TEST FOR DIFFERENCE OF PROPORTIONS :

Null hypothesis, $H_0 : P_1 = P_2$.

Test statistic, $z = \dfrac{p_1' - p_2'}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p_1' = \dfrac{x_1}{n_1}$ & $p_2' = \dfrac{x_2}{n_2}$

and $p = \dfrac{p_1' n_1 + p_2' n_2}{n_1 + n_2} = \dfrac{x_1 + x_2}{n_1 + n_2}$ & $q = 1 - p$.

1) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal, are same against that they are not, at 5% level.

Soln: Given: $n_1 = 400$, men , $x_1 = 200$

$n_2 = 600$, women , $x_2 = 325$

$p_1' = \dfrac{x_1}{n_1} = \dfrac{200}{400} = 0.5$ & $p_2' = \dfrac{x_2}{n_2} = \dfrac{325}{600} = 0.541$

$p = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{200 + 325}{400 + 600} = 0.525$ & $q = 1 - p$
$= 0.475$

step 1 : Formulating H_0 and H_1

$H_0: P_1 = P_2$, in favour of proposal (no diff. btw them)

$H_1: P_1 \neq P_2$ (two tailed test)

step 2 : LOS $\alpha = 5\% = 0.05$

step 3 : Test statistic,

$$z = \frac{p_1' - p_2'}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$= \frac{0.5 - 0.541}{\sqrt{0.525 \times 0.475\left(\frac{1}{400}+\frac{1}{600}\right)}}$$

$$= \frac{-0.041}{\sqrt{0.001039}}$$

$$= -1.269$$

$$|z| = 1.269$$

step 4 : Critical value at 5% LOS is $z_\alpha = 1.96$.

step 5 : Conclusion : $z = 1.269 < 1.96 = z_\alpha$

$\therefore$ H_0 is accepted to 5% LOS.

$\therefore$ the men & women do not differ significantly, as regards proposal of flyover is concerned.

2) In a large city A, 20% of a random sample of 900 school children had defective eye-sight. In other large city B, 15% of random sample of 1600 children had the same defect. Is this difference between the two proportions significant? [obtain 95% confidence limits for the difference in the population proportions]

Soln:

Given: In city A, $n_1 = 900$, $P_1' = 20\% = 0.20$

In city B, $n_2 = 1600$, $P_2' = 15\% = 0.15$

$$P = \frac{P_1' n_1 + P_2 n_2}{n_1 + n_2} = \frac{0.20(900) + 0.15(1600)}{900 + 1600}$$

$= 0.168$

$q = 1 - P = 1 - 0.168 = 0.832$

step 1: Formulating H_0 and H_1.

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ (two tailed test)

step 2: LOS at $\alpha = 5\% = 0.05$

step 3: Test statistic,
$$z = \frac{p_1' - p_2'}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$
$$= \frac{0.20 - 0.15}{\sqrt{0.168 \times 0.832\left(\frac{1}{900} + \frac{1}{1600}\right)}}$$

$$= \frac{0.05}{0.0156}$$

$$z = 3.21$$

step 4 : Critical value at 5% LOS is $z_\alpha = 1.96$

step 5 : Conclusion: $z = 3.21 > 1.96 = z_\alpha$

$\therefore$ H_0 is rejected at 5% LOS.

$\therefore$ The difference between the two proportions is significant.