SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

Unit 3 : FREQUENCY RESPONSE ANALYSIS

Topic 2 : Frequency Response - Bode Plot

Introduction

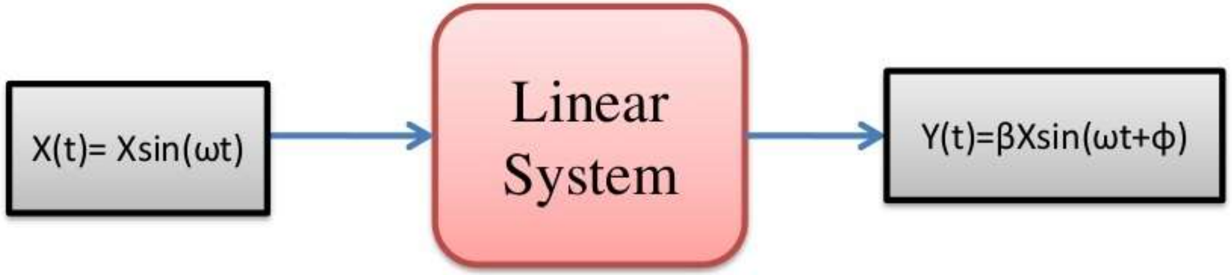
❖ <u>Time-Domain analysis:</u>

Impulse, unit step, ramp, etc. are used as input to the system

❖ <u>Frequency-Domain Analysis:</u>

Frequency Response of a system is the response of the system for sinusoidal input signal of various frequencies

Frequency Response



Need

- Extraction of Transfer function from Time domain is difficult using differential equations
- Using Frequency Response, Transfer Function can be easily obtained from the experimental data
- A system may be designed, so that effects of noise are negligible
- Analysis & Design are extended to certain non-linear control systems
- The design of controller can be easily done in the frequency domain method, as compared to time-domain method

Specifications

1. **Resonant Frequency (ωr):**

   The frequency at which the system has maximum magnitude is known as the resonant frequency. At this, **the slope of magnitude curve is zero**

2. **Resonant Peak (Mr):**

   Maximum value of magnitude.

   Mr= 1- 1.5

3. **Cut-off Frequency**:

The frequency at which the magnitude G(jω) is 0.707 times less than its, maximum value is known as cut-off frequency.

4. **Bandwidth**: For feedback control systems, the range of frequencies over which M is equal to or greater than 0.707Mr is defined as bandwidth ωb

When Mr=1, Bandwidth = Cut-off frequency

The bandwidth of a control system indicates the noise-filtering characteristics of the system.

5. Cut off Rate:

Rate of change of slope of magnitude at cut-off frequency

6. Gain Margin:

Amount of gain in decibels that can be added to the loop before the closed loop system becomes unstable

Gain Crossover: The Point at which the magnitude plot crossover 0dB

7. Phase Margin:

Amount of phase shift in degrees that can be added to the loop before the closed loop system becomes unstable

Phase Crossover: The Point at which the phase plot crosses is 180°

BODE PLOT

- Purpose:

1. To draw the frequency response of OLTF
2. To find the closed loop system stability
3. To find the relative stability by using GM & PM

