SECOND ORDER DIFFERENTIATION:

If RHS = 0 then

(i) Find auxiliary equation (ie). replace 'D' by 'm'

(ii) Find Complementary function.

$\therefore$ General soln. = complementary function alone

Complementary function – 3 types

Type: 1

If the roots are real & different.

m_1, m_2 $(m_1 \neq m_2)$ then C.F is $y = Ae^{m_1 x} + Be^{m_2 x}$

Type: 2

If the roots are real & equal

$m_1 = m_2 = m$ (say) then C.F is $y = (Ax + B)e^{mx}$

Type: 3

If the roots are imaginary (ie) $\alpha \pm i\beta$

then C.F is $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

1) Solve: $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

$(D^2+5D+6)y = 0$

The auxiliary equation is

$m^2+5m+6=0$

$(m+3)(m+2)=0$

$\Rightarrow m=-3, m=-2$

6

3 2

5

The complementary function is $y = Ae^{-2x} + Be^{-3x}$

$\therefore$ General soln is $y = Ae^{-2x} + Be^{-3x}$

2) $(D^2+4D+4)y=0$

A.E is $m^2+4m+4=0$

$(m+2)(m+2)=0$

$m=-2 ; m=-2$

General soln is $y=(Ax+B)e^{-2x}$

3) $(D^2+4D+12)y = 0$

A.E is $m^2+4m+12 = 0$

Here $a=1$; $b=4$; $c=12$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16-4\times 12}}{2}$$

$$= \frac{-4 \pm \sqrt{-32}}{2} = \frac{-4 \pm \sqrt{-16\times 2}}{2}$$

$$= \frac{-4 \pm 4\sqrt{2}\, i}{2}$$

$$= -2 \pm 2\sqrt{2}\, i$$

Here $\alpha = -2$; $\beta = 2\sqrt{2}$

General soln. is $y = e^{-2x}(A\cos 2\sqrt{2}x + B\sin 2\sqrt{2}x)$

Solve: $(D^2-4D+13)y = e^{2x}$

A.E. is $m^2-4m+13=0$

$$m = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= 2 \pm 3i$$

C.F. is $y = e^{2x}(A\cos 3x + B\sin 3x)$

$$P.I = \frac{1}{D^2-4D+13} e^{2x}$$

$$= \frac{1}{4-8+13} e^{2x}$$

$$= \frac{1}{9} e^{2x}$$

$\therefore$ Complete Soln. $y = e^{2x}(A\cos 3x + B\sin 3x) + \frac{1}{9}e^{2x}$

Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin 3x$

$$(D^2+3D+2)y = \sin 3x$$

A.E. is $m^2+3m+2=0$

$$m=-1; m=-2$$

C.F. is $y = Ae^{-x} + Be^{-2x}$

$$P.I = \frac{1}{D^2+3D+2} \sin 3x$$

$$= \frac{1}{-3^2+3D+2} \sin 3x = \frac{1}{3D-7} \sin 3x$$

$$= \frac{1}{3D-7} \times \frac{(3D+7)}{(3D+7)} \sin 3x$$

$$= \frac{(3D+7)}{9D^2-49} \sin 3x$$

$$= \frac{3D(\sin 3x) + 7\sin 3x}{9(-9)-49} = \frac{9\cos 3x + 7\sin 3x}{-130}$$

Complete Soln is $y = Ae^{-x} + Be^{-2x} - \frac{1}{130}(9\cos 3x + 7\sin 3x)$

Solve : $(D^2+4)y = \cos 2x$

AE is $m^2+4=0$

$$m = \pm 2i$$

C.F. is $y = A\cos 2x + B\sin 2x$

$$P.I = \frac{1}{D^2+4}.\cos 2x$$

$$= \frac{1}{-4+4}\cos 2x = \frac{1}{0}\cos 2x$$

$$= \frac{1}{2D}x\cos 2x$$

$$= \frac{x}{2}\frac{1}{D}(\cos 2x)$$

$$= \frac{x}{2}\frac{\sin 2x}{2} = \frac{x\sin 2x}{4}$$

$\therefore$ Complete Soln. is $y = A\cos 2x + B\sin 2x + \frac{x}{4}\sin 2x$