Reg.No: | | | | | | | |

SNS College of Technology,Coimbatore-35.
(Autonomous)
B.E/B.Tech- Internal Assessment -I
Academic Year 2022-2023 (Even Semester)
Sixth Semester
Aerospace Engineering
19ASE303– Theory of Vibrations and Aero Elasticity

B

Time: $1^{1/2}$ Hours **Maximum Marks: 50**

Answer All Questions

PART - A (5x 2 = 10 Marks)

		CO	Blooms
1.	What are the causes of vibrations? 1) Unbalanced forces in the machine: Produced within the machine itself. 2) Dry friction between the two-mating surface: Self-excited vibration produced. 3) External excitations: These excitations may be periodic, random or the nature of an impact produced external to the vibrating system. 4) Earthquakes: These are responsible for the failure of many buildings, dams. 5) Winds: These are cause the vibration of transmission and telephone line under certain conditions	CO1	Rem
2.	Define critical speed of a shaft. Why is critical speed encountered? The speed at which resonance occurs is called critical speed of the shaft. In other words, the speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite is known as critical speed. The critical speed may occur due to one or more of the following reasons: 1) Eccentric mountings like gears, flywheels, pulleys, etc., 2) Bending of the shaft due to self-weight 3) Non-uniform distribution of rotor material, etc.	CO1	Rem
3.	Write about the terms: free vibrations, forced vibrations and damped vibrations. Free or natural vibrations: When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations. Forced vibrations: When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. Damped vibrations: When there is a reduction in amplitude over every cycle of vibration, then the motion is said to be damped vibration.	CO1	App
4.	Write about Rayleigh's method of finding the natural frequency of transverse vibrations. Ans: Consider a shaft is loaded with point loads W1, W2, W3 and W4 etc. and y1, y2, y3, y4 etc. be total deflection made under these loads. According to Rayleigh's method, the maximum potential energy is equal to maximum kinetic energy. $\frac{1}{2} \Sigma m g y = \frac{1}{2} \omega^2 \Sigma m y^2$	CO2	App

	; ω = √ $g\ \Sigma\ my\ /\Sigma\ m\ y^2$ Natural frequency of transverse vibration, $fn = \omega\ ^2\pi = 1\ /2\pi$ √ $g\ \Sigma\ my\ \Sigma\ m\ y^2$		
5.	Why is it important to find the natural frequency of a vibrating system? When the frequency of externally excited system equal to natural frequency of vibration system it gets failure due to resonance. So, to avoid the resonance at vibrating system natural frequency must be known.	CO2	Rem

PART – B (13+13+14 =40 Marks)

<table>
<tr><th></th><th></th><th></th><th></th><th>CO</th><th>Blooms</th></tr>
<tr><td>6.</td><td>a</td><td>

Describe with relevant sketches, the equilibrium method to find the natural frequency of free longitudinal vibrations.

Ans:

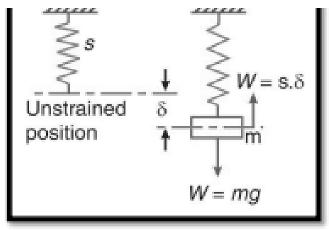
For the system shown in figure,

s = Stiffness of the constraint

m = Mass of the body

W = Weight of the body in newtons = m g

δ = Static deflection of the spring



For the equilibrium of the system, m g = s δ

Giving a displacement to the mass 'm' by a distance 'x' from its equilibrium position.

The restoring force will be

= W − s (δ+ x) = W− s δ − s x = − s x

(∵ W = s. δ)

taking upward force as negative

Force = *Mass* × Acceleration = m x $\frac{d^2x}{dt^2}$

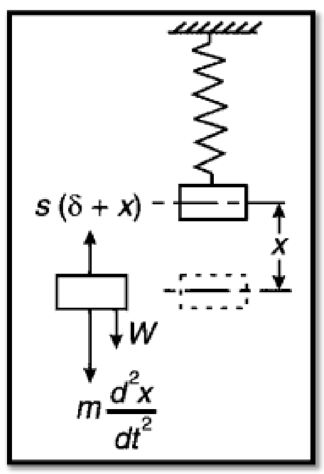
taking downward force as positive.

