Unit - II

Design of Experiments

Analysis of Variance [ANOVA]:

ANOVA is a technique that will enable us to test the significance of the difference among more than two sample mean.

Assumption:

1). The observations are random.

2). The observations are independent

3). The samples are drawn from normal populations

4). Population variances are equal.

Basic principles:

1). Randomisation

2). Replication

3). Local control

Basic Design:

* Completely Randomized Design (CRD)
  ↳ One way classification
* Randomized Block Design (RBD)
  ↳ Two way classification
* Latin Square design (LSD)
  ↳ Three way classification

Hint:

F-ratio : $F = \dfrac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$

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Procedure :

1). Formulating H_0 & H_1

2). Sum of all the terms (T) & Total no. of Sample size (N)

3). correction factor $CF = \frac{T^2}{N}$

4). TSS : Total Sum of Squares

$$= \left(\begin{array}{c}\text{Sum of the squares of} \\ \text{all the terms}\end{array}\right) - CF$$

5). SSC = Sum of Squares b/w Samples

6). SSE = Error Sum of Squares

= TSS − SSC

7). ANNOVA TABLE

8). Conclusion: $|F| < |F_\alpha|$, H_0 is accepted

Source of variation	Sum of squares	Degrees of freedom	mean sum of square	variance ratio
Between columns	SSC	C−1	$MSC = \frac{SSC}{C-1}$	$\frac{MSC}{MSE}$ or $\frac{MSE}{MSC}$
Between Errors	SSE	N−C	$MSE = \frac{SSE}{N-C}$	$F_\alpha(C-1, N-C)$ or $F_\alpha(N-C, C-1)$

I A completely randomised design experiment with 10 plots & 3 treatments gave the following result.

Plot No. :	1	2	3	4	5	6	7	8	9	10
Treatment :	A	B	C	A	C	C	A	B	A	B
Yield :	5	4	3	7	5	1	3	4	1	7

Analyse the result for treatment effects.

Soln.

Treatment	Yield			
A	5	7	3	1
B	4	4	7	-
C	3	5	1	-

	Treatment		
	A	B	C
Yield	5	4	3
	7	4	5
	3	7	1
	1	-	-

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x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
5	4	3	12	25	16	9
7	4	5	16	49	16	25
3	7	1	11	9	49	1
1	-	-	1	1	-	-
16	15	9	40	84	81	35
Σx_1	Σx_2	Σx_3		Σx_1^2	Σx_2^2	Σx_3^2

Step 1 :

Formulating H_0 & H_1

H_0 : There is no significance difference b/w the treatments

H_1 : There is a significance difference b/w the treatments.

Step 2 :

To find T & N.

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$$

$$= 16 + 15 + 9 = 40$$

$$N = n_1 + n_2 + n_3 = 4 + 3 + 3 = 10$$

Step 3 :

Correction Factor : $CF = \frac{T^2}{N} = \frac{40^2}{10} = 160$

Step 4 :

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - CF$$

$$= 84 + 81 + 35 - 160$$

$$= 40$$

Step 5 :

$$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - CF$$

$$= \frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{9^2}{3} - 160$$

$$= 6$$

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Step 6: $SSE = TSS - SSC$

$= 40 - 6$

$= 34$

Step 7: ANNOVA TABLE

Source of variation	Sum of Squares	Degrees of freedom	Mean Square	F
Between Samples (Column)	SSC: 6	$c-1 = 3-1 = 2$	$MSC = \frac{6}{2} = 3$	$F_c =$ [illegible]
Within Samples (Error)	SSE = 34	$N-C = 10-3 = 7$	$MSE = \frac{34}{7} = 4.9$	F_α [illegible] $=$ [illegible]

Step 8:

Conclusion:

$|F_c| = 1.61 < 19.35 = |F_\alpha|$

$\therefore H_0$ is accepted.

ie, there is no significance difference between the treatments.

2. 3 different machines are used for a production. On the basis of the outputs, set up one-way ANOVA table and test whether the machines are equally effective.

Outputs

Machine I	Machine II	Machine III
10	9	20
15	7	16
11	5	10
10	6	14

Given that the value of F at 5% level of significance for (2, 9) d.f. is 4.26.

x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
10	9	20	39	100	81	400
15	7	16	38	225	49	256
11	5	10	26	121	25	100
10	6	14	30	100	36	196
46	27	60		546	191	952

Step 1:

H_0 : The machines are equally effective

H_1 : The machines are not equally effective.

Step 2:

To find T & N

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 = 133$

$N = n_1 + n_2 + n_3 = 4 + 4 + 4 = 12$

Step 3:

Correction factor : $CF = \frac{T^2}{N} = \frac{133^2}{12} =$

$= 1474.08$

Step 4:

TSS = Total Sum of Squares

$= \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - CF$

$= 546 + 191 + 952 - 1474.08$

$= 214.92$

Step 5:

$$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - CF$$

$$= \frac{(46)^2}{4} + \frac{(27)^2}{4} + \frac{(60)^2}{4} - 1474.08$$

$= 529 + 182.25 + 900 - 1474.08$

$= 137.17$

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Step 6:

$$SSE = TSS - SSC$$
$$= 214.92 - 137.17$$
$$= 77.75$$

Step 7:

ANNOVA TABLE

Source of variations	Sum of squares	Degrees of freedom	mean square	F-ratio
Between samples (column)	$SSC = 137.17$	$c-1 = 3-1 = 2$	$MSC = \frac{SSC}{df} = 68.585$	$F_c = \frac{MS}{MS} = 7.$
within samples (error)	$SSE = 77.75$	$n-c = 12-3 = 9$	$MSE = \frac{SSE}{df} = 8.639$	$F_\alpha(2,$ $= 4.2$

Step 8:

Conclusion:

$|F_c| = 7.939 > 4.26 = |F_\alpha|$

$\therefore$ H_0 is rejected.

$\therefore$ The machines are not equally effective.

3]. The foll table shows the lives in hrs of 4 branch of electric bulbs

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

perform an analysis variance and test the homogenity of the mean life of the 4 branch.

Soln.

$$Avg = \frac{maxi. + min}{2} = \frac{1820 + 1460}{2} = 1640$$