Torsion

Click here to check the Animation

Introduction:

Detailed methods of analysis for determining stresses and deformations in axially loaded bars were presented in the first two chapters. Analogous relations for members subjected to torque about their longitudinal axes are developed in this chapter. The constitutive relations for shear discussed in the preceding chapter will be employed for this purpose. The investigations are confined to the effect of a single type of action, i.e., of a torque causing a twist or torsion in a member.

The major part of this chapter is devoted to the consideration of members having circular cross sections, either solid or tubular. Solution of such elastic and inelastic problems can be obtained using the procedures of engineering mechanics of solids. For the solution of torsion problems having noncircular cross sections, methods of the mathematical theory of elasticity (or finite elements) must be employed. This topic is briefly discussed in order to make the reader aware of the differences in such solutions from that for circular members. Further, to lend emphasis to the difference in the solutions discussed, this chapter is subdivided into four distinct parts. It should be noted, however, that in practice, members for transmitting torque, such as shafts for motors, torque tubes for power equipment, etc., are predominantly circular or tubular in cross section. Therefore, numerous applications fall within the scope of the formulas derived in this chapter.

In this section, discussion is limited to torsion of circular bars only.

Top

Basic Assumptions

a. *Plane sections perpendicular to the axis of a circular member before application of torque remains plane even after application of torque.*

b. *Shear strains vary linearly from the central axis reaching a maximum value at the outer surface.*

c. *For linearly elastic material, Hooke's law is valid. Hence shear stress is proportional to shear strain.*

Top

Torsion Formula

Since shear strains varies linearly across the section,

$$\gamma = \frac{\gamma_{max}}{C} R$$

where γ is the shear strains at a point of raidus R, C is the radius of the member.

$$\therefore \text{ Torque, } T = \int_A \tau R \, dA$$

$$= \int_A G\gamma R \, dA$$

where $\tau = G\,\gamma$, the shear stress at any point at a distance R (Refer Figure 6.1)

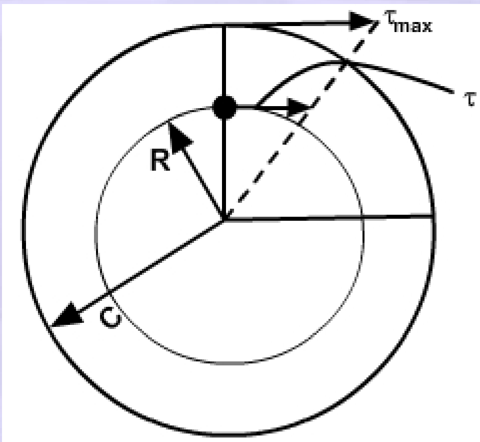


Figure 6.1

Hence writing in terms of shear stresses.

$$T = \int_A \frac{R}{C} \tau_{max} R dA$$

$$= \frac{\tau_{max}}{C} \int R^2 dA$$

$$\int R^2 dA = I_p$$

the Polar moment of Inertia of the circular section.

