Maxima or Minima

Critical point or stationary point.

Critical point or stationary point of the fn is a point (a,b) if $\frac{\partial f}{\partial x}=0$ at (a,b)

Necessary Conditional

The necessary condition for the function $f(x,y)$ to have a maxima or minima at the point (a,b) is $\frac{\partial f}{\partial x}=0$, $\frac{\partial f}{\partial y}=0$ at (a,b)

Sufficient condition

Let $A=\frac{\partial^2 f}{\partial x^2}$, $B=\frac{\partial^2 f}{\partial x \partial y}$, $C=\frac{\partial^2 f}{\partial y^2}$

1) If $AC-B^2>0, A>0$ minimum value
2) If $AC-B^2>0, A<0$ maximum value
3) If $AC-B^2<0$ Neither maximum nor minimum (saddle pt)
4) If $AC-B^2=0$ Inconclusive

1. Find the maxima & minima of the function

$f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$

$\frac{\partial f}{\partial x}=3x^2+3y^2-30x+72$

$\frac{\partial f}{\partial y}=6xy-30y$

$\frac{\partial f}{\partial x}=0 \Rightarrow 3x^2+3y^2-30x+72=0 \rightarrow (1)$

$\frac{\partial f}{\partial y}=0 \Rightarrow 6xy-30y=0 \rightarrow (2)$

$\Rightarrow 6y(x-5)=0$

$y=0, \quad x=5$

$y=0$ in (1)

$3x^2+0-30x+72=0$

$x^2-10x+24=0$

$x=4, 6$

put $x = 5$ in (1)

$$75 + 3y^2 - 150 + 72 = 0$$

$$3y^2 - 3 = 0$$

$$y^2 = 1$$

$$y = \pm 1$$

The point is $(5,-1)$ $(5,1)$

The critical points are

$(4,0)$ $(6,0)$ $(5,-1)$ $(5,1)$

$$A = \frac{\partial^2 f}{\partial x^2} = 6x - 30$$

$$B = \frac{\partial^2 f}{\partial x \partial y} = 6y$$

$$C = \frac{\partial^2 f}{\partial y^2} = 6x - 30$$

Critical point	A	B	C	$AC - B^2$	Conclusion
$(6,0)$	6	0	6	36	Minimum value
$(4,0)$	-6	0	-6	36	Maximum value
$(5,1)$	0	6	0	-36	Saddle point
$(5,-1)$	0	-6	0	36	Saddle point

Maximum value $= f(4,0) = 112$

minimum value $= f(6,0) = 108$

$f = x^3 + y^3 - 12x - 3y + 20$

$\frac{\partial f}{\partial x} = 3x^2 - 12$, $\frac{\partial f}{\partial y} = 3y^2 - 3$

$\frac{\partial f}{\partial x} = 0 \Rightarrow \quad 3x^2 - 12 = 0 \rightarrow (1)$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$\frac{\partial f}{\partial y} = 0 \Rightarrow \quad 3y^2 - 3 = 0$

$y^2 = 1$

$y = \pm 1$

The critical pts are (2,1) (2,-1) (-2,-1), (-2,1)

$A = f_{xx} = 6x$

$B = f_{xy} = 0$

$C = f_{yy} = 6y$

Critical pt	A	B	C	$AC - B^2$	Conclusion
(2,1)	12	0	6	72	Minimum pt
(2,-1)	12	0	-6	-72	Saddle pt
(-2,1)	-12	0	6	-72	Saddle pt
(-2,-1)	-12	0	-6	72	Maximum pt

Maximum value $= f(-2,-1) = 38$

Minimum value $= f(2,1) = 2$