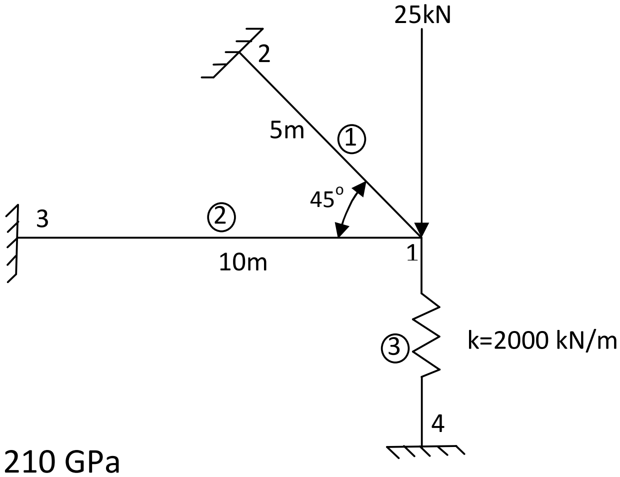
COMBINATION OF SPRING AND BAR ELEMNTS IN ONE STRUCTURE

- DARYL L LOGAN(EXAMPLE 3.7, PAGE NO.90)

To illustrate how we can combine spring and bar element in one structure, we can solve the two-bar truss supported by a spring as shown below. Both bars have E = 210 GPa and A = 5.0 x10^{-4} m^2. Bar one has a length of 5 m and bar two a length of 10 m. the spring stiffness is k = 2000 kN/m.



Solution :

Given : E = 210 GPa

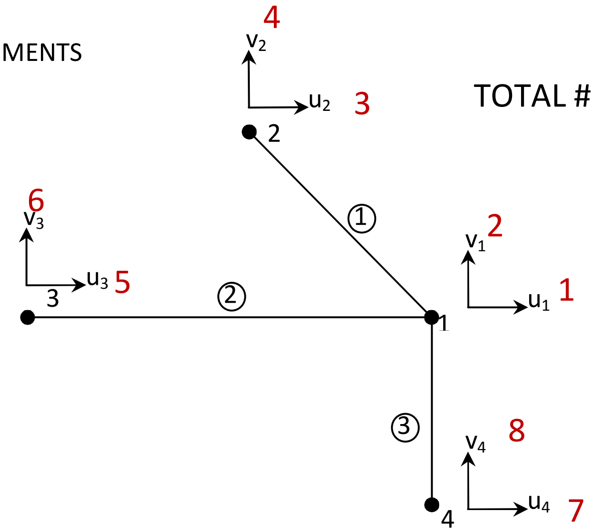
A = 5.0 x10^{-4} m^2 , L_1 = 5 m, L_2 = 10 m, K = 2 x 10^6 $\frac{N}{m^2}$

NOTE: A spring is considered as a bar element whose stiffness is 2 x 10^6 $\frac{N}{m^2}$

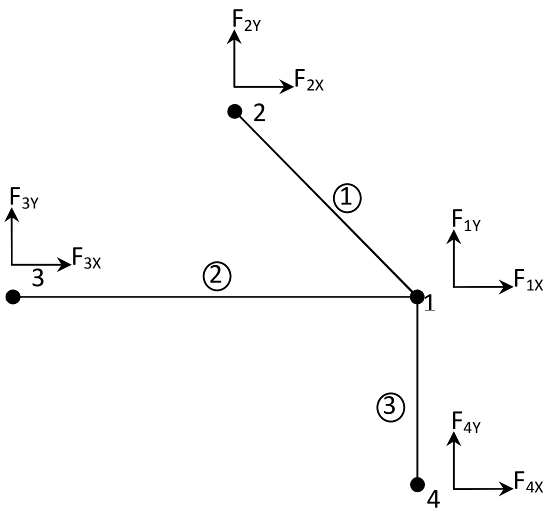
STEP 1 : FINITE ELEMENT REPRESENTATION OF FORCES AND DISPLACEMENTS

DISPLACEMENTS

TOTAL # OF DEGREES OF FREEDOM: 8



FORCES



STEP 2: FINITE ELEMENT EQUATIONS

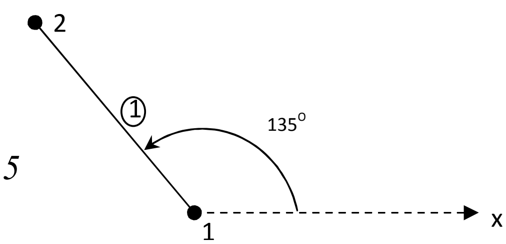
Element 1:

