



SNS COLLEGE OF TECHNOLOGY

**Coimbatore-35
An Autonomous Institution**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai



DEPARTMENT OF AUTOMOBILE ENGINEERING

16AU405 – Finite Element Methods and Analysis

IV YEAR / VII SEM

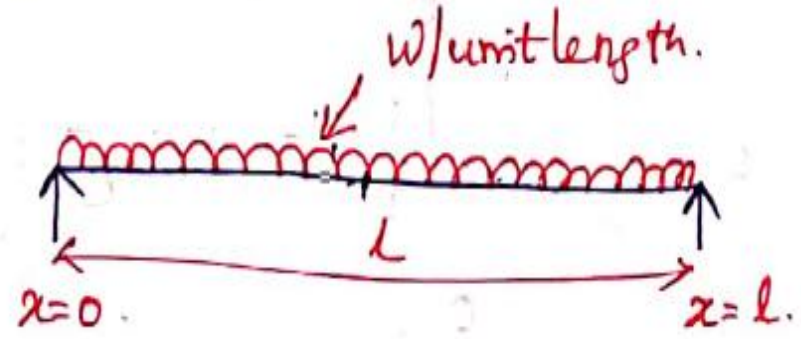
UNIT – 1 - Introduction

Topic – 4 – Rayleigh Ritz Method



Rayleigh Ritz Method- Problem

A simply supported beam subjected to udl over entire span.
Determine the bending moment and deflection at midspan by using Rayleigh-Ritz method & compare with exact solutions



To Find:

- (i) Deflection & Bending moment
- (ii) Compare with exact soln.



Solution

Soln:-

For SSB, the series series,

$$Y = \sum_{n=1,3}^{\infty} a \sin \frac{n\pi x}{l} \quad (\text{approximating fn.})$$

Let us consider

$$\text{deflection, } y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}. \quad [a_1, a_2 - \text{Ritz parameters}]$$



Total potential Energy of the beam, $\pi = U - H$
Strain Energy of the beam due to bending,
$$U = \frac{EI}{2} \int_0^L \left(\frac{d^2y}{dx^2} \right)^2 dx.$$

U - Strain Energy
H - Work done by ext. force



$$y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}$$

$$\frac{dy}{dx} = a_1 \cos \frac{\pi x}{l} \left(\frac{\pi}{l}\right) + a_2 \cos \frac{3\pi x}{l} \left(\frac{3\pi}{l}\right)$$

$$= \frac{a_1 \pi}{l} \cos \frac{\pi x}{l} + \frac{a_2 3\pi}{l} \cos \frac{3\pi x}{l}$$

$$\frac{d^2y}{dx^2} = -\frac{a_1 \pi}{l} \sin \frac{\pi x}{l} \times \frac{\pi}{l} - a_2 \frac{3\pi}{l} \sin \frac{3\pi x}{l} \times \frac{3\pi}{l}$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{\pi^2 a_1}{l^2} \sin \frac{\pi x}{l} - a_2 \frac{9\pi^2}{l^2} \sin \frac{3\pi x}{l}} \dots \rightarrow \textcircled{1}$$



$$U = \frac{EI}{2} \int_0^l \left(\frac{d^2y}{dx^2} \right)^2 dx \rightarrow \text{Sub. (i)}$$

$$= \frac{EI}{2} \int_0^l \left[\frac{a_1 \pi^2}{l^2} \sin \frac{\pi x}{l} - \frac{a_2 9\pi^2}{l^2} \sin \frac{3\pi x}{l} \right]^2 dx.$$

$$= \frac{EI}{2} \times \frac{\pi^4}{l^4} \int_0^l \left[a_1 \sin \frac{\pi x}{l} + 9 a_2 \sin \frac{3\pi x}{l} \right]^2 dx$$



