Biomedical Image Processing

Relationships Between Pixels

Some Basic Relationships Between Pixels

- Neighbors of a pixel
  - $N_4(p)$: 4-neighbors of p

$(x+1, y)$, $(x-1, y)$, $(x, y+1)$, $(x, y-1)$

$N_D(p)$: four diagonal neighbors of p

$(x+1, y+1)$, $(x+1, y-1)$, $(x-1, y-1)$,

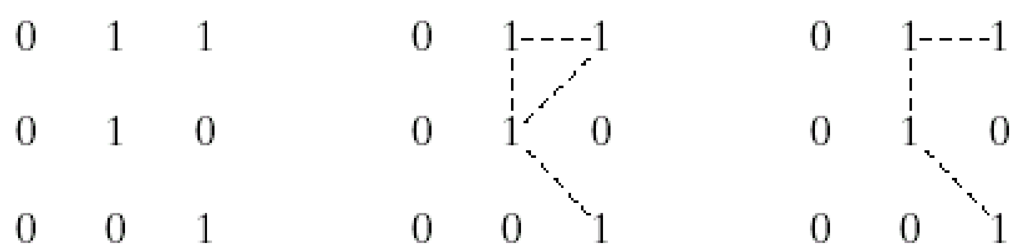
$(x-1, y+1)$

$N_8(p)$: 8-neighbors of p

$N_4(p)$ and $N_D(p)$

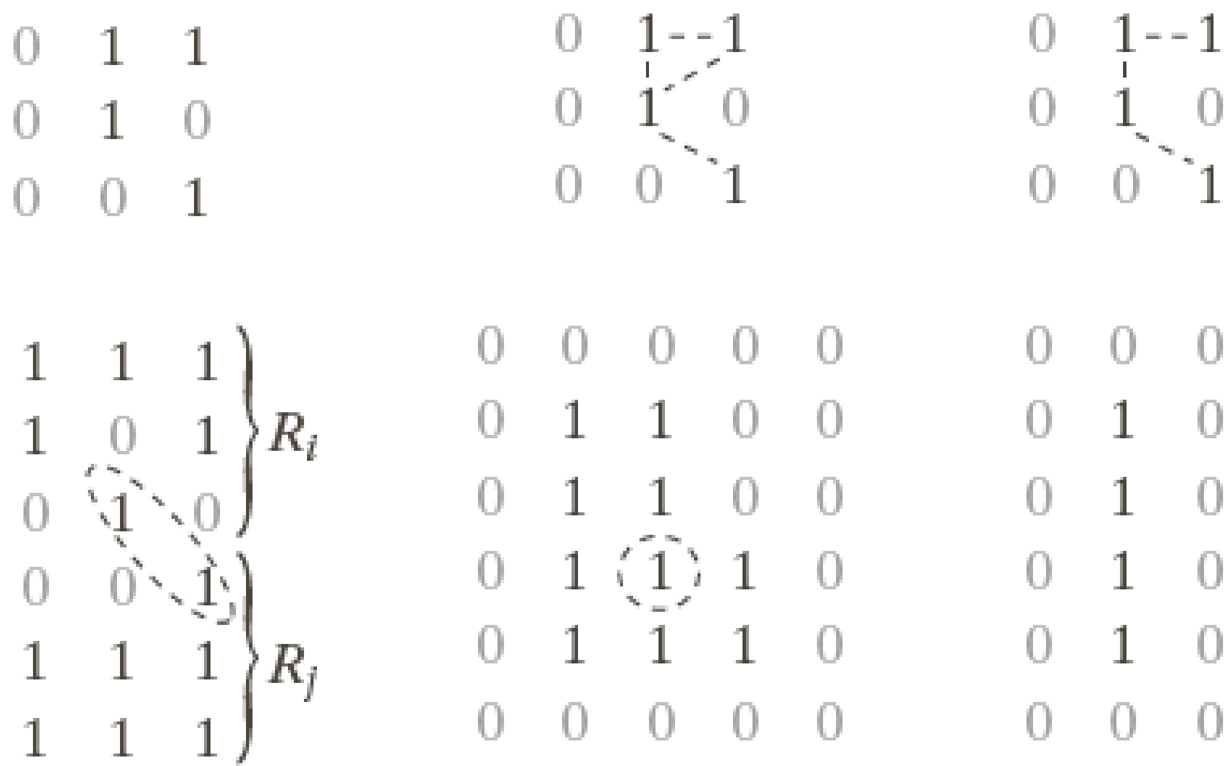
- Adjacency
  - V: The set of gray-level values used to define adjacency
  - 4-adjacency: Two pixels p and q with values from V are 4-adjacency if q is in the set $N_4(p)$
  - 8-adjacency: Two pixels p and q with values from V are 8-adjacency if q is in the set $N_8(p)$

- m-adjacency (mixed adjacency): Two pixels p and q with values from V are m-adjacency if
  - q is in $N_4(p)$, or
  - q is in $N_D(p)$ and the set $N_4(p) \bigcap N_4(q)$ has no pixels whose values are from V



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.



a	b	c
d	e	f

FIGURE 2.25 (a) An arrangement of pixels. (b) Pixels that are 8-adjacent (adjacency is shown by dashed lines; note the ambiguity). (c) *m*-adjacency. (d) Two regions that are adjacent if 8-adjecency is used. (e) The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used. (f) The inner boundary of the 1-valued region does not form a closed path, but its outer boundary does.

