3. Stefan Boltzman law.

The emissive power of black surface can be found by integrating the expression for Plank's law over all wavelengths. Thus.

$$e_{b\lambda} = \int_{\lambda=0}^{\lambda=\infty} e_{b\lambda} \cdot d\lambda = \int_{\lambda=0}^{\lambda=\infty} \frac{2\pi C_1}{\lambda^5\left[e^{(C_2/\lambda T)} - 1\right]}$$

Let $x = 1/\lambda$ $\therefore dx = (-1/x^2)d\lambda$.

$$\therefore e_b = 2\pi C_1 \int_0^\infty \frac{x^2 \cdot dx}{\left[e^{(C_2 x/T)} - 1\right]}$$

$$= 2\pi C_1 \int_0^\infty x^2 \left[e^{(C_2 x/T)} - 1\right]^{-1} dx.$$

$$= 2\pi C_1 \cdot \int_0^\infty x^3 \left[e^{-(C_2/T)x} + e^{-(C_2/T)2x} + \ldots\right] dx$$

$$= 2\pi C_1 \frac{6T^4}{C_2^4}\left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\right)$$

Try @ home.

$$= 2\pi C_1 \frac{6T^4}{C_2^4}\left(\frac{\pi^4}{90}\right)$$

$e_b \propto T^4$

$$e_b = \sigma T^4. \qquad \sigma = \frac{2\pi C_1 \times 6\pi^4}{C_2^4 \times 90}$$

σ = Stefan Boltzman constant

$\sigma = 5.67 \times 10^{-8}$ W/m^2K^4.

For gray body.

$e = \epsilon . e_b = \epsilon \sigma T^4$ T = Absolute temp

4) Kirchoff's Law:- ($\epsilon_\lambda = \alpha_\lambda$) and ($\epsilon = \alpha$)

The [monochromatic] emissivity of a surface is equal to the [monochromatic] absorptivity of the surface (emitted in a diffuse manner), at given temp. T

Proof:-> $\epsilon = \frac{e}{e_b} = \frac{\int_{\lambda=0}^{\infty} e_\lambda . d\lambda}{e_b} = \frac{\int_0^{\infty} \epsilon_\lambda . e_{b\lambda} . d\lambda}{e_b}$

$\left[\because \epsilon_\lambda = \frac{e_\lambda}{e_{b\lambda}}\right]$ $\therefore \epsilon = \epsilon_\lambda . \frac{\left(\int_0^{\infty} e_{b\lambda}.d\lambda\right)^{\cancel{e_b}}}{\cancel{e_b}}$ ϵ_λ = constant

$\epsilon = \epsilon_\lambda \longrightarrow (1)$

For black body (unity)

Topic - Laws of Radiation - Stefan-Boltzmann Law, Kirchoff Law

Also $\alpha = \frac{H_a}{H} = \frac{\int_0^\infty H_{a\lambda} d\lambda}{H_b}$ $\left[\because \epsilon_\lambda = \frac{H_{a\lambda}}{H_\lambda}\right]$

$\alpha_\lambda = \frac{H_{a\lambda}}{H_\lambda}$

$$\alpha = \frac{\int_0^\infty \epsilon_\lambda H_\lambda d\lambda}{H_b}$$

Emissive power [Irradiation]

$$\alpha = \epsilon_\lambda \frac{\int_0^\infty H_{b\lambda} d\lambda}{H_b}$$

$$\alpha = \epsilon_\lambda \frac{\cancel{H_b}}{\cancel{H_b}}$$

$$\alpha = \epsilon_\lambda \longrightarrow (2)$$

From (1) and (2) $\alpha = \epsilon = \epsilon_\lambda \longrightarrow (I)$

So a surface for which both the equation hold is called a diffuse-gray surface. Non-black surfaces are diffuse-gray.

For blackbody $\alpha = 1$, $\epsilon = 1$. $\epsilon_1 = \epsilon_2 = \epsilon$

$\alpha_1 = \alpha_2 = \alpha$.

$T_1 > T_2$.

$T_2\,\epsilon_2$ $\quad$ $T_1\,\epsilon_1$

$e_2 = \alpha_2$ $\quad$ $\epsilon_1 = \alpha_1$

$$e_1 = \epsilon_1 \sigma T_1^4 \longrightarrow (1)$$

$$e_2 = \epsilon_2 \sigma T_2^4 \longrightarrow (2)$$

$$e_1 = \alpha_1 \sigma T_1^4 \ \& \ e_2 = \alpha_2 \sigma T_2^4$$

$$\underbrace{e_1 - e_2}_{\text{Net radiation.}} = \alpha \sigma (T_1^4 - T_2^4) = \epsilon \sigma (T_1^4 - T_2^4)$$

$T_1 = T_2 \longrightarrow \alpha = \epsilon$.

References:

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3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources.