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INTERPOLATION WITH EQUAL INTERVALS.



Newton's forward interpolation formula for equal intervals: Let $x_0, x_1, \dots x_n$ be equidistant values of x and $y_0, y_1, \dots y_n$ be the corresponding values of y = f(x).

Let
$$h = \chi_i - \chi_{i-1}$$
, $i = 1, 2, \cdots n$. Then

$$y = y_{0} + \frac{u}{1!} \Delta y_{0} + \frac{u(u-1)}{2!} \Delta^{2} y_{0} + \frac{u(u-1)(u-2)}{3!} \Delta^{3} y_{0}$$

$$+ \cdots + \frac{u(u-1)\cdots(u-(n-1))}{n!} \Delta^{n} y_{0}$$

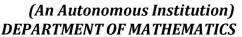
where
$$u = \frac{\chi - \chi_0}{h}$$
.

Newton's backward interpolation formula for equal intervals:

Let $x_0, x_1, x_2, \dots x_n$ be equidistant values of x and $y_0, y_1, y_2, \dots y_n$ be the corresponding values of y = f(x). Let $h = x_i - x_{i-1}$, $i = 1, 2, \dots n$. Then

where
$$V = \frac{\chi - \chi_n}{h}$$







(21.)

Problems :

1) Find the values of y at x = 21 and x = 28 from

the following data:

X: 20

23

26 29

y: 0.3+20 0.3907 0.438+ 0.48+8

iseln:

Here h = 3

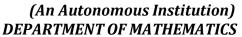
Since x = 21 is nearer to the beginning of the

table, we use Newton's forward formula.

$$y(x) = y_0 + p_\Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)_3}{3!} y_0$$

where
$$p = \frac{x - x_0}{h} = \frac{21 - 20}{3} = 0.3333$$







$$y(31) = 0.3420 + (0.33333) (0.0481) +$$

$$(0.3333) (-0.6667) (-0.001) +$$

$$2$$

$$(0.3333) (-0.6667) (-1.6667) (-0.0003)$$

Since x = 28 is nearer to the end value, we use Newton's backward interpolation formula,

$$y(x) = y_n + \nabla y_n \cdot q + \frac{q(q+1)}{2!} \nabla^2 y_n + \frac{q(q+1)(q+2)}{\sqrt{3}y_n}$$

where $q = x - x_n = 28 - 29$

+ . . .

$$9 = -0.3333$$
 $x_n = 20$

$$y(28) = 0.4848 + (-0.3333)(0.0464) + \frac{(0.33333)(0.6667)(-0.0013)}{2} +$$

6



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(27)

y(28) = 0.4848 - 0.01547 + 0.00014 + 0.00002 y(28) = 0.4695

(a) From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63

Age 2: 45 50 55 60 65

Premium y: 114.85 96.16 83.32 74.48 68.48

Soln:

χ	y	Δy	A^2y	$\Delta^3 y$	A+y
45	114 - 85	-18.69	5.85		
50	96.16	-12.84	5.00	-1.85	
55	83.32		4	-1.16	0.69
60	74.48	-8.84	2.84	- 1.16	
65	68.48	- 6			

Here h = 5.

Since x = 4b is nearer to the beginning of the table, we use Newton's forward difference formula.



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$$y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0$$

where
$$p = \frac{x - x_0}{h} = \frac{4b - 45}{5} = 0.2$$
 $p = 0.2$

$$y(46) = 114.85 + (0.2)(-18.69) + (0.2)(0.2+1)$$

Since
$$x = 63$$
 is nearer to $x = 65$, we use

Newton's backward formula

$$\frac{9(9+1)(9+2)}{3!} \quad \forall^{3}y_{n} + \frac{9(9+1)(9+2)(9+3)}{4!} \quad \forall^{4}y_{n}$$



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(28)

$$Y = \frac{x - x_n}{h} = \frac{63 - 65}{5} = -0.4$$

$$Y (63) = 68.48 + (-0.4)(-6) + \frac{(-4)(.6)(2.84)}{2}$$

$$+ \frac{(-0.4)(0.6)(1.6)}{6} (-1.16) + (-0.4)(0.6)$$

$$\frac{(1.6)(2.6)(.69)}{24}$$

$$= 68.48 + 2.4 - 0.3408 + 0.07424 - .0287$$

4(63) = 70.5847

 $\Delta y \quad \Delta^2 y \quad \Delta^3 y \quad \Delta^4 y \quad \Delta^5 y$ y = tan x° x' 45 0.03553 0.00131 1.03553 46 0.00009 0.03684 0.00003 0.0014 47 1.07237 -0.00005 0.00012 0.03824 -0.00002 0.00152 1-11061 48 0.03976 0.0001 0.00162 49 1. 15037 0.04138 1.19175 50