- Subset adjacency
  - S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2
- Path
  - A path from p with coordinates (x, y) to pixel q with coordinates (s, t) is a sequence of distinct pixels with coordinates
  - (x_0, y_0), (x_1, y_1), …, (x_n, y_n)

  where $(x_0, y_0) = (x, y)$, $(x_n, y_n) = (s, t)$, and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent

- Region
  - We call R a region of the image if R is a connected set
- Boundary
  - The boundary of a region R is the set of pixels in the region that have one or more neighbors that are not in R
- Edge
  - Pixels with derivative values that exceed a preset threshold

- Distance measures
  - Euclidean distance

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{\frac{1}{2}}$$

  - City-block distance

$$D_4(p,q) = |(x-s)| + |(y-t)|$$

  - Chessboard distance

$$D_8(p,q) = \max(|(x-s)|, |(y-t)|)$$

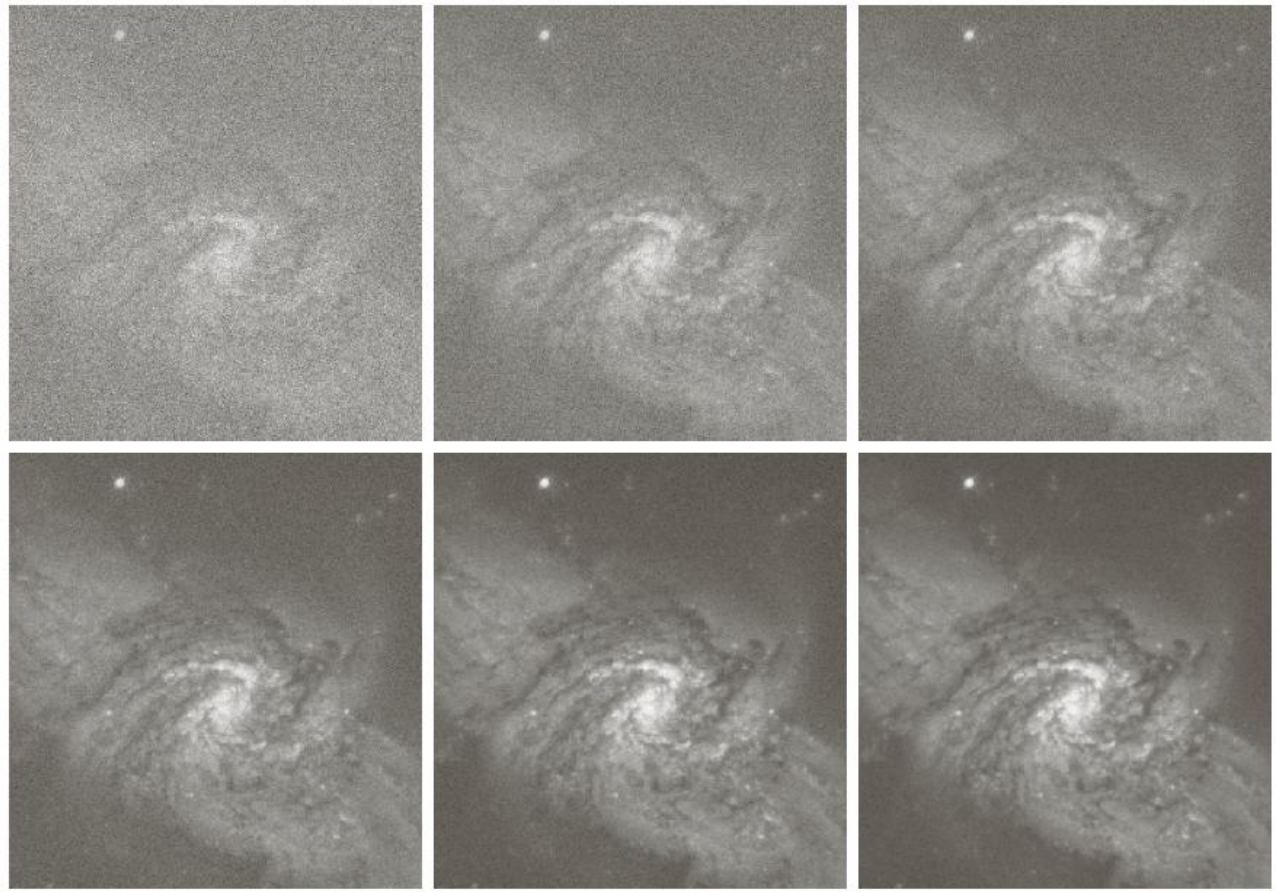
- D_m distance: The shortest m-path between the points

An Introduction to the Mathematical Tools Used in Digital Image Processing

- Linear operation
  - H is said to be a linear operator if, for any two images f and g and any two scalars a and b,