$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$U = \frac{EI}{2} \times \frac{\pi^4}{l^4} \int_0^l \left[a_1^2 \sin^2 \frac{\pi x}{l} + 81 a_2^2 \sin^2 \frac{3\pi x}{l} + 2 \left(a_1 \sin \frac{\pi x}{l} \right) \left(a_2 \sin \frac{3\pi x}{l} \right) \right] dx$$

$$U = \frac{EI}{2} \frac{\pi^4}{l^4} \int_0^l \left[a_1^2 \sin^2 \frac{\pi x}{l} + 81 a_2^2 \sin^2 \frac{3\pi x}{l} + 18 a_1 a_2 \sin \frac{\pi x}{l} \sin \frac{3\pi x}{l} \right] dx$$

$$\int_0^l a_1^2 \sin^2 \frac{\pi x}{l} dx = a_1^2 \int_0^l \frac{1}{2} \left(1 - \cos \frac{2\pi x}{l} \right) dx$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$



$$= \frac{q_1^2}{2} \int_0^l \left(1 - \cos \frac{2\pi x}{l} \right) dx.$$

$$= \frac{q_1^2}{2} \left[\int_0^l dx - \int_0^l \cos \frac{2\pi x}{l} dx \right]$$

$$= \frac{q_1^2}{2} \left[(x)_0^l - \left(\frac{\sin \frac{2\pi x}{l}}{\frac{2\pi}{l}} \right)_0^l \right]$$

$$= \frac{q_1^2}{2} \left[l - 0 - \frac{l}{2\pi} \left(\sin \frac{2\pi l}{l} - \sin 0 \right) \right]$$



$$= \frac{a_1^2}{2} \left[l - 0 - \frac{1}{2\pi} \left(\sin \frac{2\pi l}{l} - \sin 0 \right) \right]$$

$$\int_0^l a_1^2 \sin^2 \frac{\pi x}{l} dx = \frac{a_1^2 l}{2}$$

$$\sin 2\pi = 0; \sin 0 = 0$$

$$\frac{a_1^2}{2} \left[l - \frac{1}{2\pi} (0 - 0) \right]$$

$$\int_0^l 81 a_2^2 \sin^2 \frac{3\pi x}{l} = 81 a_2^2 \int_0^l \frac{1}{2} \left(1 - \cos \frac{6\pi x}{l} \right) = \frac{a_1^2 l}{2}$$

$$= \frac{81 a_2^2}{2} \left[(x)_0^l - \left(\frac{\sin \frac{6\pi x}{l}}{\frac{6\pi}{l}} \right)_0^l \right]$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$



$$= \frac{81 a_2^2}{2} \left[l - 0 - \frac{1}{6\pi} \left(\sin \frac{6\pi l}{l} - \sin 0 \right) \right]$$

$$= \frac{81 a_2^2}{2} \left[l - \frac{1}{6\pi} (\sin 6\pi - \sin 0) \right]$$

$$\boxed{\sin 6\pi = 0; \sin 0 = 0}$$

$$\frac{81 a_2^2}{2} [l - 0]$$

$$= \frac{81 a_2^2 l}{2}$$



$$\int_0^l 189,92 \sin \frac{\pi x}{l} \sin \frac{3\pi x}{l}$$
$$= 189,92 \int_0^l \sin \frac{\pi x}{l} \sin \frac{3\pi x}{l} = 189,92 \int_0^l \sin \frac{3\pi x}{l} \sin \frac{\pi x}{l}$$
$$= \frac{189,92}{2} \left[\int_0^l \cos \frac{2\pi x}{l} dx - \int_0^l \cos \frac{4\pi x}{l} dx \right]$$

$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$



$$= \frac{q_1 q_2}{2} \left[\left(\frac{\sin \frac{2\pi x}{l}}{\frac{2\pi}{l}} \right)^2 - \left(\frac{\sin \frac{4\pi x}{l}}{\frac{4\pi}{l}} \right)^2 \right]$$

$$\begin{aligned} \sin 2\pi &= 0 \\ \sin 4\pi &= 0 \\ \sin 0 &= 0 \end{aligned}$$

$$= q_1 q_2 [0 - 0]$$

$$= 0 //$$



Thank You