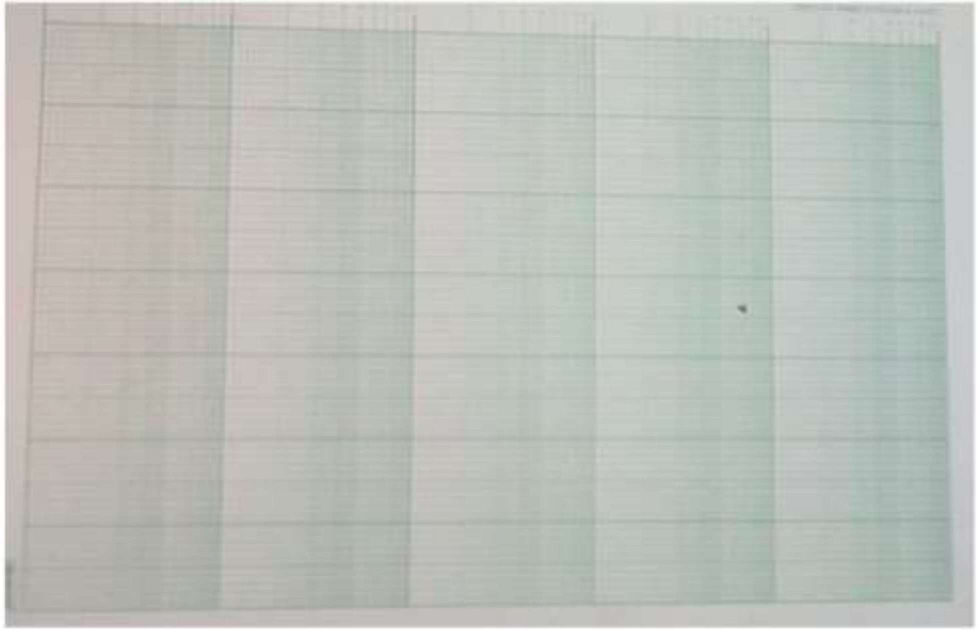
https://www.youtube.com/watch?v=Apm1D9Tie9s
https://www.youtube.com/watch?v=CYaRer4vsdo
https://www.youtube.com/watch?v=9S5HVn63rFU-ANIMATION

BODE PLOT

- The Bode Plot consists of two parts:

1. Magnitude Plot
2. Phase Plot

Semilog paper



Basic Procedure

1. Replace s→jω, to convert it into frequency domain
2. Write the magnitude & Convert it into dB

   Magnitude = 20log G(j ω)

3. Find the ϕ angle
4. Vary 'ω' from min to max value & draw the approximate magnitude & Phase plot

Basic Factors

1. Gain K
2. Pole at the origin (integral factor) $1/(j\omega)$
3. Multiple poles at the origin $1/(j\omega)^n$
4. Zero at the origin (derivative factor) $j\omega$
5. Multiple zeros at the origin $(j\omega)^n$
6. Factors of the form $K/(j\omega)^r$
7. First-order pole on the real axis $1/(1+ j\omega T)$
8. Multiple poles on the real axis $1/(1+ j\omega T)^n$
9. First-order zero on the real axis $(1+ j\omega T)$
10. Multiple zeros on the real axis $(1+ j\omega T)^n$
11. Quadratic poles $1/[1 + 2\xi(j\omega/\omega_n)+(j\omega/\omega_n)^2]$
12. Quadratic zeros $[1 + 2\xi(j\omega/\omega_n)+(j\omega/\omega_n)^2]$

Constant Gain K (Shift K)

