PART B

1. From the following distribution of (X,Y) find. (i) $P(X \leq 1)$ (ii) $P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv)$P\left(X \leq \frac{1}{Y} \leq 3\right)$ (v) $P\left(Y \leq \frac{3}{X} \leq 1\right)$ (vi) $P(X + Y \leq 4)$.

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

2. The joint probability function (X,Y) is given by $P(x, y) = k(2x + 3y) \quad x = 0,1,2;\ y = 1,2,3$
   (i) Find the marginal distributions.
   (ii) Find the probability distributions of (X+Y)
   (iii) Find all conditional probability distributions.

3. The joint p.d.f of the random variable (X,Y) is given by

$$f(x, y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2,\ 0 < y < 1 \\ 0 & otherwise \end{cases}$$

   Find
   (i) Marginal density function s of X and Y
   (ii) Conditional density of X given Y
   (iii) $P\left(\frac{1}{4} < X < \frac{\frac{1}{2}}{Y} = \frac{1}{3}\right)$

4. The joint p.d.f of the two dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{8xy}{9} & : 1 \le x \le y \le 2 \\ 0 & : otherwise \end{cases}$$

Find

(i) Marginal densities of X and Y

(ii) The conditional density functions $f(x/y)$ and $f(y/x)$.

5. If the joint p.d.f of a two dimensional random variable (X,Y) is given by $f(x,y) =$

$$\begin{cases} x^2 + \frac{xy}{3}: & 0 < x < 1; 0 < y < 2 \\ 0: & otherwise \end{cases}$$

Find (i) $P(X > {}^{1}/_{2})$ (ii) $P(Y > 1)$ (iii) $P(Y < X)$

(iii) $P\left(Y < \frac{1}{2} \middle/ X < \frac{1}{2}\right)$ (v) $P(X + Y \ge 1)$

(vi) find the conditional density functions.

(vii) Check whether the conditional density functions are valid.

6. The joint p.d.f of the random variable (X,Y) is given by $f(x,y) = kxye^{-(x^2+y^2)}$ $x > 0, y > 0$

(i) Find k (ii) Prove that X and Y are independent.

7. Given $f_{XY}(x,y) =$

$$\begin{cases} cx(x-y) & , 0 < x < 2, -x < y < x \\ 0 & otherwise \end{cases}$$

(i) Evaluate c

(ii) Find $f_X(x)$

(iii) $f_{\frac{Y}{X}}(y/x)$

(iv) $f_Y(y)$.

8. Two random variables X and Y have the following joint probability density functions

$$f(x,y) = \begin{cases} 2-x-y : 0 \le x \le 1, 0 \le y \le 1 \\ 0 \qquad\qquad\quad : otherwise \end{cases}$$

(i) Find the marginal density functions of X and Y
(ii) Conditional density function
(iii) Var X and Var Y
(iv) Correlation coefficient between X and Y.

9. Given the joint p.d.f of X and Y is

$$f(x,y) = \begin{cases} 8xy : 0 < x < y < 1 \\ 0 \quad\quad : otherwise \end{cases}$$

Find the marginal and conditional p.d.f's X and Y.Are X and Y independent?

10. Let (X,Y) be the two dimensional random variable described by the joint p.d.f

$$f(x,y) = \begin{cases} 8xy : 0 \le x \le 1, 0 \le y \le x \\ 0 \quad\quad : otherwise \end{cases}$$

Find the Cov(X,Y).

11. The joint p.d.f of the random variable (X,Y) is $f(x,y) = 3(x+y) : 0 \le x \le 1, 0 \le y \le 1, x+y \le 1$. Find Cov(X,Y).

12. If X and Y are uncorrelated random variables with variances 16 and 9, find the correlation co-efficient between x+y and x-y.

13. Calculate the correlation coefficient for the following heights(in inches) of fathers X and their sons Y.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

14. Marks obtained by 10 students in Mathematics(x) and statistics(y) are given below

x:	60	34	40	50	45	40	22	43	42	64

y:	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find y when x=55.

15. In a correlation analysis the equations of the two regression lines are $3x + 12y = 9; and\ 3y + 9x = 46$. Find (i) The value of the correlation coefficient (ii) Mean value of X and Y.
16. Find the correlation coefficient and the equation of the regression lines for the following values of X and Y.

X	1	2	3	4	5
Y	2	5	3	8	7

17. Find the most likely price in City A corresponding to the price of Rs.70 at City B from the following:

	City B	City A
Average Price	65	67
S.D. of Price	2.5	3.5

Correlation coefficient is 0.8.