1. A random variable $\boldsymbol{X}$ has the following probability distribution.

X	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find

(1) The value of $\boldsymbol{k}$

(2) Evaluate P(X<6),P(0<X<5)

(3) The smallest value of a for which $P(X \leq a) > \frac{1}{2}$.

(4) The Cumulative distribution function.

2. A random variable X has the following probability function

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) Determine the value of ‘a’

(ii) Find P(X<3), P(X≥ 3), P(0<X<5)

(iii) Find the distribution function of X.

3. A random variable X has the following probability distribution

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

Find (1) The value of K (2) Evaluate P(X< 2) and P(-2<X<2) (3) Find the Cumulative distribution of X (4) Find the mean of X.

4. If the Random variable X takes the value 1,2,3,4 such that 2P(X=1) =3P(X=2)=P(X=3)=5P(X=4). Find the probability distribution.

5. A continuous R.V X has the p.d.f $f(x) = 3x^2,\ 0 \leq x \leq 1$. Find the value of a, such that $P(X \leq a) = P(X > a)$. Find the value b such that P(X > b) = 0.05.

6. A continuous R.V. $\boldsymbol{X}$ has the p.d.f. $f(x) =$

$$\begin{cases} \frac{k}{1+x^2} & -\infty < x < \infty \\ 0 & otherwise \end{cases}$$

. Find

(1) The value of $\boldsymbol{k}$

(2) Distribution function of $\boldsymbol{X}$

(3) $\boldsymbol{P(X \geq 0)}$

7. The probability function of an infinite discrete distribution is given by $P(X = j) = \frac{1}{2^x}$ $(x = 1,2,3, \ldots)$
   (1) Mean and variance of $\boldsymbol{X}$
   (2) $M.G.F$
   (3) $P(X$ is even$)$

8. A random variable has the pdf $(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & otherwise \end{cases}$. Obtain the mgf and first four moments about the origin. Also find the mean and variance.

9. Find the M.G.F of the random variable with the probability law $P(X = x) = q^{x-1}p,\ x = 1,2,3, \ldots.$ Find the mean and variance.

10. A continuous Random variable X has the distribution function $F(x) = \begin{cases} 0 & x \leq 1 \\ k(x-1)^4 & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$.
    (1) Find K (2) p.d.f f(x) (3) P(X< 2).

11. The diameter of an electric cable say X, is assumed to be a continuous Random variable with P.d.f
    $f(x) = 6x(1-x)\ , 0 \leq x \leq 1$

(i) Check that the above is a P.d.f

(ii) Determine a and b such that $P(X < b) = P(X > b)$

(iii) Find the distribution function of X

(iv) Find $P(X \leq \frac{1}{2} / \frac{1}{3} < X < \frac{2}{3})$

12. If the probability density of X is given by

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find its rth moment. Hence evaluate $E[(2x+1)^2]$

13. If the cumulative distribution function of X is given by $F(x) = \begin{cases} 1 - \frac{4}{x^2}, x > 2 \\ 0 \quad , x \leq 2 \end{cases}$

Find (i) P(X<3) (ii) P(4<X<5) (iii) $P(X \geq 3)$.

14. Experience has shown that walking in a certain park, the time X(in mins) , between seeing two people smoking has a density function of the form $f(x) = \begin{cases} \lambda x e^{-x} & x > 0 \\ 0 & elsewhere \end{cases}$

(a) Calculate the value of λ

(b) Find the distribution function of X

(c) What is the probability that a person who has just seen a person smoking will see another person smoking in 2 to 5 minutes? In atleast 7 minutes?

15. The density function of a random variable X is given by $f(x) = \begin{cases} kx(2-x)^2 & 0 < x < 2 \\ 0 & otherwise \end{cases}$ find (i) k (ii) Mean and variance of the distribution.

16. Find the M.G.F for the distribution $f(x) = \begin{cases} \frac{x}{4} \; e^{-\frac{x}{2}} & x > 0 \\ 0 & otherwise \end{cases}$ find (i) M.G.F (ii) First Four moments obout the origin

17. A random variable has the p.d.f given by

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find (a) The moment generating function

(b) First four moments about the origin.