



DEPARTMENT OF MATHEMATICS

UNIT - IV FUNCTIONS OF SEVERAL VARIABLES

TOTAL DERIVATIVE

1) If $u = f(x, y)$ then total differential of u is

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

2) If $u = f(x, y)$ where $x = g_1(t)$, $y = g_2(t)$ then

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t}$$

3) If $u = f(x, y)$ where y is a function of x then

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dx}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$\frac{du}{dy} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dy} + \frac{\partial u}{\partial y}$$

Differentiation of Implicit function :

If $f(x, y) = c$ where c may be zero or non-zero
is an implicit function of x & y then

$$\frac{dy}{dx} = - \frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$



DEPARTMENT OF MATHEMATICS

UNIT - IV FUNCTIONS OF SEVERAL VARIABLES

type - I

1) If $u = x^2 + y^2$ then find total differential of u .

Soln: $u = x^2 + y^2$

$$\frac{\partial u}{\partial x} = 2x,$$

$$\frac{\partial u}{\partial y} = 2y.$$

$$\begin{aligned} du &= \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy \\ &= 2x dx + 2y dy. \end{aligned}$$

type - II

3) Find $\frac{du}{dt}$ if $u = x^2 + y^2 + z^2$ where $x = e^t$, $y = e^t \sin t$,

$$z = e^t \cos t.$$

Soln: $u = x^2 + y^2 + z^2$

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial u}{\partial y} = 2y, \quad \frac{\partial u}{\partial z} = 2z$$

Given: $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t$

$$\frac{dx}{dt} = e^t, \quad \frac{dy}{dt} = e^t \cos t + e^t \sin t, \quad \frac{dz}{dt} = -e^t \sin t + e^t \cos t$$



DEPARTMENT OF MATHEMATICS

UNIT - IV FUNCTIONS OF SEVERAL VARIABLES

$$\begin{aligned}\frac{du}{dt} &= \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt} \\ &= 2x \cdot e^t + 2y \cdot (e^t \cos t + e^t \sin t) + 2z \cdot (e^t \cos t - e^t \sin t) \\ &= 2e^t \cdot e^t + 2e^t \sin t (e^t \cos t + e^t \sin t) + 2e^t \cos t (e^t \cos t - e^t \sin t) \\ &= 2e^{2t} + 2e^{2t} \sin t / \cos t + 2e^{2t} \sin^2 t + 2e^{2t} \cos^2 t - 2e^{2t} \cos t \sin t \\ &= 2e^{2t} + 2e^{2t} = 4e^{2t}\end{aligned}$$

TYPE - 2

10) If $x^3 + y^3 = 3axy$ find $\frac{dy}{dx}$

Soln: $x^3 + y^3 - 3axy = 0$

Let $f(x, y) = x^3 + y^3 - 3axy$

$$\frac{\partial f}{\partial x} = 3x^2 - 3ay$$

$$\frac{\partial f}{\partial y} = 3y^2 - 3ax$$

$$\frac{dy}{dx} = - \frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)} = - \frac{3(x^2 - ay)}{3(y^2 - ax)} = \frac{ay - x^2}{y^2 - ax}$$