22/10/19

Centre and circle of curvature:

The centre of curvature $\bar{x}, \bar{y}$ at any point x, y on the curve $y = f(x)$ is $(\bar{x}, \bar{y})$ where

$$\left[\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2}\right.$$

$$\left.\bar{y} = y + \frac{(1+y_1^2)}{y_2}\right]$$

where $y_1 = \frac{dy}{dx}$; $y_2 = \frac{d^2y}{dx^2}$

The circle of curvature is $\boxed{(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2}$

eg: 1

① Find the centre a circle of curvature for the curve $xy = c^2$ at the point c, c.

$$xy = c^2 \rightarrow ①$$

Diff eqn w.r.t 'x'.

$$x\left(\frac{dy}{dx}\right) + y(1) = 0$$

$$\frac{dy}{dx} = \frac{-y}{x} \rightarrow ②$$

Diff eqn ② w.r.t 'x',

$$\frac{d^2y}{dx^2} = -\left[\frac{x \cdot \frac{dy}{dx} - y(1)}{x^2}\right] \rightarrow ③$$

Now eqn ② & ③ at (c,c)

$$y_1 = \left(\frac{dy}{dx}\right)_{(c,c)} = -1$$

$$y_2 = \left(\frac{d^2y}{dx^2}\right)_{(c,c)} = -\left[\frac{c(-1)-c}{c^2}\right]$$

$$\boxed{y_2 = 2/c}$$

$$P = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{2^{3/2}}{2/c} = \frac{\cancel{2}\cdot 2^{1/2}}{\cancel{2}}\cdot c$$

$$\boxed{\rho = c\sqrt{2}}$$

The centre of curvature $(\bar{x}, \bar{y})$

$$\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$\bar{x} = \frac{c - (-1)(1+(-1)^2)}{2/c} = c + \frac{\cancel{2}c}{\cancel{2}}$$

$$\boxed{\bar{x} = 2c}$$

$$\bar{y} = y + \frac{(1+y_1^2)}{y_2}$$

$$= \frac{c + (1+(-1)^2)}{2/c} = c + \frac{\cancel{2}\times c}{\cancel{2}}$$

$$\boxed{\bar{y} = 2c}$$

$$C = (2c, 2c)$$

The circle of curvature is $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

$$(x-2c)^2 + (y-2c)^2 = (c\sqrt{2})^2.$$

* * (2) Find the centre and circle of curvature

$\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $(a/4, a/4)$

* * (3) find the centre and circle of curvature

$x^3+y^3=3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

Normal - it is a perpendicular line