$$H(af + bg) = aH(f) + bH(g)$$

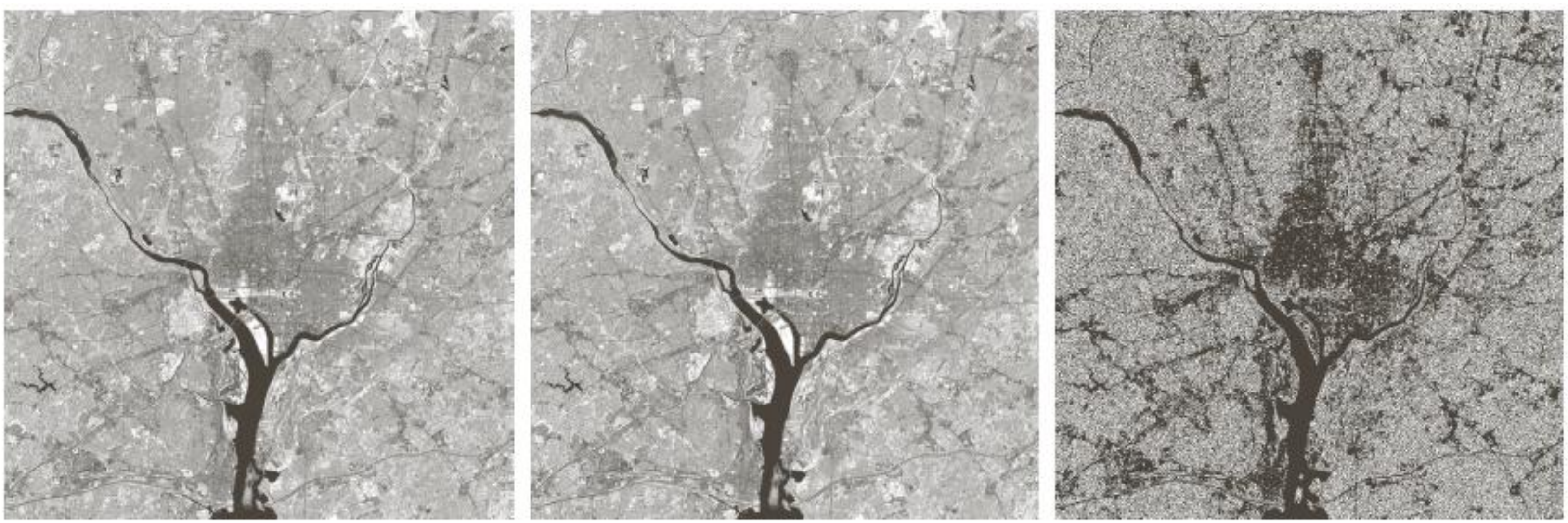
- Arithmetic operations
  - Addition



a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

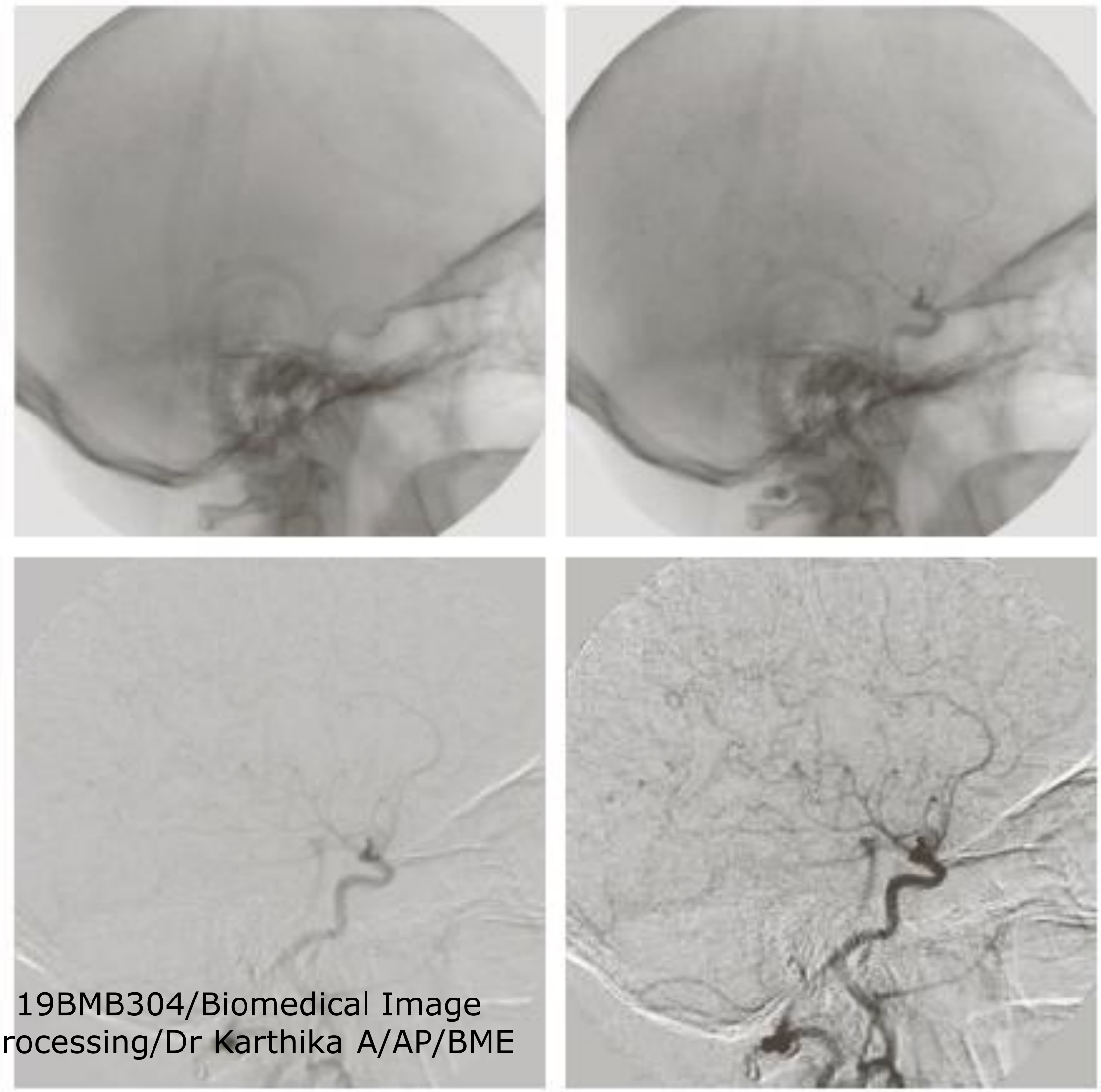
- Arithmetic operations
  - Subtraction



a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

- Digital subtraction angiography



a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

- Shading correction

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

- Image multiplication

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

- Set operations

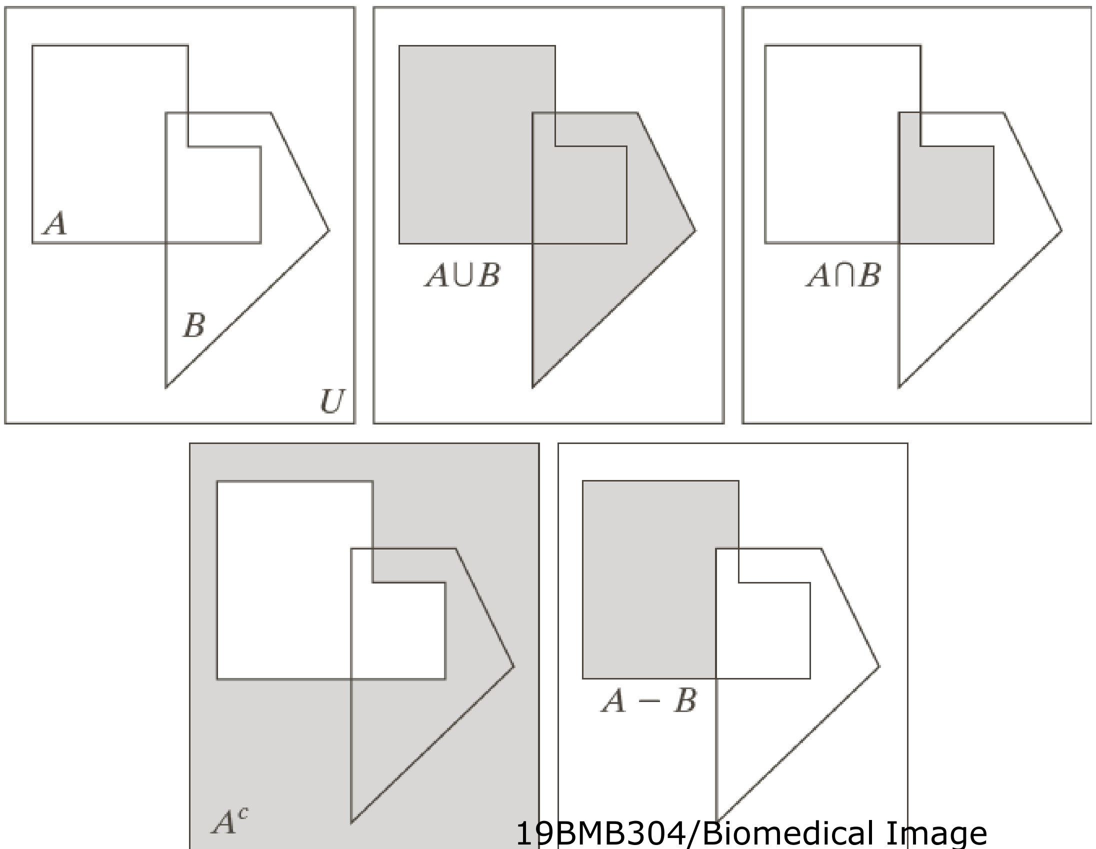
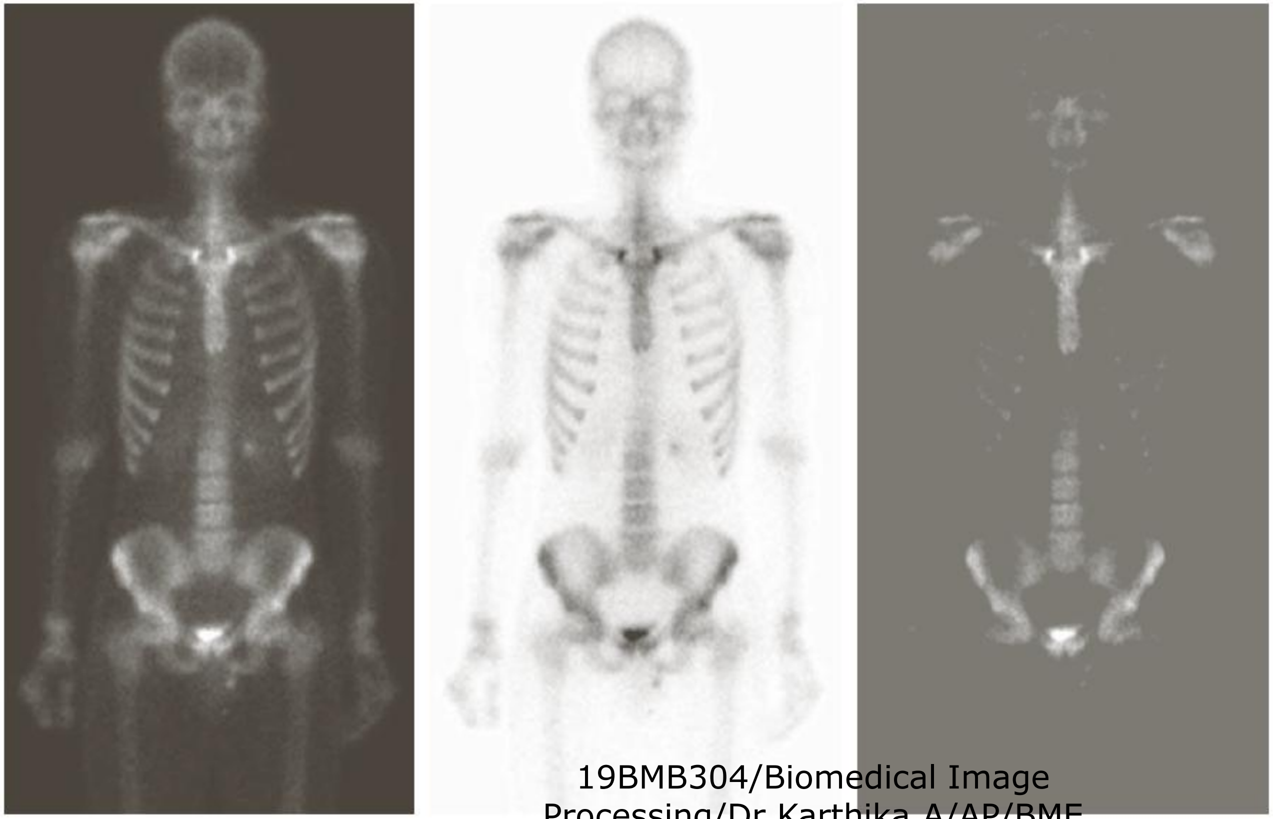


FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

- Complements



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

- Logical operations

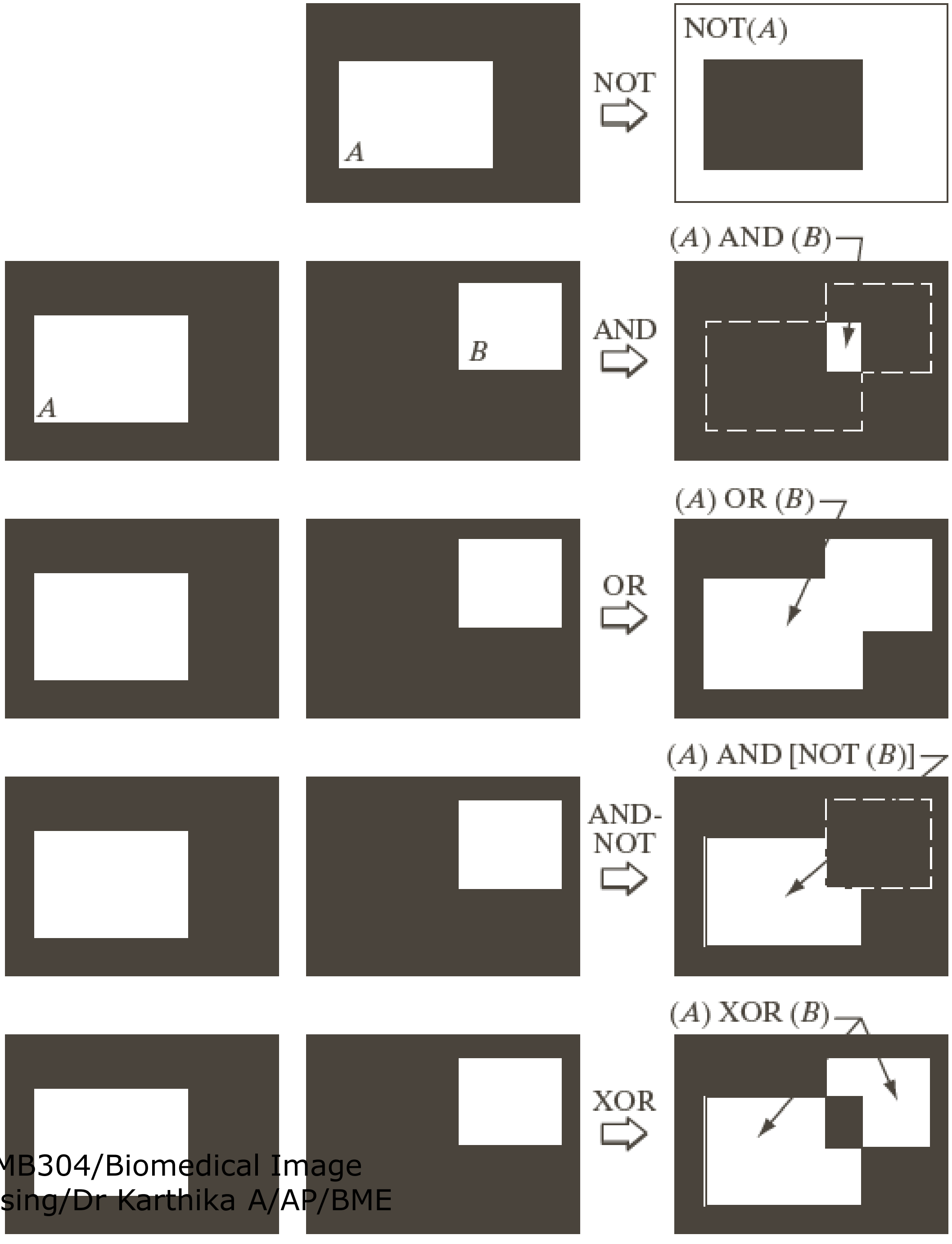


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

- Single-pixel operations

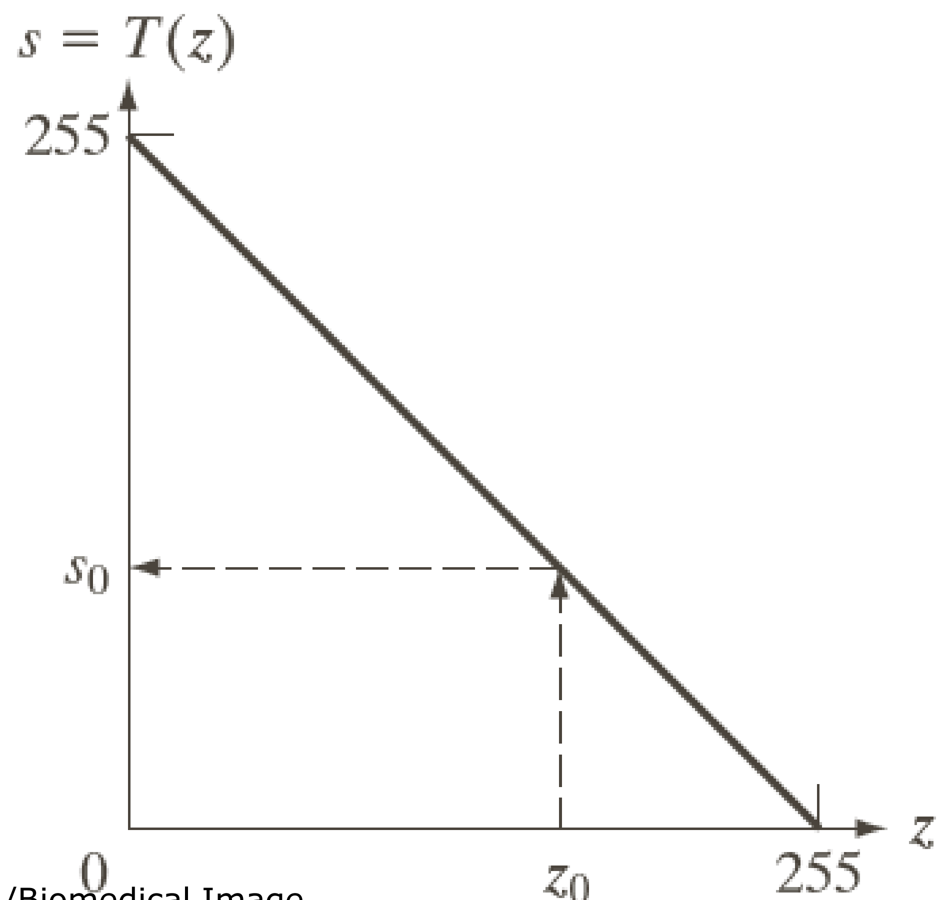