From the above two equations,

motion of the body of mass m after time t is given by

$$m \times \frac{d^2x}{dt^2} = -s \times x$$

$$m \times \frac{d^2x}{dt^2} + s \times x = 0$$



</td><td>13</td><td>CO1</td><td>Ana</td></tr>
</table>

		∴ Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$ The natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ where mg = s δ taking g = 9.81 m/s^2 and δ in meters, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} Hz$ where the δ is the static deflection. This can be determined from the following equations $\frac{Stress}{Strain} = E$ $\frac{W}{A} \times \frac{l}{\delta} = E$ $\delta = \frac{W.l}{A.E}$			
		(or)			
	b	Find the frequency of transverse vibrations of a shaft which is simply supported at the ends and is of 40 mm in diameter. The length of the shaft is 5 m. The shaft carries three, point loads of masses 15 kg, 35 kg, and 22.5 kg at 1m, 2m and 3.4m respectively from the left support. The Young's modulus for the material of the shaft is 200 GN/m^2. The weight of the shaft is 18.394N per meter length. Solution: d = 40 mm = 0.04 m; l = 5 m; I = $\frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times (0.04)^4 = 0.1257 \times 10^{-6}\ m^4$ $f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \cdots\ldots + \frac{\delta_s}{1{,}27}}}$ $\delta = \frac{m\,g\,a^2 b^2}{3\,E\,I\,l}$ here for δ_1, m = 15 kg; a = 1 m; b = 4 m $\delta_1 = \frac{15 \times 9.81 \times 1^2\ 4^2}{3 \times 200 \times 10^9 \times 0.1257 \times 10^{-6} \times 5}$ = 0.00624 m	13	CO1	Eva

		for δ_2, m = 35 kg; a = 2 m; b = 3 m $\delta_2 = \frac{35 \times 9.81 \times 2^2\, 3^2}{3 \times 200 \times 10^9 \times 0.1257 \times 10^{-6} \times 5} = 0.03277$ m for δ_3, m = 22.5 kg; a = 3.4 m; b = 1.6 m $\delta_3 = \frac{22.5 \times 9.81 \times 3.4^2\, 1.6^2}{3 \times 200 \times 10^9 \times 0.1257 \times 10^{-6} \times 5} = 0.01732$ m $\delta_s = \frac{5\, W\, l^4}{384\, E\, I} = \frac{5 \times 18.394 \times 5^4}{384 \times 200 \times 10^9 \times 0.1257 \times 10^{-6}} = 0.00595$ m $f_n = \frac{0.4985}{\sqrt{0.00624+0.03277+0.01732+\frac{0.00595}{1.27}}} = 2.02$ Hz			
7.	a	A single degree of freedom spring mass damper has a mass of 60kg and spring stiffness of 6000N/m. Determine the following (i)Critical damping coefficient (ii) Damped natural frequency when c=800Ns/m (iii)Logarithmic decrement (a) m = 60 kg. K = 6000 N/m. $C_c = 2m\omega_n = 2m.\sqrt{\frac{K}{m}} = 2\sqrt{Km}$. $= 2 \times \sqrt{6000 \times 60} = 1200$ Ns/m. (b) Now. $c = 2 \cdot \frac{C_c}{3} = 800$ Ns/m. Damped natural frequency $\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{\frac{K}{m}}\sqrt{1-\left(\frac{c}{c_c}\right)^2}$ $= \sqrt{\frac{6000}{60}}\sqrt{1-\left(\frac{800}{1200}\right)^2}$ $= 7.45$ rad/sec. (c) Logarithmic decrement $\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi\left(\frac{2}{3}\right)}{\sqrt{2\pi - (2/3)^2}}$ $= 5.6198$	13	CO2	Eva
		(or)			

<table>
<tr><td></td><td>b</td><td>Derive the Differential Equation of free damped vibrations

Spring-mass-damper system:

