TAYLOR'S SERIES EXPANSION

If any point (a, b) . $h = x - a$, $k = y - b$ Then
point (a, b) $h = (x - a$ $k = y - b$
The Taylor's expansion is

$$f(x, y) = f(a, b) + \frac{1}{1!}\left[h f_x + k f_y\right]$$

$$+ \frac{1}{2!}\left[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}\right]$$

$$+ \frac{1}{3!}\left[h^3 f_{xxx} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy}\right] + \dots$$

Example 1: Expand $e^x \cos y$ as a Taylor's series in power of x & y into third degree.

Given $f(x, y) = e^x \cos y$

$\Rightarrow (a, b) = (0, 0)$, $h = x$, $k = y$

$$f(a, b) = f(0, 0)$$
$$= e^0 \cos 0 = 1$$

	At point $(0,0)$
$f(x, y) = e^x \cos y$	
$f_x = e^x \cos y$	1
$f_y = -e^x \sin y$	0
$f_{xx} = e^x \cos y$	1
$f_{yy} = -e^x \cos y$	-1
$f_{xy} = -e^x \sin y$	0
$f_{xxx} = e^x \cos y$	1
$f_{xxy} = -e^x \sin y$	0
$f_{xyy} = -e^x \cos y$	-1
$f_{yyy} = e^x \sin y$	0

Taylor's series

$$f(x,y) = f(a,b) + \frac{1}{1!}(hf_x + kf_y) + \frac{1}{2!}(h^2 f_{xx} + 2h f_x + k^2 f_{yy})$$

$$+ \frac{1}{3!}(h^3 f_{xxx} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy} + kf_{yyy})$$

$$= 1 + \frac{1}{1!}[x(1) + y(0)] + \frac{1}{2!}[x^2(1) + 2xy(0) + y^2(-1)]$$

$$+ \frac{1}{3!}[x^3(1) + 3x^2y(0) + 3xy^2(-1) + y^3(0)]$$

$$= 1 + x + \frac{1}{2}(x^2 - y^2) + \frac{1}{6}(x^3 - 3xy^2) + \ldots\ldots$$

(2) Expand $e^x \sin y$ as a Taylor series in powers of x & y

$$f(x,y) = e^x \sin y$$

$$(a,b) = (0,0)$$

$$h = x, \quad k = y$$

$$f(a,b) = f(0,0)$$

$$= e^0 \sin 0$$

$$= 0$$

$f(x,y) = e^x \sin y$		at $(0,0)$
$f_x = e^x \sin y$		0
$f_y = e^x \cos y$		1
$f_{xx} = e^x \sin y$		0
$f_{yy} = -e^x \sin y$		0
$f_{xy} = e^x \cos y$		1
~~$f_{yx} = e^x \cos y$~~		~~1~~
$f_{xxx} = e^x \sin y$		0
$f_{yyy} = -e^x \cos y$		-1
$f_{xxy} = e^x \cos y$		1
$f_{xyy} = -e^x \sin y$		0

$$f(x,y) = f(a,b) + \frac{1}{1!}(hf_x + kf_y) + \frac{1}{2!}(h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy})$$

$$+ \frac{1}{3!}(h^3 f_{xxx} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy})$$

$$= 0 + \frac{1}{1!}[x(0) + y(1)] + \frac{1}{2!}\, x^2(0) + 2xy(1) + y^2(0)]$$

$$+ \frac{1}{3!}[x^3(0) + 3x^2 y(1) + 3xy^2(0) + y^3(-1)] + \cdots$$

$$= y + \frac{1}{2}(2xy) + \frac{1}{6}(3x^2 y - y^3) + \cdots$$

$$= y + xy + \frac{1}{6}(3x^2 y - y^3) + \cdots$$

3. Expand $e^x \log(1+y)$ in power of x & y upto 2nd degree.

$$f(x,y) = e^x \log(1+y)$$

$$(a,b) = (0,0), \quad h = x, \quad k = y$$

$$f(a,b) = f(0,0)$$
$$= e^0 \log 1$$
$$= 1$$

	At pt $(0,0)$
$f(x,y) = e^x \log(1+y)$	
$f_x = e^x \log(1+y)$	0
$f_y = e^x \frac{1}{1+y}(1)$	1
$f_{xx} = e^x \log(1+y)$	0
~~$f_{xxx} = e^x \log(1+y)$~~	~~0~~
$f_{yy} = e^x \frac{-1}{(1+y)^2}$	-1
$f_{xy} = e^x \frac{1}{1+y}$	1

$$f(x,y) = f(a,b) + \frac{1}{1!}(hf_x + kf_y) + \frac{1}{2!}(h^2 f_{xx} + 2hk f_{xy} + k^2 f_y)$$

$$= 0 + [(x)(0) + y(1)] + \frac{1}{2}(x^2(0) + 2xy(1) + y^2(-1)]$$

$$= y + \frac{1}{2}(2xy - y^2 + \cdots$$

4. Expand $xy^2+2x-3y$ in power of $(x+2)$ & $(y-1)$ upto 2nd degree

$f(x,y) = xy^2+2x-3y$ $\qquad h = x-a$

$(a,b) = (-2,1)$

$h = x+2 \quad , \quad k = y-1$

$f(a,b) = f(-2,1)$

$= (-2)(1)^2 + 2(-2) - 3(1)$

$= -2-4-3 = -9$

$f(x,y) = xy^2+2x-3y$ $\qquad$ At $(-2,1)$

$f_x = y^2+2$ $\qquad 3$

$f_y = 2xy-3$ $\qquad -7$

$f_{xx} = 0$ $\qquad 0$

$f_{yy} = 2x$ $\qquad -4$

$f_{xy} = 2y$ $\qquad 2$

$$f(x,y) = f(a,b) + \frac{1}{1!}(h f_x + k f_y) + \frac{1}{2!}(h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy})$$

$$= -9 + [(x+2)3 + (y-1)(-7)] + \frac{1}{2}\left[(x+2)^2 \cdot 0 + 2(x+2)(y-1)(2) + (y-1)^2(-4)\right] + \dots$$

$$= -9 + (3x+6-7y+7) + \frac{1}{2}\left[4(x+2)(y-1) - 4(y-1)^2\right]$$

$$= -9 + (3x-7y+13) + \frac{1}{2}\left[4(x+2)(y-1) - 4(y-1)^2\right]$$