$\theta = 135^0$

$l^2 = \cos^2\theta = 0.5$

$m^2 = \sin^2\theta = 0.5$

$lm = \cos\theta \sin\theta = -0.5$



$$[K]^{(1)} = \frac{\left(5\times10^{-4}\,\text{m}^2\right)\left(210\times10^6\,\text{kN/m}^2\right)}{5\text{ m}} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} \end{array}$$

$$[K]^{(1)} = 105\times10^5 \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \end{array}$$

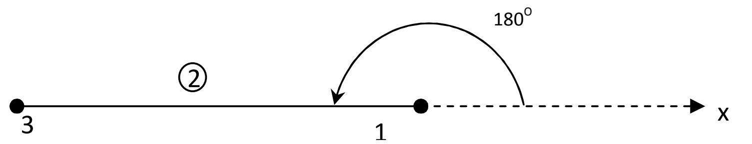
Element 2:

$\theta = 180^0$

$l^2 = \cos^2\theta = 1$

$m^2 = \sin^2\theta = 0$

$lm = \cos\theta\sin\theta = 0$



$$[K]^{(2)} = \frac{\left(5\times10^{-4}\,\text{m}^2\right)\left(210\times10^{6}\,\text{kN/m}^2\right)}{10\ \text{m}} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$[K]^{(2)} = 105\times10^{5} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

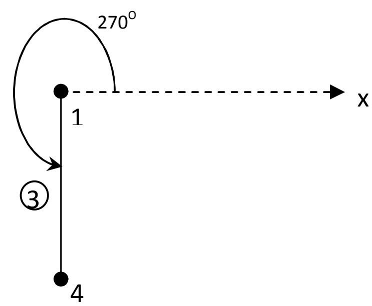
Element 3:

$\theta = 270^0$

$l^2 = \cos^2\theta = 0$

$m^2 = \sin^2\theta = 1$

$lm = \cos\theta\sin\theta = 0$



$$[K]^{(3)} = 2\times10^{6} \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

STEP 3: COMBINATION OF FINITE ELEMENT EQUATIONS

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix}$$

$$\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{Bmatrix} = 10^5 \times \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} \begin{bmatrix} 210 & -105 & -105 & 105 & -105 & 0 & 0 & 0 \\ -105 & 125 & 105 & -105 & 0 & 0 & 0 & -20 \\ -105 & 105 & 105 & -105 & 0 & 0 & 0 & 0 \\ 105 & -108 & -105 & 105 & 0 & 0 & 0 & 0 \\ -105 & 0 & 0 & 0 & 105 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

STEP 4: APPLYING BOUNDARY CONDITIONS:

Since nodes 1, 2, and 3 are fixed, we have

$u_2 = v_2 = 0$; $u_3 = v_3 = 0$; $u_4 = v_4 = 0$;

$F_{1x} = 0$ and $F_{1y} = -25$ kN

$$\begin{Bmatrix} 0 \\ -25 \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{Bmatrix} = 10^5 \times \begin{bmatrix} 210 & -105 & -105 & 105 & -105 & 0 & 0 & 0 \\ -105 & 125 & 105 & -105 & 0 & 0 & 0 & -20 \\ -105 & 105 & 105 & -105 & 0 & 0 & 0 & 0 \\ 105 & -108 & -105 & 105 & 0 & 0 & 0 & 0 \\ -105 & 0 & 0 & 0 & 105 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Check whether there are as many unknowns as knowns.

STEP 5: SOLVING THE EQUATIONS:

Reduced matrix:

$$\begin{Bmatrix} 0 \\ -25 \end{Bmatrix} = 10^5 \times \begin{bmatrix} 210 & -105 \\ -105 & 125 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

On solving,

u_1=-1.724 x 10^{-3} m

v_1=-3.4482 x 10^{-3} m

Find the reactions at supports by substituting the known nodal values

F_{2x} = -18.104 kN

F_{2y} = 18.1041 kN

F_{3x} = 18.102 kN

F_{3y} = 0

F_{4x} = 0

F_{4y} = 6.89 kN

STEP 6: Post Processing

Stress in element 1:

$$\sigma^{(1)} = \frac{E}{L}\begin{bmatrix} -l & -m & l & m \end{bmatrix} 10^{-3} \begin{Bmatrix} -1.724 \\ -3.4482 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma^{(1)} = 51.2 \text{ MPa (Tensile)}$$

Stress in element 2:

$$\sigma^{(1)} = \frac{E}{L}\begin{bmatrix} -l & -m & l & m \end{bmatrix} 10^{-3} \begin{Bmatrix} -1.724 \\ -3.4482 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma^{(2)} = -36.2 \text{ MPa (Compressive)}$$