TAYLOR SERIES EXPANSION.

Taylor's expansion for the function $f(x,y)$ at the pt. a,b is given by

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right]$$
$$+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$
$$+ \frac{1}{3!}\left[(x-a)^3 f_{xxx}(a,b) + 3(x-a)^2(y-b)f_{xxy}(a,b) + 3(x-a)(y-b)^2 f_{xyy}(a,b) + (y-b)^3 f_{yyy}(a,b)\right] + \dots$$

① Find the Taylor's series for the function $e^x \sin y$ at $(0, \pi/2)$ upto second degree.

$f = e^x \sin y$	$f(0,\pi/2) = e^0 \sin \pi/2 = 1$
$f_x = e^x \sin y$	$f_x(0,\pi/2) = e^0 \sin \pi/2 = 1$
$f_{xx} = e^x \sin y$	$f_{xx}(0,\pi/2) = e^0 \sin \pi/2 = 1$
$f_{xy} = e^x \cos y$	$f_{xy}(0,\pi/2) = e^0 \cos \pi/2 = 0$
$f_y = e^x \cos y$	$f_y(0,\pi/2) = e^0 \cos \pi/2 = 0$
$f_{yy} = e^x(-\sin y)$	$f_{yy}(0,\pi/2) = e^0(-\sin \pi/2) = -1$

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] + \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$

$$f(x,y) = f(0,\pi/2) + \frac{1}{1!}\left[(x-0)f_x(0,\pi/2) + (y-\pi/2)f_y(0,\pi/2)\right] + \frac{1}{2!}\left[(x-0)^2 f_{xx}(0,\pi/2) + 2(x-0)(y-\pi/2)f_{xy}(0,\pi/2) + (y-\pi/2)^2 f_{yy}(0,\pi/2)\right]$$

$$= 1 + \frac{1}{1!}\left[x(1) + (y-\pi/2)(0)\right] + \frac{1}{2!}\left[x^2(1) + 2(x)(y-\pi/2)(0) + (y-\pi/2)^2(-1)\right]$$

$$= 1 + \frac{1}{1!}(x) + \frac{1}{2!}\left[x^2 - (y-\pi/2)^2\right] + \cdots$$