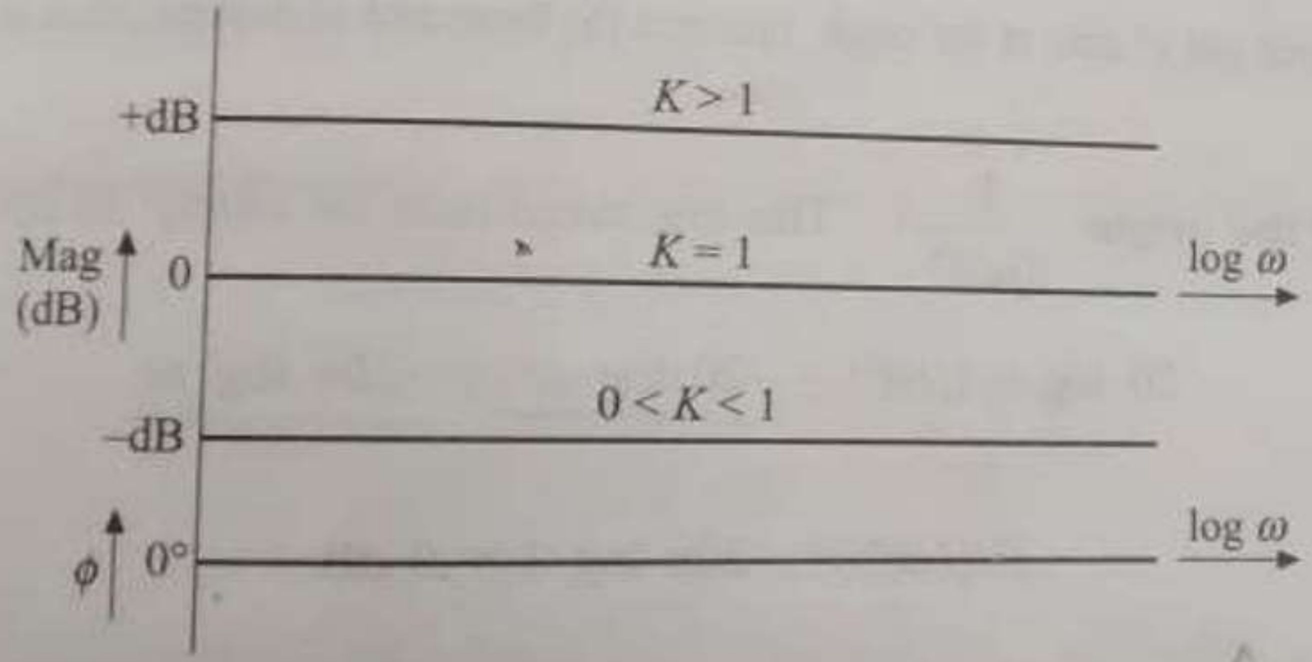


Figure 7.17 Bode plot of constant gain K.

Pole at Origin

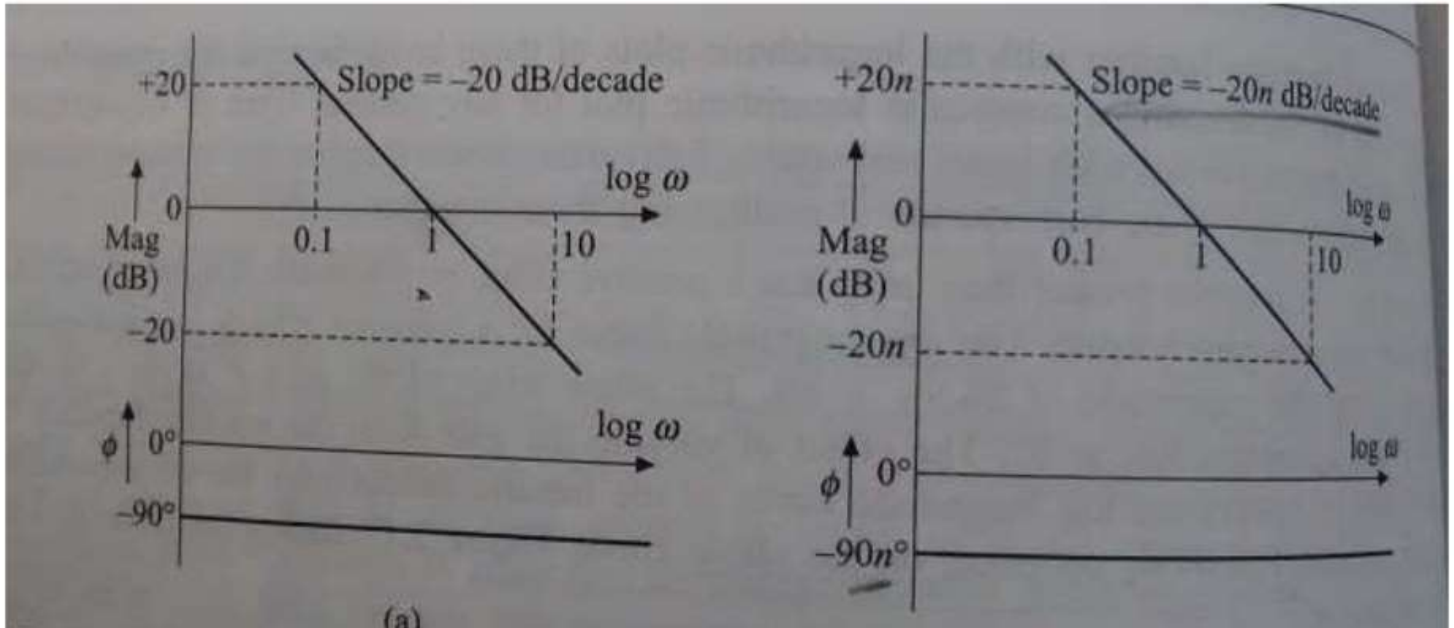


Figure 7.18 (a) Bode plot of pole at the origin 1/jω and (b) Bode plot of multiple poles at the origin 1/(jω)^n

