**Defn: Curvature:**

The rate of bending of a curve at any point on it is called the curvature of the curve at that point.

**Radius of curvature:**

The reciprocal of the curvature of the curve at any pt. is called the radius of curvature at that point. It is denoted by $\rho$.

**Formula for radius of curvature:**

[Cartesian Co-ordinates]

Let $y = f(x)$ be the eqn. curve when

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

If $\frac{dy}{dx} = \infty$ at a pt. on the curve $y = f(x)$ then radius of curvature

$$\rho = \frac{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}}$$

Note:

1) The general form of eqn. of circle is $x^2+y^2+2gx+2fy+c=0$ where centre $(-g,-f)$ and radius $= \sqrt{g^2+f^2-c}$

2) The radius of curvature at any pt. on the circle = radius of the circle.
Curvature of the circle $= \frac{1}{r}$ where r is the radius of the circle.

3) Curvature of the st. line is zero.

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① Find the curvature at any pt. on the curve $x^2+y^2-6x-4y+10=0$.

The gn. curve is circle in the form $x^2+y^2+2gx+2fy+c=0$

Here $2g=-6 \Rightarrow g=-3$
$2f=-4 \Rightarrow f=-2$

Centre $= (-g,-f) = (3,2)$

Radius $= \sqrt{g^2+f^2-c} = \sqrt{9+4-10} = \sqrt{3}$

$\rho = r = \sqrt{3}$

Curvature $= \frac{1}{\rho} = \frac{1}{\sqrt{3}}$

(2) Find the curvature of $2x^2+2y^2+5x-2y+1=0$.

The general form is $x^2+y^2+2gx+2fy+c=0$.

$\Rightarrow x^2+y^2+\frac{5}{2}x-y+\frac{1}{2}=0$

Here $2g=\frac{5}{2} \Rightarrow g=\frac{5}{4}$

$2f=-1 \Rightarrow f=-\frac{1}{2}$

Centre $=(-g,-f)=(-\frac{5}{4},\frac{1}{2})$

Radius $=\sqrt{g^2+f^2-c}=\sqrt{\frac{25}{16}+\frac{1}{4}-\frac{1}{2}}=\sqrt{\frac{21}{16}}=\frac{\sqrt{21}}{4}$

$\rho=r=\frac{\sqrt{21}}{4}$

Curvature $=\frac{1}{\rho}=\frac{4}{\sqrt{21}}$

(3) Find the curvature of $x^2+y^2=5$

The general form is $x^2+y^2+2gx+2fy+c=0$

Here $2g=0 \Rightarrow g=0$

$2f=0 \Rightarrow f=0$

Centre $=(0,0)$

Radius $=\rho=\sqrt{5}$ ; Curvature $=\frac{1}{\rho}=\frac{1}{\sqrt{5}}$