The Assignment Problem

There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *i* to job *j* is C[*i*,*j*]. Find an assignment that minimizes the total cost.

	Job 0	Job 1	Job 2	Job 3
Person 0	9	2	7	8
Person 1	6	4	3	7
Person 2	5	8	1	8
Person 3	7	6	9	4

Algorithmic Plan:

Generate all legitimate assignments

Compute costs

Select cheapest

Assignment Problem: Exhaustive Search

$$C = \begin{matrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{matrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
…	…

(For this instance, the optimal assignment can be easily found by exploiting the specific features of the numbers given. It is:)

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…	…

(For this instance, the optimal assignment can be easily found by exploiting the specific features of the numbers given. It is: (2, 1, 3, 4)

Example 3: The Assignment Problem

There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *i* to job *j* is C[*i*,*j*]. Find an assignment that minimizes the total cost.

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Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

How many assignments are there?

Describe sol'n using cost matrix:

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There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *i* to job *j* is C[*i*,*j*]. Find an assignment that minimizes the total cost.

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Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

How many assignments are there: permutations of 1..n = n!

Sol’n using cost matrix: select one from each row/col. Min sum.

Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time only on very small instances

- In some cases, there are much better alternatives!
  - Euler circuits
  - shortest paths
  - minimum spanning tree
  - assignment problem

- In many cases, exhaustive search or its variation is the only known way to get exact solution

Brute Force: Review

- Based on problem statement and definitions

- Typically slow, but may be only known algorithm

- Useful to consider first
  - better algorithm frequently known

- Examples:
  - Sorting and Searching
  - Exhaustive Search:
    - Pattern Match, TSP, Knapsack, Assignment,
  - Graph (DFS, BFS)

Brute-Force Strengths and Weaknesses

- Strengths
  - Wide applicability
  - Simplicity
  - Yields reasonable algorithms for some important problems (e.g., matrix multiply, sorting, searching, string matching)
  - Algorithm may be good enough for small problem
  - Improvement may be too hard
  - Provides yardstick for comparison

- Weaknesses
  - Rarely yields efficient algorithms
  - Some brute-force algorithms are unacceptably slow
  - Not as constructive as some other design techniques