Centre of Curvature & Circle of Curvature.

Centre of curvature at any pt. on the curve.

$y = f(x)$ is $C(\bar{x}, \bar{y})$

where $\bar{x} = x - \dfrac{\dfrac{dy}{dx}\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]}{\dfrac{d^2y}{dx^2}}$

$$\bar{y} = y + \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$

Circle of curvature at any point is $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$.

where ρ is the radius of curvature.

① Find the circle of curvature at (c, c) on $xy = c^2$.

$xy = c^2$.

Here $y_1 = -1$ & $y_2 = \dfrac{2}{c}$

$\rho = c\sqrt{2}$.

To find $\bar{x}$ & $\bar{y}$:

$$\bar{x} = x - \frac{dy}{dx}\frac{\left[1 + y_1^2\right]}{y_2} = x + \frac{[1+1]}{\frac{2}{c}}$$

$$= x + \frac{2}{\frac{2}{c}} = x + c.$$

$$\bar{y} = y + \frac{[1+y_1^2]}{y_2} = y + \frac{[1+1]}{\frac{2}{c}} = y + c.$$

At (c,c) : $\bar{x} = c+c = 2c.$

$\bar{y} = c+c = 2c.$

Centre of curvature $C(\bar{x},\bar{y}) = C(2c, 2c)$

Circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

$(x-2c)^2 + (y-2c)^2 = (c\sqrt{2})^2$

(2) $y^2 = 12x$ at $(3,6)$.

Here $2y\frac{dy}{dx} = 12.$

$\frac{dy}{dx} = \frac{6}{y}.$

At $(3,6)$ $\frac{dy}{dx} = 1$

$\frac{d^2y}{dx^2} = -\frac{6}{y^2}.$

At $(3,6) = -\frac{1}{6}$

$\rho = \frac{[1+1]^{3/2}}{-1/6} = -12\sqrt{2}.$ (or) $\rho = 12\sqrt{2}.$

$\bar{x} = x - y_1\frac{[1+y_1^2]}{y_2} = x + 1\frac{[1+1]}{y_6} = x + 12.$

At $(3,6)$ $\bar{x} = 15$

$\bar{y} = y + \frac{[1+y_1^2]}{y_2} = y + \frac{[1+1]}{-1/6} = y - 12.$

At $(3,6)$ $\bar{y} = -6$

Centre of curvature $C(\bar{x},\bar{y}) = C(15,-6)$

Circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

$(x-15)^2 + (y+6)^2 = (12\sqrt{2})^2$

③ $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(a/4, a/4)$

WKT $y_1 = -1$ & $y_2 = \frac{4}{a}$.

$\rho = \frac{a}{\sqrt{2}}$.

To find $\bar{x}$ & $\bar{y}$:

$\bar{x} = x - (-1)\frac{[1+1]}{\frac{4}{a}} = x + \frac{2}{\frac{4}{a}} = x + \frac{a}{2}.$

At $(a/4, a/4)$ $\bar{x} = \frac{a}{4} + \frac{a}{2} = \frac{3a}{4}$

5) $\bar{y} = y + \frac{(1+1)}{\frac{4}{a}} = y + \frac{2}{\frac{4}{a}} = y + \frac{a}{2}$.

At $(a/4, a/4)$, $\bar{y} = \frac{3a}{4}$

Centre of curvature $c(\bar{x}, \bar{y}) = c\left(\frac{3a}{4}, \frac{3a}{4}\right)$

Circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$.

$$\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2.$$

Find the centre & circle of curvature on

(4) $xy = 12$ at $(3, 4)$

(5) $x^3 + y^3 = 3axy$ at $(3a/2, 3a/2)$

(6) $y = x^3$ at $(3, 27)$