# UNIT - 3

## MATHEMATICAL LOGIC

**Proposition** → [statement]

A Proposition is a declarative sentence which either true or false. (or) Truth value.

Eg:-
(i) $1+1=2$ → ✓ True

(ii) $9<6$ → ✗ False

(iii) London is in Denmark → ✗ False

(iv) Paris is in France → ✓ True

(v) where are you going? → question type

(vi) sit down → Command type.

**Compound Proposition :-**

when one or more prositions are connected through various connectivities is called Compound Proposition.

eg: Roses are red and violets are blue.
Proposition 1    compound.    Proposition 2

A Proposition is said to be Primitive if it cannot be broken down into simpler Propositions.

eg Roses are red. → unbreakable.

## Basic Logical Operators:-

(i) Conjunction → AND → $\wedge$

(ii) Disjunction → OR → $\vee$

(iii) Negation → NOT → $\sim$ ($\sim$)

### Truth Table:-

1. Conjunction:-

Any two Propositions can be combined by the word 'and' to form a Compound Proposition called 'Conjunction'

let p and q be two Proposition.

$(p \wedge q)$ → is called Conjunction.

Truth Table:-

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

## 2. Disjunction:-

Any two Proposition combined by the word 'or' is called disjunction. it is denoted by $(p \vee q)$.

### Truth Table:-

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## 3. Negation:-

A single Proposition which value can be changed by using 'Negation'. it is denoted by $\sim p$ (or) $\sim q$

~~$\sim p, \sim q$~~

Truth Table.

| p | $\sim p$ |
|---|---|
| T | F |
| F | T |

(or)

| q | $\sim q$ |
|---|---|
| T | F |
| F | T |

Examples:-

(1) Find the Negation for "Vandana's smartphone has at least 32 GB of memory"

Ans:

"Vandana's smartphone does not have at least 32 GB of memory"

Truth Table of Compound Proposition:-

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ | $p \wedge q$ | $(p \vee \sim q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | T | T | F | F |
| F | T | F | F | F | T |
| F | F | T | T | F | F |

The Truth Table for the exclusive or of Two Propositions:-

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

Truth Table for Conditional statement $p \rightarrow q$

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

## Truth Table for Biconditional $(p \leftrightarrow q)$

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

| Truth value | Bit |
|---|---|
| T | 1 |
| F | 0 |

## Table for the Bit operator OR, AND, and XOR.

| $x$ | $y$ | $x \vee y$ | $x \wedge y$ | $x \oplus y$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

Ex:!

Construct a Truth Table for each of these compound proposition.

(a) $(q \to \sim p) \vee (\sim p \to \sim q)$

(b) $(p \vee \sim t) \wedge (p \vee \sim s)$

(c) ~~[illegible]~~

(d) ~~[illegible]~~

(a)

| $p$ | $q$ | $\sim p$ | $\sim q$ | $q \to \sim p$ | $\sim p \to \sim q$ ~~$(p \vee \sim p)$~~ | $(q \to \sim p) \vee (\sim p \to \sim q)$ ~~$(q \to \sim p) \vee (\sim p \to \sim q)$~~ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | T | F | T |
| F | F | T | T | T | T | T |

(b)

| $p$ | $t$ | $s$ | $\sim t$ | $\sim s$ | $p \vee \sim t$ | $p \vee \sim s$ | $(p \vee \sim t) \wedge (p \vee \sim s)$ |
|---|---|---|---|---|---|---|---|
| T | F | F | F | T | T | T | T |
| F | T | T | T | F | T | F | F |
| T | T | F | F | T | T | T | F |
| F | F | T | T | F | T | F | F |

Ex: 2 Let p and q be the propositions.

p: I bought a lottery ticket this week.

q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

(i) $\sim p$

I have not bought a lottery ticket this week.

(ii) $p \vee q$

~~I bought~~ a ~~lottery ticket this~~

I buy a lottery ticket this week (or) I win million dollar jackpot.

(iii) $p \rightarrow q$

I bought a lottery ticket this week.

(iv) $p \wedge q$

I bought a lottery ticket and I won the million dollar jackpot.

(v) $p \leftrightarrow q$

~~I bought~~

I won the million dollar jackpot if and only if I bought a lottery ticket this week.

## Propositional Equivalence:-

~~Compoun~~

**Tautology** → Truth values are true for any truth value of variables.

**Contradiction:** → Truth values are false for any truth value of variables.

**Contigency** → Some truth values are true and some truth values are false.

**Definition:-**

The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes p and q are logically equivalent.

| $p$ | $\sim p$ | tautology $(p \vee \sim p)$ | contradiction. $(p \wedge \sim p)$ |
|---|---|---|---|
| T | F | T | F |
| F | T | T | F |

**De Morgan's Law:-**

$$\boxed{\begin{aligned} \sim(p \wedge q) &\equiv \sim p \vee \sim q \\ \sim(p \vee q) &\equiv \sim p \wedge \sim q \end{aligned}}$$

Ex: 1 show that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent.

| $p$ | $q$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

Home work:

Ex: 2 show that $p \to q$ and $\sim p \vee q$ are logically equivalent.

## Basic laws:-

1. Identity laws $\Rightarrow$ $p \wedge T \equiv p$
   $p \wedge F \equiv p$

2. Domination laws $\Rightarrow$ $p \vee T \equiv T$
   $p \wedge F \equiv F$

3. Idempotent laws $\Rightarrow$ $p \vee p \equiv p$
   $p \wedge p \equiv p$

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5. Commutative laws ⇒ $p \vee q \equiv q \vee p$

$$p \wedge q \equiv q \wedge p$$

6. Associative laws ⇒ $(p \vee q) \vee r \equiv p \vee (q \vee r)$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

7. Distributive laws ⇒ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

8. De-Morgan's laws ⇒ $\sim(p \wedge q) \equiv \sim p \wedge \sim q$

$$\sim(p \vee q) \equiv \sim p \vee \sim q$$

9. Absorption law ⇒ $p \vee (p \wedge q) \equiv p$

$$p \wedge (p \vee q) \equiv p$$

10. Negation laws ⇒ $p \vee \sim p \equiv T$

$$p \wedge \sim p \equiv F$$

**Ex:1** Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$(p \wedge q) \rightarrow (p \vee q) \equiv \sim(p \wedge q) \vee (p \vee q)$ (by ~~above~~ result ~~example~~)

$\equiv (\sim p \wedge \sim q) \vee (p \vee q)$ by I (De morgan's law)

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$ (by associative and commutative laws)

$\equiv T \vee T$ (by result.)