$$\therefore \; T = \frac{\tau_{max} I_p}{C}$$

$$\tau_{max} = \frac{TC}{I_p}$$

and

$$\tau = \frac{TR}{I_p}$$

Top

Stresses on Inclined Planes

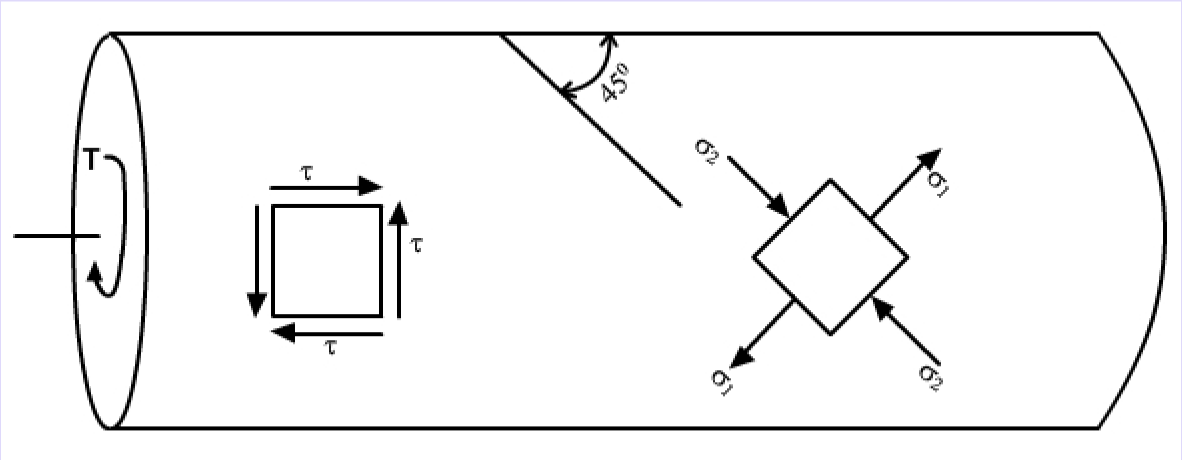


Figure 6.2

The shear stress at a point, on the surface acting on a plane perpendicular to the axis of the rod can be found out from the preceding analysis.

Using transformation law for, stresses the state of stress at a point on a plane at 45^0 to the axis can be found out. These stresses are found out to be

$$\sigma_1 = \tau_{max}$$
$$\sigma_2 = -\tau_{max}$$

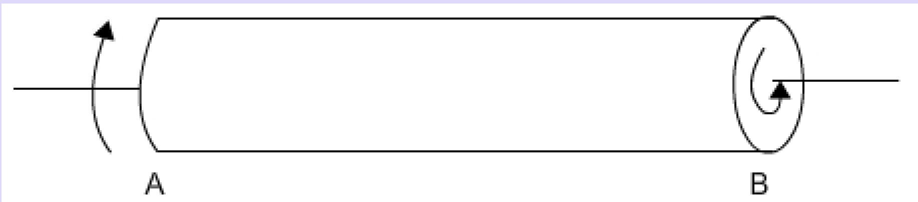
Ductile materials have lesser shear strength than tensile strength and hence fail through planes perpendicular to the axis.

Brittle materials have lesser tensile strength than shear strength. Hence they fail through planes inclined at 45^0 to the axis.

Top

Angle of Twist in Torsion

Consider a circular shaft subjected to torque. It is assumed that plane sections perpendicular to the axis remain plane even after loading. Isolating an element form such a member, (Refer Figure 6.3).



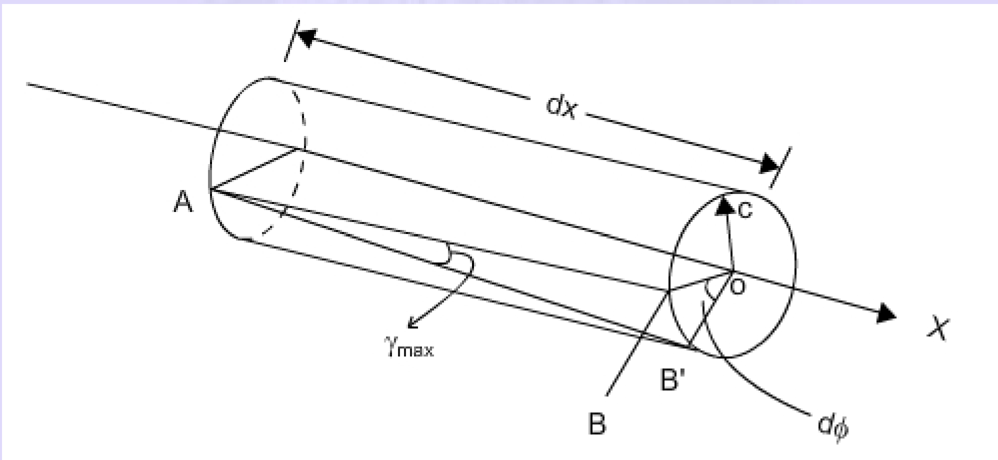


Figure 6.3

A line segment in the surface of the shaft, AB is initially parallel to the axis of the shaft. Upon application of torque, it occupies a new position AB'. The segment OB now occupy the position OB'.

From figure 16,

BB' = C $d\phi$

Also

$BB' = dx\,\gamma_{max}$

$$\frac{d\phi}{dx} = \frac{\gamma_{max}}{C}$$

Since $\tau = G\gamma$

$$\tau_{max} = G\,\gamma_{max}$$
$$\tau_{max} = \frac{TC}{GI_p}$$

$$\therefore \gamma_{max} = \frac{TC}{GI_p}$$

$$\therefore \frac{d\phi}{dx} = \frac{T}{GI_p}$$

This equation gives the relative angle of twist between any two sections of a shaft distance dx apart.

