

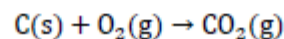


DEPARTMENT OF MECHANICAL ENGINEERING, 16ME306/ Heat and Mass Transfer –
UNIT V - MASS TRANSFER
Topic - Tutorial steady state molecular diffusion

A fluidized coal reactor has been proposed for a new power plant. If operated at 1145 K, the process will be limited by the diffusion of oxygen countercurrent to the carbon dioxide, CO_2 , formed at the particle surface. Assume that the coal is pure solid carbon with a density of $1.28 \times 10^3 \text{ kg/m}^3$ that the particle is spherical with an initial diameter of $1.5 \times 10^{-4} \text{ m}$ ($150 \mu\text{m}$). Air (21% O_2 and 79% N_2) exists several diameters away from the sphere. Under the conditions of the combustion process, the diffusivity of oxygen in the gas mixture at 1145 K is $1.3 \times 10^{-4} \text{ cm}^2/\text{s}$. If a steady-state process is assumed,

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calculate the time necessary to reduce the diameter of the carbon particle to $5 \times 10^{-5} \text{ m}$ ($50 \mu\text{m}$). The surrounding air serves as an infinite source for O_2 transfer, whereas the oxidation of the carbon at the surface of the particle is the sink for O_2 mass transfer. The reaction at the surface is:





At the surface of the coal particle, the reaction is so rapid.

Solution:

The pure carbon particle is the source for the CO_2 flux and the sink for O_2 flux. As the coal particle is oxidized, there will be an output of carbon as stipulated by the stoichiometry of the reaction.

Number of moles of oxygen transferred = number of moles of carbon reacted

Number of moles transferred of oxygen = mole flux \times area

$$\text{Number of moles transferred of oxygen} = N_{\text{O}_2\text{-mix}} \times 4\pi r^2$$

$N_{\text{O}_2\text{-mix}}$ can be obtained by using the general differential equation with the Fick's equation as follows:

By applying the following assumptions on the general differential equation of mass transfer:

$$\nabla \cdot \vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

1. Steady state oxygen diffusion
2. One dimensional mass transfer in r direction
3. No homogenous reaction
4. Instantaneous heterogeneous reaction

$$\therefore \nabla \cdot \vec{N}_A = 0$$

For diffusion of oxygen in r-direction

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{\text{O}_2}) = 0$$



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$$\therefore \frac{d}{dr}(r^2 N_{O_2}) = 0$$

The above equation specifies that $r^2 N_{O_2}$ is constant over the diffusion path in the r direction, so that

$$r^2 N_{O_2} \Big|_r = R^2 N_{O_2} \Big|_R$$

From Fick's equation:

$$N_{O_2} = -cD_{O_2-mix} \frac{dy_{O_2}}{dr} + y_{O_2}(N_{O_2} + N_{CO_2})$$

But from the stoichiometry of the reaction

$$N_{O_2} = -N_{CO_2}$$

i.e. equimolar counter diffusion

$$\therefore N_{O_2} = -cD_{O_2-mix} \frac{dy_{O_2}}{dr}$$

$$N_{O_2} \int_R^\infty dr = -cD_{O_2-mix} \int_0^{y_{O_2,\infty}} dy_{O_2}$$

$$N_{O_2} r^2 \int_R^\infty \frac{dr}{r^2} = -cD_{O_2-mix} \int_0^{y_{O_2,\infty}} dy_{O_2}$$

$$N_{O_2} r^2 \left(\frac{1}{R}\right) = -cD_{O_2-mix}(y_{O_2,\infty} - 0)$$

$$N_{O_2} r^2 = -RcD_{O_2-mix} y_{O_2,\infty}$$

$$\text{number of moles of oxygen transferred per unit time} = N_{O_2} \times 4\pi r^2$$

$$\text{number of moles of oxygen transferred per unit time} = -4\pi R cD_{O_2-mix} y_{O_2,\infty}$$

The negative sign because the transfer of oxygen is in the opposite direction of r

$$\therefore \text{number of moles of carbon consumed per unit time} = 4\pi R cD_{O_2-mix} y_{O_2,\infty}$$



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By applying the law of conservation of mass on the carbon:

Input – output + generation – consumption = accumulation(rate of change)

– consumption = accumulation(rate of change)

$$-4\pi R c D_{O_2-mix} y_{O_2,\infty} = \frac{dN}{dt} = \frac{\rho_c}{M. wt} \frac{dV}{dt}$$

$$V = \frac{4}{3}\pi R^3$$

$$-4\pi R c D_{O_2-mix} y_{O_2,\infty} = \frac{\rho_c}{M. wt} 4\pi R^2 \frac{dR}{dt}$$

$$dt = - \frac{\rho_c}{M. wt} \frac{R dR}{c D_{O_2-mix} y_{O_2,\infty}}$$

by integrating the above equation between the limits:

$$\text{at } t = 0 \quad R = R_i = 7.5 \times 10^{-5} \text{ m}$$

$$\text{at } t = t \quad R = R_f = 2.5 \times 10^{-5} \text{ m}$$

$$\int_0^t dt = - \frac{\rho_c}{M. wt} \frac{1}{c D_{O_2-mix} y_{O_2,\infty}} \int_{R_i}^{R_f} R dR$$

$$t = \frac{\rho_c}{2 M. wt} \frac{(R_i^2 - R_f^2)}{c D_{O_2-mix} y_{O_2,\infty}}$$

$$c = \frac{p}{RT} = 0.0106 \text{ kmol/m}^3$$

$$y_{O_2,\infty} = 0.21$$

$$\therefore t = 0.92 \text{ s}$$



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References:

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3. MIT open courseware – <https://ocw.mit.edu/courses/mechanical-engineering>

Other web sources