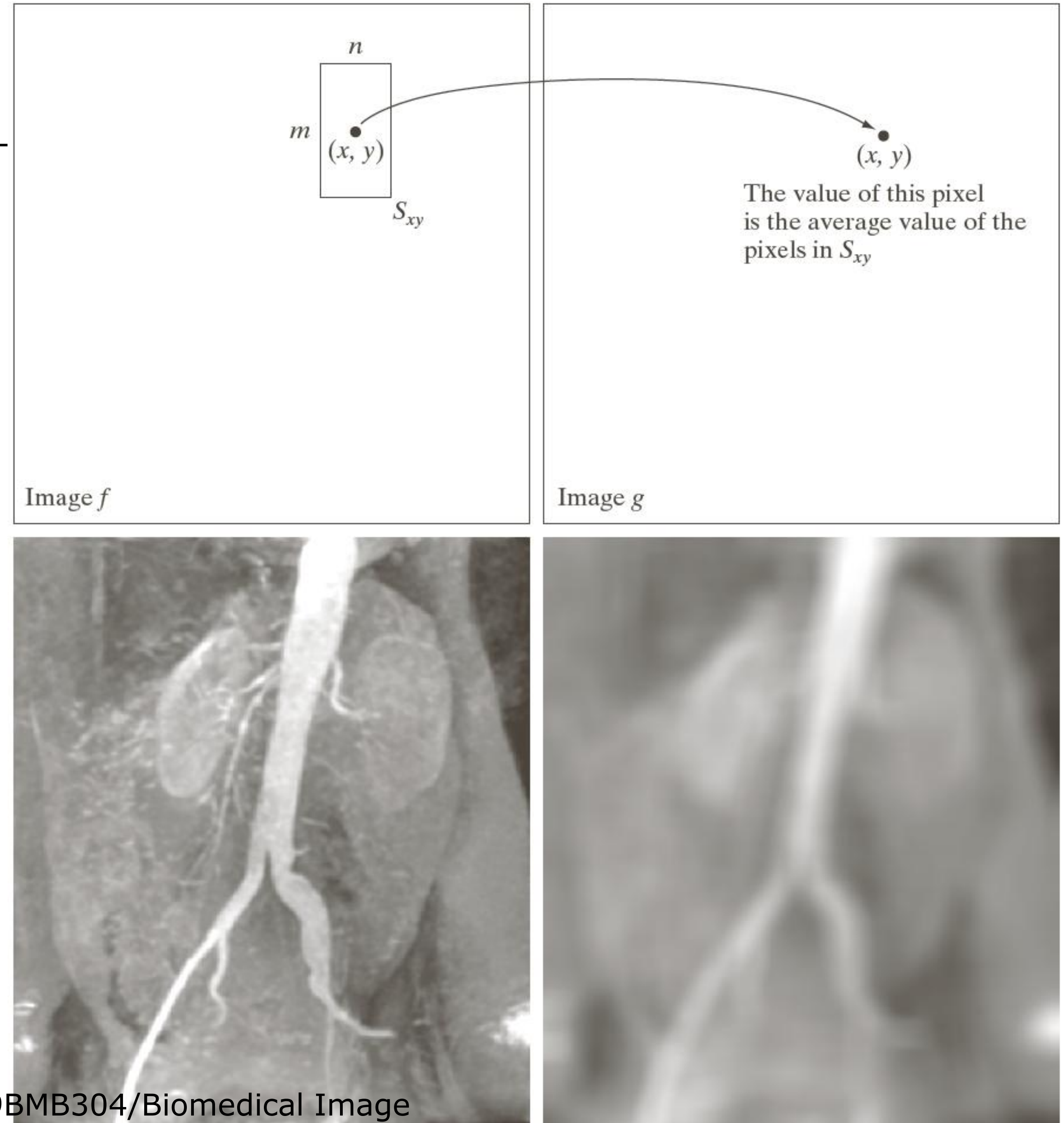


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Neighborhood operations

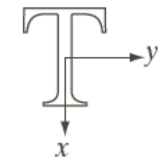
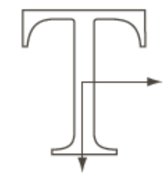
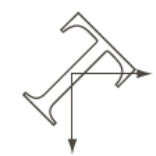
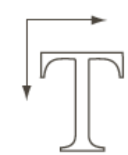
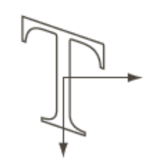


a b
c d

FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

Affine transformations

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Inverse mapping

a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Registration

a b
c d

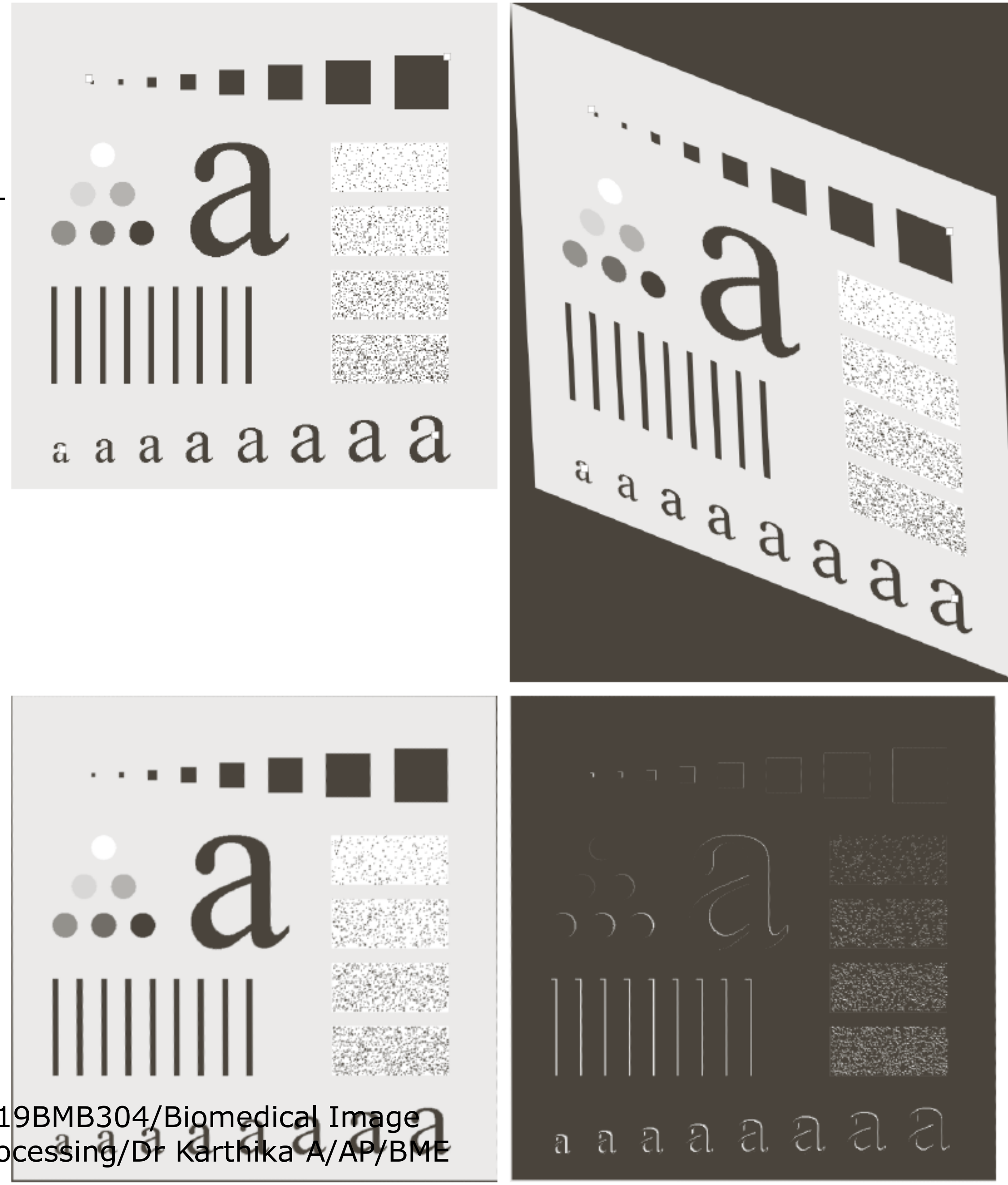
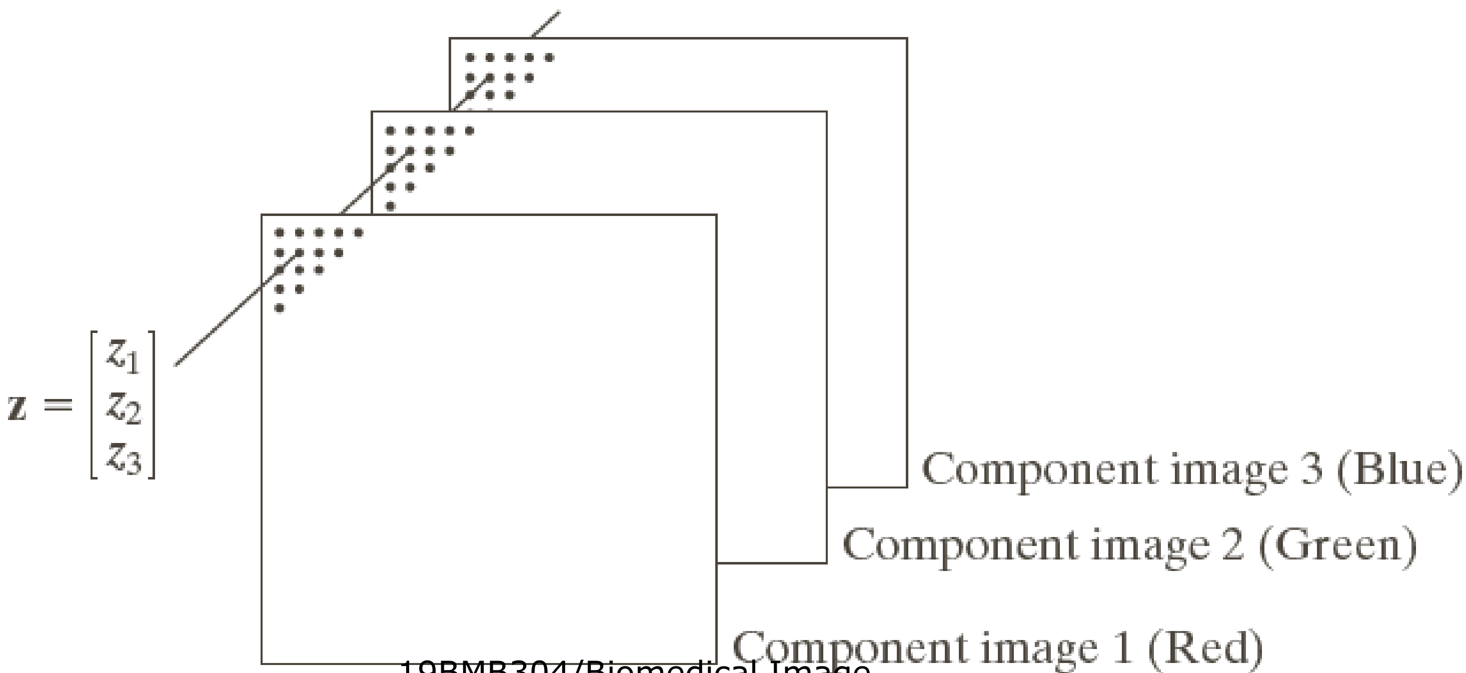


FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

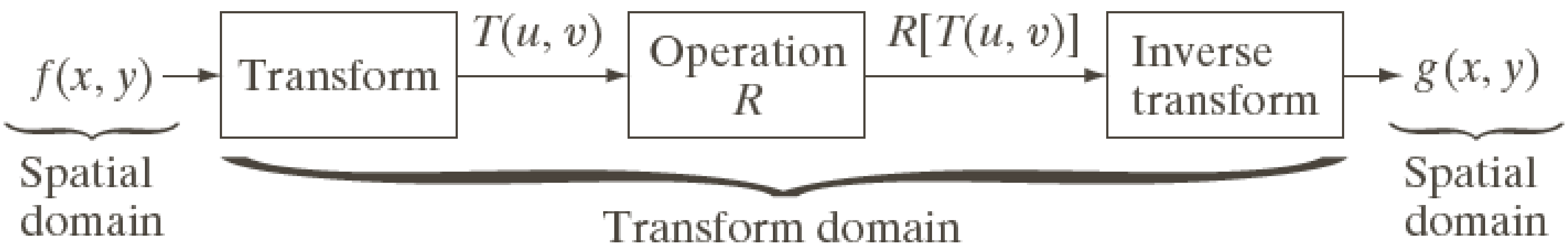
- Vector operations

FIGURE 2.38 Formation of a vector from corresponding pixel values in three RGB component images.

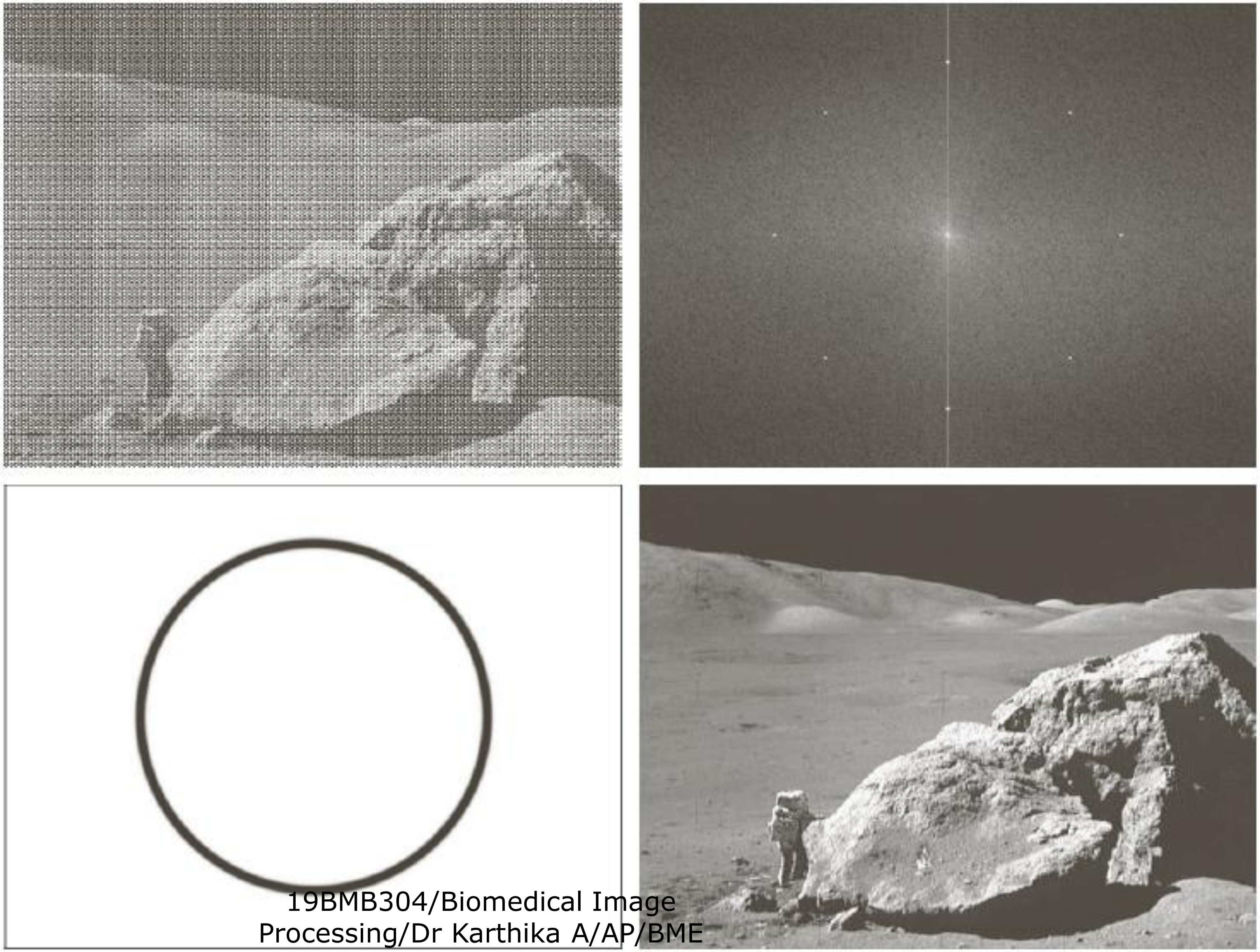


- Image transforms

FIGURE 2.39 General approach for operating in the linear transform domain.



Fourier transform



a b
c d

FIGURE 2.40
(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

- Probabilistic methods

a b c

FIGURE 2.41 Images exhibiting (a) low contrast, (b) medium contrast, and (c) high contrast.