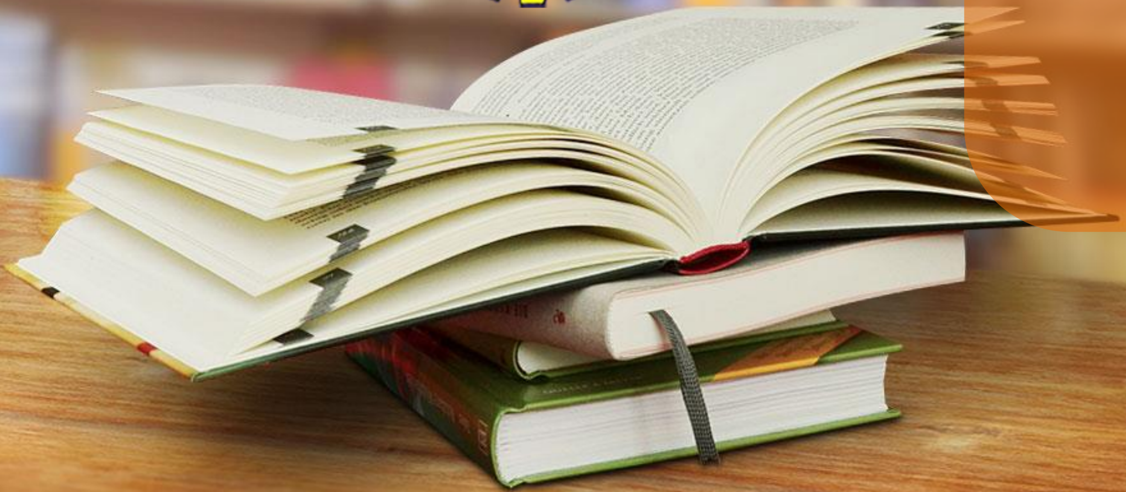
EUCLID'S ALGORITHM

Subject :Design and Analysis of Algorithm
Unit :I



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GCD(Greatest common divisor)

- The greatest common divisor of two nonnegative, not-both-zero integers *m* and *n*, denoted gcd*(m, n)*, is defined as the largest integer that divides both *m* and *n* evenly, i.e., with a remainder of zero

- Euclid of Alexandria (third century B.C.) outlined an algorithm for solving this problem in one of the volumes of his *Elements* most famous for its systematic exposition of geometry

Euclid's algorithm

Euclid's algorithm is based on applying repeatedly the equality

gcd(m, n) = gcd$(n, m \bmod n)$,

Where m mod n is the remainder of the division of m by n, until m mod n is equal to 0. Since gcd$(m, 0) = m$ (why?), the last value of m is also the greatest common divisor of the initial m and n.

For example, gcd$(60, 24)$ can be computed as follows:

gcd$(60, 24)$ = gcd$(24, 12)$ = gcd$(12, 0)$ = 12.

Euclid's algorithm for computing gcd*(m, n)*

Step 1 If $n=0$, return the value of m as the answer and stop;

otherwise, proceed to Step 2.

Step 2 Divide m by n and assign the value of the remainder to r.

Step 3 Assign the value of n to m and the value of r to n. Go to Step

Alternatively, we can express the same algorithm in pseudocode:

ALGORITHM *Euclid(m, n)*

//Computes gcd*(m, n)* by Euclid’s algorithm

//Input: Two nonnegative, not-both-zero integers *m* and *n*

//Output: Greatest common divisor of *m* and *n*

while *n*!= 0 **do**

$r \leftarrow m$ mod n

$m \leftarrow n$

$n \leftarrow r$

return *m*

Consecutive integer checking algorithm

Consecutive integer checking algorithm for computing gcd(m, n)

Step 1 Assign the value of min$\{m, n\}$ to t.

Step 2 Divide m by t. If the remainder of this division is 0, go to Step 3; otherwise, go to Step 4.

Step 3 Divide n by t. If the remainder of this division is 0, return the value of t as the answer and stop; otherwise, proceed to Step 4.

Step 4 Decrease the value of t by 1. Go to Step 2.

Middle-school procedure

Middle-school procedure for computing gcd(m, n)

Step 1 Find the prime factors of m.

Step 2 Find the prime factors of n.

Step 3 Identify all the common factors in the two prime expansions found in Step 1 and Step 2. (If p is a common factor occurring p_m and p_n times in m and n, respectively, it should be repeated $\min\{p_m, p_n\}$ times.)

Step 4 Compute the product of all the common factors and return it as the greatest common divisor of the numbers given.

Thus, for the numbers 60 and 24, we get

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$\gcd(60, 24) = 2 \cdot 2 \cdot 3 = 12.$

sieve of Eratosthenes

As an example, consider the application of the algorithm to finding the list of primes not exceeding $n = 25$:

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	3		5		7		9		11		13		15		17		19		21		23		25
2	3		5		7				11		13				17		19				23		25
2	3		5		7				11		13				17		19				23		

ALGORITHM *Sieve(n)*

//Implements the sieve of Eratosthenes

//Input: A positive integer $n > 1$

//Output: Array L of all prime numbers less than or equal to n

for $p \leftarrow 2$ **to** n **do** $A[p] \leftarrow p$
for $p \leftarrow 2$ **to** $\lfloor \sqrt{n} \rfloor$ **do** //see note before pseudocode
  if $A[p] \neq 0$ //p hasn't been eliminated on previous passes
    $j \leftarrow p * p$
    while $j \leq n$ **do**
      $A[j] \leftarrow 0$ //mark element as eliminated
      $j \leftarrow j + p$
//copy the remaining elements of A to array L of the primes
$i \leftarrow 0$
for $p \leftarrow 2$ **to** n **do**
  if $A[p] \neq 0$
    $L[i] \leftarrow A[p]$
    $i \leftarrow i + 1$
return L

Thank you!