Zero at Origin

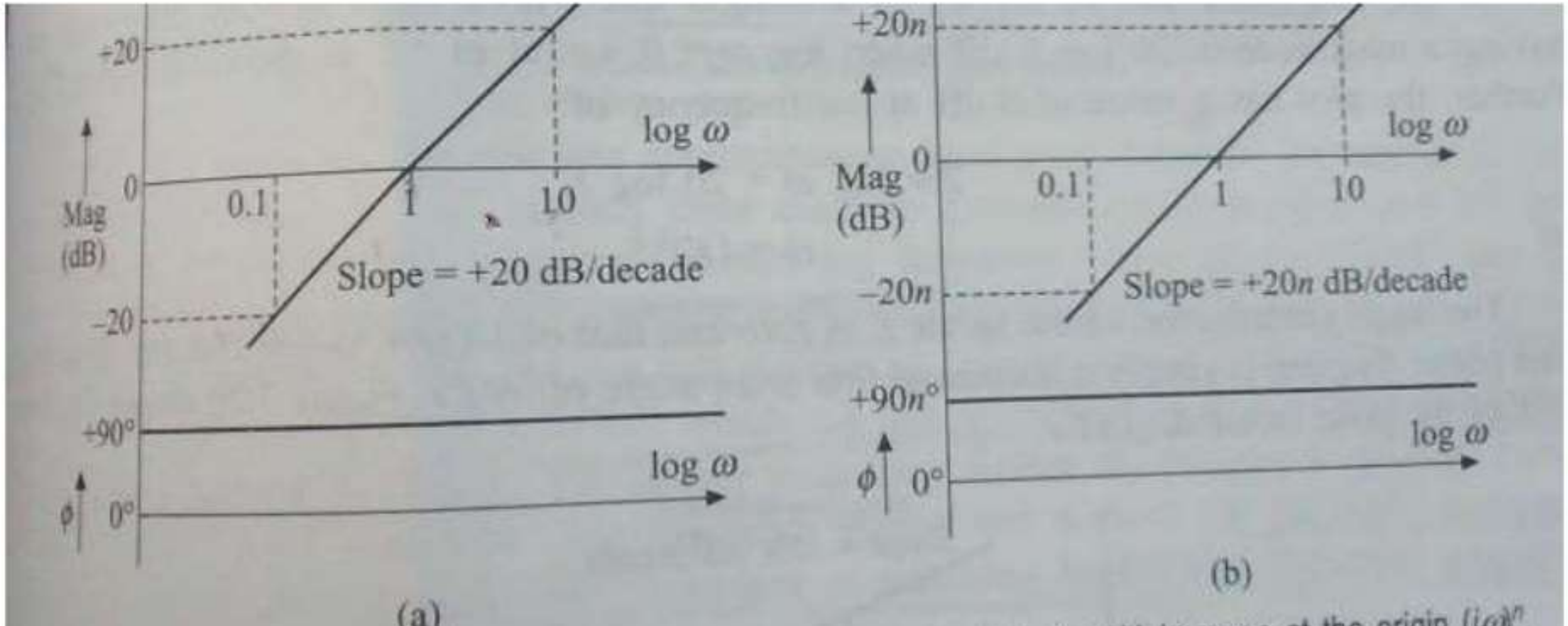


Figure 7.19 (a) Bode plot of zero at the origin $j\omega$ and (b) Bode plot of multiple zeros at the origin $(j\omega)^n$.