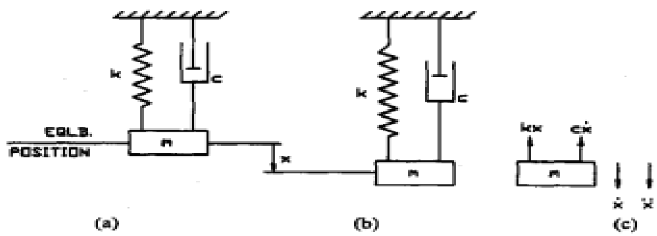


Fig 3

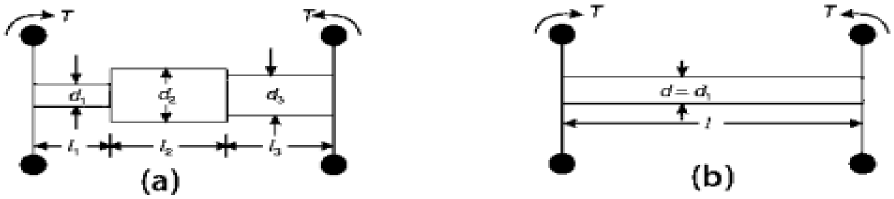
Fig.3 shows the schematic of a simple spring-mass-damper system, where, m is the mass of the system, k is the stiffness of the system and c is the damping coefficient.
If x is the displacement of the system, from Newton's second law of motion, it can be written
$m\ddot{x} = -c\dot{x} - kx$
Ie $m\ddot{x} + c\dot{x} + kx = 0$ (1)
This is a linear differential equation of the second order and its solution can be written as
$x = e^{st}$ (2)
Differentiating (2), $\frac{dx}{dt} = \dot{x} = se^{st}$
$\frac{d^2x}{dt^2} = \ddot{x} = s^2e^{st}$
Substituting in (1), $ms^2e^{st} + cse^{st} + ke^{st} = 0$
$(ms^2 + cs + k)e^{st} = 0$
Or $ms^2 + cs + k = 0$ (3)
Equation (3) is called the characteristic equation of the system, which is quadratic in s.
The two values of s are given by
$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$ (4)

The general solution for (1) may be written as
$x = C_1e^{s_1t} + C_2e^{s_2t}$ (5)
Where, C_1 and C_2 are arbitrary constants, which can be determined from the initial conditions.
In equation (4), the values of $s_1 = s_2$, when $\left(\frac{c}{2m}\right)^2 = \frac{k}{m}$
Or, $\left(\frac{c}{2m}\right) = \sqrt{\frac{k}{m}} = \omega_n$ (6)
Or $c = 2m\omega_n$, which is the property of the system and is called critical damping coefficient and is represented by c_c.
Ie, critical damping coefficient = $c_c = 2m\omega_n$
The ratio of actual damping coefficient c and critical damping coefficient c_c is called damping factor or damping ratio and is represented by ζ.
.
Ie, $\zeta = \frac{c}{c_c}$ (7)
In equation (4), $\frac{c}{2m}$ can be written as $\frac{c}{2m} = \frac{c}{c_c} \times \frac{c_c}{2m} = \zeta.\omega_n$
Therefore, $s_{1,2} = -\zeta.\omega_n \pm \sqrt{(\zeta.\omega_n)^2 - \omega_n^2} = \left[-\zeta \pm \sqrt{\zeta^2 - 1}\right]\omega_n$ (8)

The system can be analyzed for three conditions.
(i) $\zeta > 1$, ie, $c > c_c$, which is called over damped system.
(ii) $\zeta = 1$. ie, $c = c_c$, which is called critically damped system.
(iii) $\zeta < 1$, ie, $c < c_c$, which is called under damped system.

Depending upon the value of ζ, value of s in equation (8), will be real and unequal, real and equal and complex conjugate respectively.</td><td>13</td><td>CO2</td><td>Ana</td></tr>
<tr><td>8.</td><td>a</td><td>Derive the length of torsionally equivalent shaft.</td><td>14</td><td>CO</td><td>Ana</td></tr>
</table>

Consider a shaft of varying cross-sections as shown in below fig. Let this shaft is replaced by an equivalent shaft of uniform diameter *d* and length *l* as shown figure. 1



(a) Shaft of varying diameter (b)Torsionally Equivalent shaft

These two shafts must have the same total angle of a twist when equal opposing torques *T* are applied at the opposite ends.

d_1, d_2 and d_3 = Diameters for the lengths l_1, l_2 and l_3 respectively.

θ_1, θ_2 and θ_3 = Angles of twist for the lengths l_1, l_2 and l_3 respectively.

θ = Angle of twist for the diameter *d* and length *l*.

J_1, J_2 and J_3 = Polar moment of inertia for the shaft of diameters d_1, d_2 *and* d_3 respectively.