To find the total angle of twist ϕ between any two sections 1 and 2, all rotations of all elements between 1 and 2 should be summed up

$$\therefore \phi = \phi_2 - \phi_1 = \int \frac{Tdx}{I_p G}$$

Where

$$T = T(x)$$
$$I_p = I_p(x)$$
$$G = G(x)$$

When τ, I_p and G vary along the length of the shaft.

Top

Torsion of Circular Elastic Bars:

Formulae:

1. For solid circular member

Polar Moment of Inertia,

$$\boxed{I_p = \frac{\pi C^4}{2} = \frac{\pi D^4}{32}}$$

where, C is radius of the circular member. D is Diameter of the circular member.

2. For a circular tube:

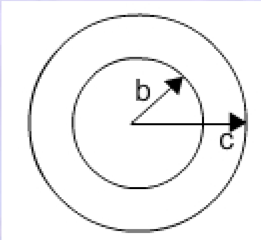


Figure 6.4

Polar moment of Inertia,

$$\boxed{I_p = \frac{\pi c^4 - \pi b^4}{2}}$$

where, c = outer radius of the tube.

b = Inner radius of the tube.

3. For very thin tubes:

where thickness t = c-b is very less. Then I_P can be approximated to,

$$\boxed{I_p = 2\pi R_{avg} t}$$

where

$$R_{avg} = \frac{b+c}{2},$$

4. (i) $\boxed{\tau_{max} = \frac{TC}{I_P}}$

where τ_{max} = Maximum shear stress in a circular cross section.

T = Torque in that section.

C = Radius of the section.

I_P = Polar Moment of Inertia.

Note: Shear stress linearly with radius.

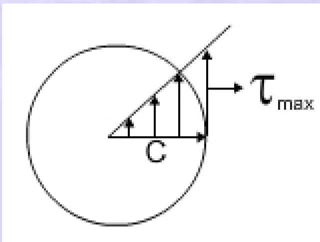


Figure 6.5

(ii) Shear Stress (τ) at a distance R from the centre.

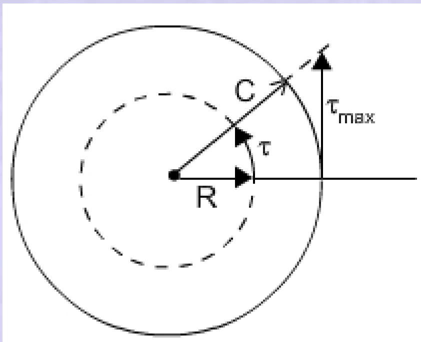


Figure 6.6

$$\tau = \tau_{max} \frac{R}{C}$$

$$\tau = \frac{TR}{I_P}$$

Note: J is also used to denote the polar moment of Inertia.

Click here to check the Animation

Assumptions

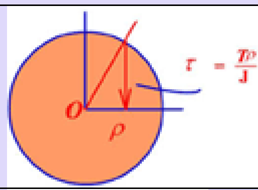
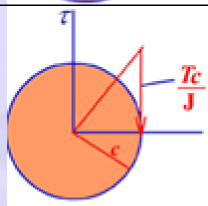
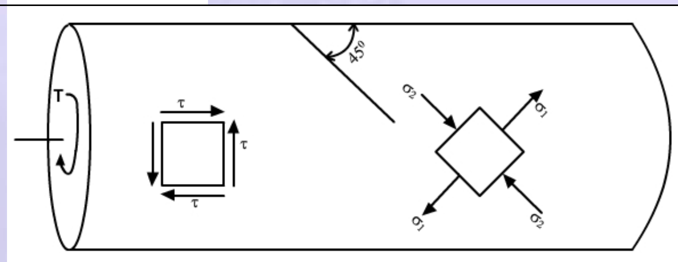
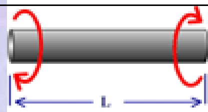
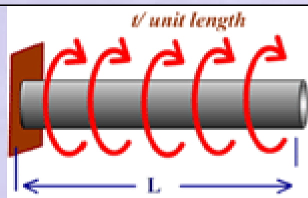
Stress Formula

Angle of twist

Maximum Stress

Top

Table of Formulae

S.No	Quantity	Formula	Diagram
1.	τ (Shear stress)	$\tau = \frac{T\rho}{J}$	
2.	τ_{max} (Max. shear stress)	$\tau_{max} = \frac{Tc}{J}$	
3.	σ_{max} (Max. normal stress)	$\boxed{\sigma_{max} = \tau_{max}}$	
4.	θ (angle of twist)	$\theta = \frac{TL}{GJ}$	
5.	θ (angle of twist)	$\theta = \int_L \frac{TL}{GJ}$	

Top