CHI - SQUARE TEST :

$$\chi^2 = \frac{\Sigma [O_i - E_i]^2}{E_i}$$

where $O_i \to$ Observed frequency

$E_i \to$ Experimental frequency or Expected frequency $= \frac{\Sigma O_i}{n}$

degrees of freedom, $\nu = n-1$

properties :

i) The mean of χ^2 dist. is equal to the no. of degrees of freedom

ii) The variance of χ^2 dist. is twice the degrees of freedom

iii) If χ^2 is a chi-square variate with ν degrees of freedom, then $\chi^2/2$ is a gamma variate with parameter $\nu/2$.

iv) standard χ^2 variate tends to standard normal variate as $n \to \infty$.

Applications :

i) To test if the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$

ii) To test the goodness of fit.

iii) To test the independence of attributes.

iv) To test the homogeniety of indep. estimates of the population variance.

Degrees of freedom: No. of values in a set which may be assigned arbitrarily.

1) The table below gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	14	18	12	11	15	14

Soln:

Given, Total no. of accidents $= 84$

No. of days $= 6$

$\therefore$ Expected frequencies of the accidents (ie) $E_i = \frac{84}{6} = 14$

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
14	14	0	$0/14 : 0$
18	14	16	$16/14 : 1.14$
12	14	4	$4/14 : 0.285$
11	14	9	$9/14 : 0.642$
15	14	1	$1/14 : 0.071$
14	14	0	$0/14 : 0$

$$\sum \frac{(O_i-E_i)^2}{E_i} = 2.14285$$

Step 1: Formulate H_0 & H_1:

H_0: The accidents are uniformly distributed.

H_1: The accidents are not uniformly distributed.

Step 2: LOS at $\alpha = 5\%$.

Step 3: Test statistic, $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 2.1428$

Step 4: Degrees of freedom, $\nu = n-1$

$$= 6-1$$
$$= 5$$

Tab value is $11.07 = \chi^2_\alpha$

Step 5: Conclusion:

$$\chi^2 = 2.1428 < 11.07 = \chi^2_\alpha$$

$\therefore$ H_0 is accepted at 5% LOS. (ie) the accident are uniformly distributed.

2) A die was thrown 498 times. Denoting x to be the number appearing on the top face of it, the observed frequency of x is given below:

x:	1	2	3	4	5	6
f:	69	78	85	82	86	98

what opinion you would form for the accuracy of the die?

Soln:

Given, Expected frequency, $E_i = \dfrac{\text{Total frequenc}}{6}$

$$= \frac{498}{6} = 83$$

x	O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
1	69	83	196	2.3614
2	78	83	25	0.3012
3	85	83	4	0.0481
4	82	83	1	0.0120
5	86	83	9	0.1084
6	98	83	225	2.4108

$$\therefore \sum_{}^{6} \frac{(O_i-E_i)^2}{E_i} = 5.5419$$

step 1: Formulate H_0 & H_1:

H_0: A die is unbiased

H_1: A die is not unbiased i.e. biased.

step 2: LOS at $\alpha = 5\%$.

step 3: Test statistic, $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 5.542$.

step 4: Degrees of freedom, $\nu = n-1 = 6-1 = 5$

$$\therefore \chi^2_\alpha = 11.07$$

step 5: Conclusion;

$$\chi^2 = 5.542 < 11.07 = \chi^2_\alpha$$

$\therefore H_0$ is accepted at 5% LOS (i.e.) A die is unbiased

CHI SQUARE TEST FOR INDEPENDENCE OF ATTRIBUTES.

$$\chi^2 = \Sigma\left[\frac{(O_i - E_i)^2}{E_i}\right]$$

where $O_i \rightarrow$ Observed frequency

$E_i \rightarrow$ Expected frequency

$$E_i = \frac{(\underset{B_i}{\text{Row total}})(\underset{A_j}{\text{column total}})}{\text{whole total}}, \quad i = 1 \text{ to } s, \ j = 1 \text{ to } t$$

Degrees of freedom, $\nu = (s-1)(t-1)$.

1) On the basis of information noted below, find out whether the new treatment is comparatively superior to the conventional one.

	Favourable	Not Favourable	total
New	60	30	90
Conventional	40	70	110
total	100	100	200

Soln:

To find E_i:

$\frac{90 \times 100}{200} = 45$	$\frac{90 \times 100}{200} = 45$
$\frac{110 \times 100}{200} = 55$	$\frac{110 \times 100}{200} = 55$

O_i	E_i	$O_i - E_i$	$(O_i-E_i)^2/E_i$
60	45	15	5
30	45	-15	5
40	55	-15	4.09
70	55	15	4.09

$$\sum \frac{(O_i - E_i)^2}{E_i} = 18.18$$

step 1: Formulating H_0 & H_1:

H_0: There is no difference between new & conventional treatment.

H_1: There is difference between new & conventional treatment.

step 2: Los at $\alpha = 5\%$.

step 3: Test statistics, $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$= 18.18$

step 4: Degrees of freedom, $\nu = ((s-1)*(t-1))$

$\nu = (2-1 * 2-1)$

$= 1 * 1$

$= 1$

$\therefore$ tab value, $\chi^2_\alpha = 3.841$

step 5: Conclusion:

$\chi^2 = 18.18 > 3.841 = \chi^2_\alpha$

$\therefore$ H_0 is rejected at 5% Los

(i.e) there is difference between new & conventional treatment.

2) Two researchers A and B adopted different techniques while rating the students level. Can you say that the techniques adopted by them are significant?

Researchers:	Below avg.	Avg.	Above Avg.	Genius	Total
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300

Soln:

To find E_i

$\frac{100\times126}{300} = 42 \quad \frac{100\times93}{300} = 31 \quad \frac{100\times69}{300} = 23 \quad \frac{100\times12}{300} = 4$

$\frac{200\times126}{300} = 84 \quad \frac{200\times93}{300} = 62 \quad \frac{200\times69}{300} = 46 \quad \frac{200\times12}{300} = 8$

O_i	E_i	$O_i - E_i$	$(O_i-E_i)^2/E_i$
40	42	-2	0.0952
33	31	2	0.129
25	23	2	0.173
2	4	-2	1
86	84	2	0.047
60	62	-2	0.064
44	46	-2	0.086
10	8	2	0.5

$$\sum \frac{(O_i-E_i)^2}{E_i} = 2.097$$

Step 1 : Formulating H_0 and H_1:

H_0: There is no difference between the two researchers.

H_1: There is difference between the two researchers

Step 2 : LOS at $\alpha = 5\%$.

Step 3 : Test statistics, $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$= 2.097$

Step 4 : Degrees of freedom, $\nu = ((4-1) * (2-1))$

$= (3 * 1)$

$= 3$

$\therefore$ Tab value is $\chi^2_\alpha = 7.115$

Step 5 : Conclusion.

$\chi^2 = 2.097 < 7.115 = \chi^2_\alpha$

$\therefore$ H_0 is accepted at 5% LOS

(a) There is no difference between the two researchers.