Simple pole at origin

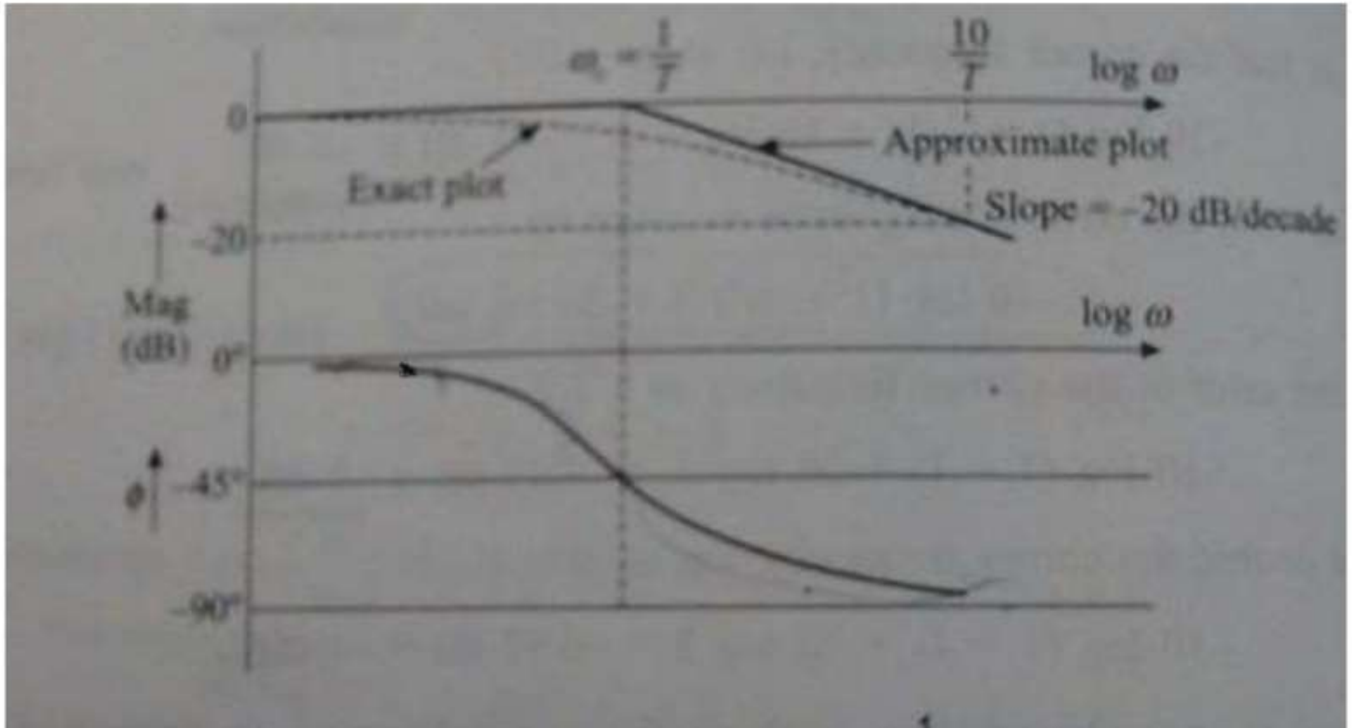


Figure 7.21 Bode plot of the factor $\frac{1}{1+j\omega T}$ (simple pole).