Since the total angle of the twist of the shaft is equal to the sum of angle of twists of the different lengths.

$$\theta = \theta_1 + \theta_2 + \theta_3$$

From the torsion Equation

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{JG}$$

where,

τ = Shear stress (MPa)

r = Radius of the shaft (mm)

T = Torque (Nm)

J = Polar moment of inertia

G = Modulus of rigidity (MPa)

θ = Angle of twist (rad)

l = length of the shaft

We can write $\theta = \theta_1 + \theta_2 + \theta_3$ as following

$$\frac{Tl}{JG} = \frac{Tl_1}{J_1G} + \frac{Tl_2}{J_2G} + \frac{Tl_3}{J_3G}$$

$$\frac{l}{J} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\frac{l}{\frac{\pi}{32}d^4} = \frac{l_1}{\frac{\pi}{32}(d_1)^4} + \frac{l_2}{\frac{\pi}{32}(d_2)^4} + \frac{l_3}{\frac{\pi}{32}(d_3)^4}$$

$$\frac{l}{d^4} = \frac{l_1}{(d_1)^4} + \frac{l_2}{(d_2)^4} + \frac{l_3}{(d_3)^4}$$

As we want to make the multiple cross-sectional shafts into a uniform diametral shaft, so we have to assume that diameter d of the equivalent shaft should be equal to the one of the diameters of the actual shaft. So, we can assume d = d_1

Substitute

$$\frac{l}{(d_1)^4} = \frac{l_1}{(d_1)^4} + \frac{l_2}{(d_2)^4} + \frac{l_3}{(d_3)^4}$$

$$l = \frac{l_1(d_1)^4}{(d_1)^4} + \frac{l_2(d_1)^4}{(d_2)^4} + \frac{l_3(d_1)^4}{(d_3)^4}$$

$$l = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

		(or)			
	b	Write an equation about logarithmic decrement equation of free damped vibrations.	14	CO2	App

LOGARITHMIC DECREMENT

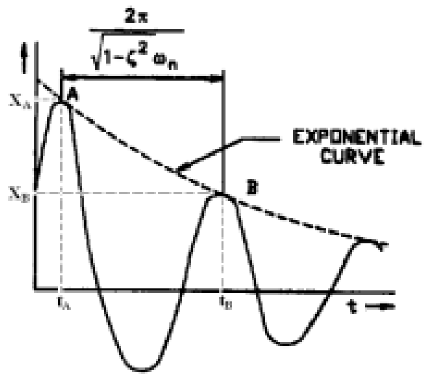


Fig.7 Logarithmic decrement

Referring to Fig.7, points A & B represent two successive peak points on the response curve of an under damped system. X_A and X_B represent the amplitude corresponding to points A & B and t_A & t_B represents the corresponding time.

We know that the natural frequency of damped vibration = $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$ rad/sec.

Therefore, $f_d = \frac{\omega_d}{2\pi}$ cycles/sec

Hence, time period of oscillation = $t_B - t_A = \frac{1}{f_d} = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\zeta^2}\,\omega_n}$ sec (16)

From equation (15), amplitude of vibration

$$X_A = \frac{X_o}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_A}$$

$$X_B = \frac{X_o}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_B}$$

Or, $$\frac{X_A}{X_B} = e^{-\zeta\omega_n(t_A - t_B)} = e^{\zeta\omega_n(t_B - t_A)}$$

Using eqn. (16), $$\frac{X_A}{X_B} = e^{\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Or, $$\log_e \frac{X_A}{X_B} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

This is called logarithmic decrement. It is defined as the logarithmic value of the ratio of two successive amplitudes of an under damped oscillation. It is normally denoted by δ.

Therefore, $$\delta = \log_e \frac{X_A}{X_B} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad (17)$$

This indicates that the ratio of any two successive amplitudes of an under damped system is constant and is a function of damping ratio of the system.

For small values of ζ, $\delta \approx 2\pi\zeta$

If X_0 represents the amplitude at a particular peak and X_n represents the amplitude after 'n' cycles, then, logarithmic decrement = $\delta = \log_e \frac{X_0}{X_1} = \log_e \frac{X_1}{X_2} = \ldots\ldots = \log_e \frac{X_{n-1}}{X_n}$

Adding all the terms, $$n\delta = \log_e \frac{X_0}{X_1} \times \frac{X_1}{X_2} \ldots\ldots \frac{X_{n-1}}{X_n}$$

Or, $$\delta = \frac{1}{n}\log_e \frac{X_0}{X_n} \quad (18)$$