



## DEPARTMENT OF MATHEMATICS

### UNIT - V SECOND ORDER LINEAR ORDINARY DIFFERENTIAL EQUATIONS

#### SECOND ORDER DIFFERENTIATION:

If RHS = 0 then

(i) Find auxiliary equation (ie). replace 'D' by 'm'

(ii) Find Complementary function.

∴ General soln. = complementary function alone  
Complementary function - 3 types

Type: 1

If the roots are real & different.

$m_1, m_2$  ( $m_1 \neq m_2$ ) then C.F. is  $y = Ae^{m_1x} + Be^{m_2x}$

Type: 2

If the roots are real & equal

$m_1 = m_2 = m$  (say) then C.F. is  $y = (Ax + B)e^{mx}$

Type: 3

If the roots are imaginary ( $\alpha \pm i\beta$ )

then C.F. is  $y = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$



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1) Solve:  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

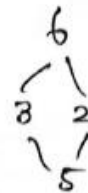
$$(D^2 + 5D + 6)y = 0$$

The auxiliary equation is

$$m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$\Rightarrow m = -3, m = -2$$



The complementary function  $y = Ae^{-2x} + Be^{-3x}$

$\therefore$  General soln is  $y = Ae^{-2x} + Be^{-3x}$

2)  $(D^2 + 4D + 4)y = 0$

A.E. is  $m^2 + 4m + 4 = 0$

$$(m+2)(m+2) = 0$$

$$m = -2; m = -2$$

General soln. is  $y = (Ax + B)e^{-2x}$



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$$3) (D^2 + 4D + 12)y = 0$$

$$\text{A.E. is } m^2 + 4m + 12 = 0$$

$$\text{Here } a = 1 ; b = 4 ; c = 12$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4 \times 12}}{2}$$

$$= \frac{-4 \pm \sqrt{-32}}{2} = \frac{-4 \pm \sqrt{-16 \times 2}}{2}$$

$$= \frac{-4 \pm 4i\sqrt{2}}{2}$$

$$= -2 \pm 2\sqrt{2}i$$

$$\text{Here } \alpha = -2 ; \beta = 2\sqrt{2}$$

$$\text{General Sol. is } y = e^{-2x} (A \cos 2\sqrt{2}x + B \sin 2\sqrt{2}x)$$



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Solve:  $(D^2 - 4D + 13)y = e^{2x}$

A.E. is  $m^2 - 4m + 13 = 0$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$
$$= 2 \pm 3i$$

C.F. is  $y = e^{2x}(A \cos 3x + B \sin 3x)$

P.I. =  $\frac{1}{D^2 - 4D + 13} e^{2x}$

$$= \frac{1}{4 - 8 + 13} e^{2x}$$

$$= \frac{1}{9} e^{2x}$$

$\therefore$  Complete Soln.  $y = e^{2x}(A \cos 3x + B \sin 3x) + \frac{1}{9} e^{2x}$

Solve:  $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = \sin 3x$

$$(D^2 + 3D + 2)y = \sin 3x$$

A.E. is  $m^2 + 3m + 2 = 0$

$$m = -1; m = -2$$

C.F. is  $y = Ae^{-x} + Be^{-2x}$



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$$\begin{aligned} P.I &= \frac{1}{D^2 + 3D + 2} \sin 3x \\ &= \frac{1}{-3^2 + 3D + 2} \sin 3x = \frac{1}{3D - 7} \sin 3x \\ &= \frac{1}{3D - 7} \times \frac{(3D + 7)}{(3D + 7)} \sin 3x \\ &= \frac{(3D + 7) \sin 3x}{9D^2 - 49} \\ &= \frac{3D(\sin 3x) + 7\sin 3x}{9(-9) - 49} = \frac{9 \cos 3x + 7\sin 3x}{-130} \end{aligned}$$

Complete soln is  $y = Ae^{-x} + Be^{-2x} - \frac{1}{130} (9 \cos 3x + 7 \sin 3x)$



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$$\text{Solve : } (D^2 + 4)y = \cos 2x$$

$$\text{A.E. is } m^2 + 4 = 0$$

$$m = \pm 2i$$

$$\text{C.F. is } y = A \cos 2x + B \sin 2x$$

$$\text{P.I.} = \frac{1}{D^2 + 4} \cdot \cos 2x$$

$$= \frac{1}{-4 + 4} \cos 2x = \frac{1}{0} \cos 2x$$

$$= \frac{1}{2D} \cos 2x$$

$$= \frac{x}{2} \cdot \frac{1}{D} (\cos 2x)$$

$$= \frac{x}{2} \cdot \frac{\sin 2x}{2} = \frac{x \sin 2x}{4}$$

$$\therefore \text{Complete Soln. is } y = A \cos 2x + B \sin 2x + \frac{x}{4} \sin 2x$$