the function is maximum at that point.

(b) If $rt-s^2>0$ and $r>0$, then the function is minimum at that point.

(c) If $rt-s^2<0$, then the function is neither maximum nor minimum at that point. This Point is called as saddle point.

(d) If $rt-s^2=0$, then the case is inconclusive. Hence further investigation is required.

NECESSARY CONDITION:

The necessary condition for the function $f(x,y)$ to have a maxima minima at a point (a,b) is

$\frac{\partial f}{\partial x}=0$, $\frac{\partial f}{\partial y}=0$ at (a,b).

SUFFICIENT CONDITION:

Write 3 and 4 Step in working rule.

This is the sufficient condition for the function to be maxima or minima.

PROBLEMS :

① Examine $f(x,y) = x^3 + y^3 - 12x - 3y + 20$ for its extreme values.

Soln:

$$f(x,y) = x^3 + y^3 - 12x - 3y + 20$$

$$\frac{\partial f}{\partial x} = 3x^2 - 12 \quad ; \quad \frac{\partial f}{\partial y} = 3y^2 - 3.$$

$$\frac{\partial f}{\partial x} = 0 \qquad \qquad \frac{\partial f}{\partial y} = 0$$

$$3x^2 - 12 = 0 \qquad \qquad 3y^2 - 3 = 0$$

$$x^2 = 4 \qquad \qquad y^2 = 1$$

$$x = \pm 2 \qquad \qquad y = \pm 1$$

Hence the stationary points are $(2,1)$, $(2,-1)$, $(-2,1)$, $(-2,-1)$.

$$r = \frac{\partial^2 f}{\partial x^2} = 6x$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6y$$

Critical points	$r = 6x$	s	t	$rt - s^2$	Conclusion
$(2,1)$	$12 > 0$	0	6	$72 > 0$	Minimum point
$(2,-1)$	$12 > 0$	0	-6	$-72 < 0$	Neither max nor min point - Saddle point
$(-2,1)$	$-12 < 0$	0	6	$-72 < 0$	"
$(-2,-1)$	$-12 < 0$	0	-6	$72 > 0$	Maximum point

Min value $= [f(x,y)]_{(2,1)} = 2$

Max value $= [f(x,y)]_{(-2,-1)} = 38$

(2) Find the maximum and minimum values of $x^2 - xy + y^2 - 2x + y$.

Soln:

$$\frac{\partial f}{\partial x} = 0, \quad \frac{\partial f}{\partial y} = 0$$

$$2x - y - 2 = 0, \quad -x + 2y + 1 = 0$$

$$\Rightarrow x = 1, \quad y = 0$$

$(1,0)$ is the stationary point.

$$r = \frac{\partial^2 f}{\partial x^2} = 2 > 0$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = -1 < 0$$

$$t = \frac{\partial^2 f}{\partial y^2} = 2 > 0$$

$$rt - s^2 = 4 - 1 = 3 > 0$$

$(1,0)$ is a minimum point.

Min. value $= [f(x,y)]_{(1,0)} = -1$.