



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

EXPONENTIAL DISTRIBUTION



Exponential Distribution:

If  $x$  is a continuous random variable which follows an exponential distribution, then

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where  $\lambda$  is a parameter,  $\lambda > 0$ .

MGF, mean & variance:

MGF:

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$= \int_0^{\infty} e^{tx} \lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^{\infty} e^{-x(\lambda-t)} dx$$

$$= \lambda \left[ \frac{e^{-(\lambda-t)x}}{-(\lambda-t)} \right]_0^{\infty}$$

$$= \frac{-\lambda}{\lambda-t} [e^{-\infty} - e^0]$$

$$= \frac{-\lambda}{\lambda-t} (0-1)$$

$$M_x(t) = \frac{\lambda}{\lambda-t}$$

mean:

$$E(x) = \left[ \frac{d}{dt} M_x(t) \right]_{t=0}$$

$$= \left[ \frac{d}{dt} \left( \frac{\lambda}{\lambda-t} \right) \right]_{t=0}$$



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$$= \left[ \lambda \left( \frac{-1}{(\lambda - t)^2} (-1) \right) \right]_{t=0} \quad \because \frac{d}{dt} \frac{1}{t} = -\frac{1}{t^2}$$

$$= \left[ \frac{\lambda}{(\lambda - t)^2} \right]_{t=0}$$

$$= \frac{\lambda}{\lambda^2}$$

$$E(x) = \frac{1}{\lambda}$$

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \left[ \frac{d^2}{dt^2} m_x(t) \right]_{t=0}$$

$$= \left[ \frac{d}{dt} \left( \frac{\lambda}{(\lambda - t)^2} \right) \right]_{t=0}$$

$$\frac{d}{dt} \left( \frac{1}{t^2} \right) = -\frac{2}{t^3}$$

$$= \lambda \left( \frac{-2(-1)}{(\lambda - t)^3} \right)_{t=0}$$

$$= \frac{2\lambda}{\lambda^3}$$

$$E(x^2) = \frac{2}{\lambda^2}$$

$$\therefore \text{var}(x) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$$m_x(t) = \frac{\lambda}{\lambda - t}$$

$$\text{mean} = \frac{1}{\lambda}$$

$$\text{variance} = \frac{1}{\lambda^2}$$



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Memoryless Property:

If  $x$  is exponential distribution, then for any integers  $s$  and  $t$ , prove that

$$P(x > s+t | x > s) = P(x > t)$$

Soln.

WKT

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Now,

$$P(x > s+t | x > s) = \frac{P(x > s+t \cap x > s)}{P(x > s)}$$

$$= \frac{P(x > s+t)}{P(x > s)} \rightarrow (1)$$

$$\begin{aligned} P(x > s) &= \int_s^{\infty} \lambda e^{-\lambda x} dx \\ &= \lambda \left[ \frac{e^{-\lambda x}}{-\lambda} \right]_s^{\infty} \\ &= - [e^{-\infty} - e^{-\lambda s}] \\ &= - [0 - e^{-\lambda s}] \\ &= e^{-\lambda s} \end{aligned}$$

$$\begin{aligned} \text{Hly } P(x > s+t) &= e^{-\lambda(s+t)} \\ &= e^{-\lambda s - \lambda t} \\ &= e^{-\lambda s} \cdot e^{-\lambda t} \end{aligned}$$



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$$\begin{aligned} (1) \Rightarrow P(x > s+t / x > s) &= \frac{e^{-\lambda s} e^{-\lambda t}}{e^{-\lambda s}} \\ &= e^{-\lambda t} \\ &= P(x > t) \end{aligned}$$

Hence proved

11. The time in hours required to repair a machine is exponential distribution with parameter  $\lambda = \frac{1}{2}$ .
- What is the probability that repair time exceed 2 hrs.
  - What is the probability that repair takes atleast 11 hrs. given that its duration exceed 8 hrs.

Soln.

$$\text{Given } \lambda = \frac{1}{2}$$

$$\begin{aligned} \text{i). } P(x > 2) &= e^{-\lambda(2)} & \because P(x > s) &= e^{-\lambda s} \\ &= e^{-\frac{1}{2}(2)} \\ &= e^{-1} \\ &= 0.368 \end{aligned}$$

$$\text{ii). } P(x > 8+3 / x > 8) = P(x > 3)$$

$$\begin{aligned} &= e^{-\lambda(3)} & \because P(x > s+t / x > s) &= P(x > t) \\ &= e^{-\frac{3}{2}} \\ &= 0.223 \end{aligned}$$

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Q1. The mileage which the car owners get with certain kind of radial tyre is a random variable having an exponential distribution with mean 4000 km. Find the probability that one of these tyre will last

i). At least 2000 km

ii). At most 3000 km

Soln.

Given mean = 4000

$$\frac{1}{\lambda} = 4000$$

$$\lambda = \frac{1}{4000}$$

$$\begin{aligned} \text{i). } P(X > 2000) &= e^{-\lambda(2000)} \\ &= e^{-\frac{1}{4000}(2000)} \\ &= e^{-1/2} \\ &= 0.607 \end{aligned}$$

$$\begin{aligned} \text{ii). } P(X \leq 3000) &= 1 - P(X > 3000) \\ &= 1 - e^{-\lambda(3000)} \\ &= 1 - e^{-\frac{1}{4000}(3000)} \\ &= 1 - e^{-3/4} \\ &= 0.528 \end{aligned}$$

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