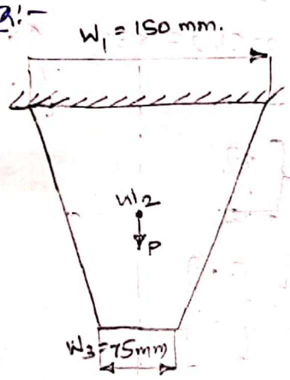
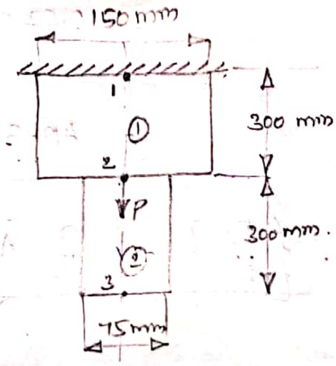
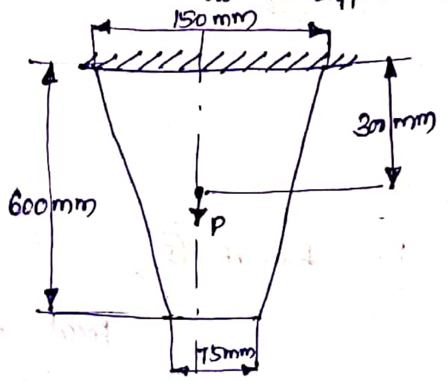
(6) Consider a taper steel plate of uniform thickness, $t = 25$ mm as shown below. The Young's modulus of the plate, $E = 2 \times 10^5$ N/mm^2 and weight density $\rho = 0.82 \times 10^{-4}$ N/mm^3. In addition to its selfweight, the plate is subjected to a point load $P = 100$ N at its mid point. Calculate the following by modeling the plate with two finite elements. (i) Global force vector $\{F\}$. (ii) Global stiffness matrix $[K]$. (iii) Displacements in each element $\{u\}$ (iv) stress in each element, σ (v) Reaction forces at the support $\{R\}$

Given Data:-







$$W_2 = \left[\frac{W_1 + W_3}{2}\right] = \frac{150 + 75}{2}$$

Solution:-

* Nodal areas,

Area at node 1, A_1 = width × Thickness

$= 150 \times 25$

$A_1 = 3750$ mm^2

Area at node 2, $A_2 = W_2 \times t$

$= 112.5 \times 25$

$A_2 = 2812.5$ mm^2

Area at node 3, $A_3 = W_3 \times t$

$= 75 \times 25$

$A_3 = 1875$ mm^2

Average area of element ①;

$$\bar{A}_1 = \frac{\text{Area at node, 1} + \text{Area at node, 2}}{2} = \frac{3750 + 2812.5}{2}$$

$\bar{A}_1 = 3281.25$ mm^2

Average area of element ②

$$\bar{A}_2 = \frac{\text{Area at node, 2} + \text{Area at node 3}}{2} = \frac{2812.5 + 1875}{2}$$

$\bar{A}_2 = 2343.75$ mm^2

Soln:-

The steel plate is subjected to self weight, so we have to find the force acting at nodal points 1, 2 & 3.

Body Force Vector, $\{F\} = \frac{\rho A L}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

For Element ①,

Force vector, $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\rho_1 \bar{A}_1 l_1}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= \frac{0.82 \times 10^{-4} \times 3281.25 \times 300}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= 40.359 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ ---> ①

For Element ②,

Force vector, $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{\rho_2 \bar{A}_2 l_2}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= \frac{0.82 \times 10^{-4} \times 2343.75 \times 300}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= 28.828 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ ---> ②

Assembling Body Force vector, ① & ②

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 40.359 + 28.828 \\ 28.828 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 69.187 \\ 28.828 \end{Bmatrix}$$

A point load of 100 N is acting at node 2, so $F_2 = 69.187$ (+) 100.

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 69.187 + 100 \\ 28.828 \end{Bmatrix}$$

Global Force Vector, $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{Bmatrix}$ //