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Since x = 45'' 15' is nearer to the beginning of the table, we use Newton's forward difference formula,

 $y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0$

 $p = \frac{x - x_0}{h} \qquad x_0 = 45^\circ, \ x = 45^\circ 15^\prime, \ h = 1^\circ$ $= \frac{45^\circ 15^\prime - 45^\circ}{1^\circ} = \frac{15^\prime}{1^\circ} = \frac{15^\prime}{60^\prime} = 0.25$

 $y(45^{\circ}15') = 1 + (0.25)(0.03553) + \frac{(0.25)(-0.75)(0.00131)}{2} +$

(0.25)(-0.75)(-1.75) (0.00009) + (0.25) (-0.75) 6 (-1.75)(-2.75)

+ (0.25) (-0.75) (-1.75) (-2.75) (-3.75) (-0.00005)

= 1 + 0.00888 - 0.00012 + 0.000005 - 0.000001

9 (45° 15') = 1.008763

120

- 0. 000001



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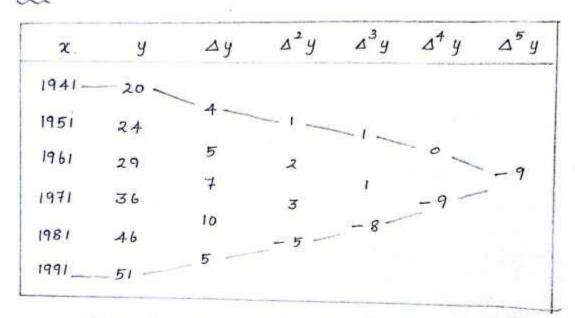
29)

1 The population of a Lown is as follows:

Year : 1941 1951 1961 1971 1981 1991

Population: 20 24 29 36 46 51

Estimate the population increase during the period 1946 to soln:



Here h = 10.

Since x = 1946 is nearer to the beginning of the table, we use Newton's forward difference table formula.

$$y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} y_0$$

Where
$$p = \frac{x - x_0}{h} = \frac{1946 - 1941}{10} = \frac{1}{2} = 0.5$$



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$$y(1946) = 20 + (0.5) 4 + (0.5) (-0.5) + \frac{2}{2} + \frac{(0.5)(-0.5)(-1.5)(-1.5)}{6} + \frac{(0.5)(-0.5)(-1.5)(-1.5)(-2.5)(0)}{24} + \frac{(0.5)(-0.5)(-1.5)(-2.5)(-3.5)(-9)}{24}$$

$$=$$
 20 + 2 - 0.125 + 0.0625 - 0.2461

Ne use backward difference formula to find

$$y(x) = y_n + q \nabla y_n + \underline{q(q+1)}_{2!} \nabla^2 y_n + \cdots$$

Where
$$c_V = \frac{\chi - \chi_0}{h} = \frac{1976 - 1991}{10} = -1.5$$

$$y(1976) = 51 + (-1.5)(5) + (-1.5)(-0.5)(-5)$$

$$+ \frac{(-1.5)(-0.5)(0.5)(-8)}{6} + \frac{(-1.5)(-0.5)(0.5)}{(1.5)(-9)}$$



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(30)

y (1976) = 51 - 7.5 - 1.875 - 0.5 - 0.2109 - 0.1055

Increase in Population during the period 1946 to 1976 is = 40.8086 - 21.6914 = 19.1172 lakhs.

(5) From the following table, find 0 at x = 43 and x = 84.

x: 40 50 60 70 80 90

0: 184 204 226 250 276 304

-50 ln:

X	Θ	40	1º 0	430
40	184			
50	204	20 _	- 2 _	
60	226	22	2	0
70	250	24	2	0
80	276	26	2 —	- 0
90	304	- 28 —		

Here h = 10.

To find x = 43, let us use Newton's forward difference formula.



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$$g(x) = \theta_0 + \beta \Delta \theta_0 + \frac{\beta(\beta-1)}{2} \Delta^2 \theta_0 + \cdots$$

where
$$b = \frac{x - x_0}{h} = \frac{43 - 40}{10} = 0.3$$

$$\theta(43) = 184 + (0.3)(20) + (0.3)(-0.7)(2)$$

$$= 184 + 6 - 0.21$$

To find $\chi = 84$, let us use Newton's Backward difference formula.

$$\theta(x) = \theta_n + q \nabla \theta_n + \frac{q(q+1)}{2!} \nabla^2 \theta_n + \cdots$$

Where
$$v = \frac{x - x_n}{h} = \frac{84 - 90}{10} = \frac{-6}{10} = -0.6$$

$$\theta(84) = 304 + (-0.6)(28) + (-0.6)(0.4)(2)$$