ACTIVITY

Simple Zero at origin

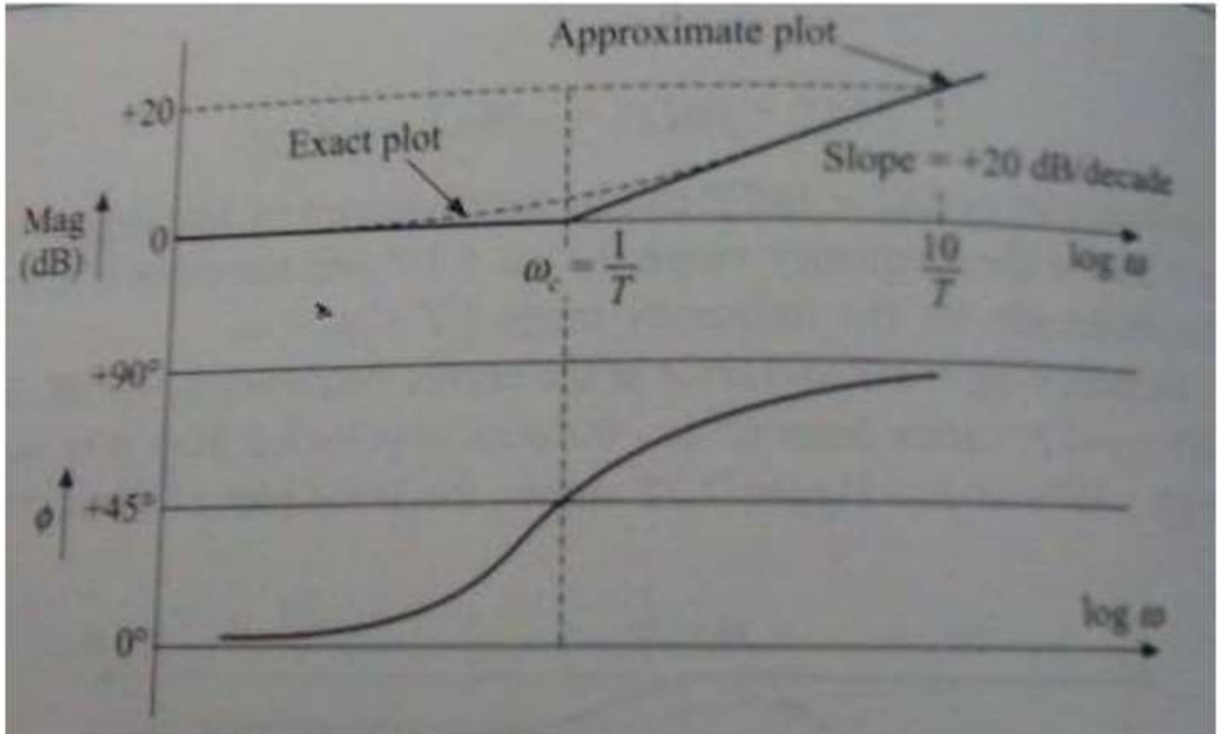


Figure 7.23 Bode plot of the factor $(1 + j\omega T)$ (simple zero).

Comparison

n-finite poles

$$GH(s) = \frac{1}{(s\tau+1)^n}$$

$$s = j\omega \rightarrow GH(j\omega) = \frac{1}{(j\omega\tau+1)^n}$$

$$M = \left(\frac{1}{\sqrt{(j\omega)^2+1}}\right)^n$$

$$M_{indB} = -20n\log\sqrt{(\omega\tau)^2+1}$$

$$\phi_{actual} = \frac{\angle 1+j0}{\angle(j\omega\tau+1)\ldots n\text{ times}}$$

$$\phi_{actual} = n\tan^{-1}(\omega\tau)$$

n-finite zero

$$GH(s) = (s\tau+1)^n$$

$$s = j\omega \rightarrow GH(j\omega) = (j\omega\tau+1)^n$$

$$M = \left(\sqrt{(j\omega)^2+1}\right)^n$$

$$M_{indB} = 20n\log\sqrt{(\omega\tau)^2+1}$$

$$\phi_{actual} = -n\tan^{-1}(\omega\tau)$$

Comparison

N poles at Origin

$GH(s) = 1/s^n$

$s \rightarrow j\omega$

$$GH(j\omega) = \frac{1}{(j\omega)^n}$$

$$M = \frac{1}{\omega^n}$$

$M_{in\,dB} = -20n \log \omega$

$$S = \frac{dM}{d \log \omega} = -20n \text{ dB/decade.}$$

$$\phi = \frac{\angle(1+j0)}{\angle(j\omega) \ldots n\text{ times}} = \frac{0^\circ}{90^\circ \ldots n\text{ times}} = -90^\circ n$$

N zeros at origin

$GH(s) = s^n$

$s \rightarrow j\omega$

$GH(j\omega) = (j\omega)^n$

$M = (\omega)^n$

$M_{in\,dB} = +20n \log(\omega)$

$$S = \frac{dM}{d \log \omega} = 20n \text{ db/decade}$$

$\phi = +90^\circ n$

Steps to sketch Bode Plot

1. Express given G(s)H(s) in time constant form
2. Draw a line of 20LogK dB
3. Draw a line of appropriate slope representing poles or zeros at the origin, passing through intersection point of ω=1 and 0 dB
4. Shift this intersection point on 20LogK line and draw parallel line to the line drawn in step 3
5. Change the slope of this line at various corner frequencies by appropriate value. **For a pole, slope must be changed by -20dB/Decade, For a simple zero slope changed by +20dB/Decade .**

- **Continue this slope line, till it intersects last corner frequency**

6. Prepare the phase angle plot and table & obtain the table of ω and resultant phase angle φR by actual calculation.
7. Plot these points , & Draw a smooth curve obtaining necessary phase angle plot

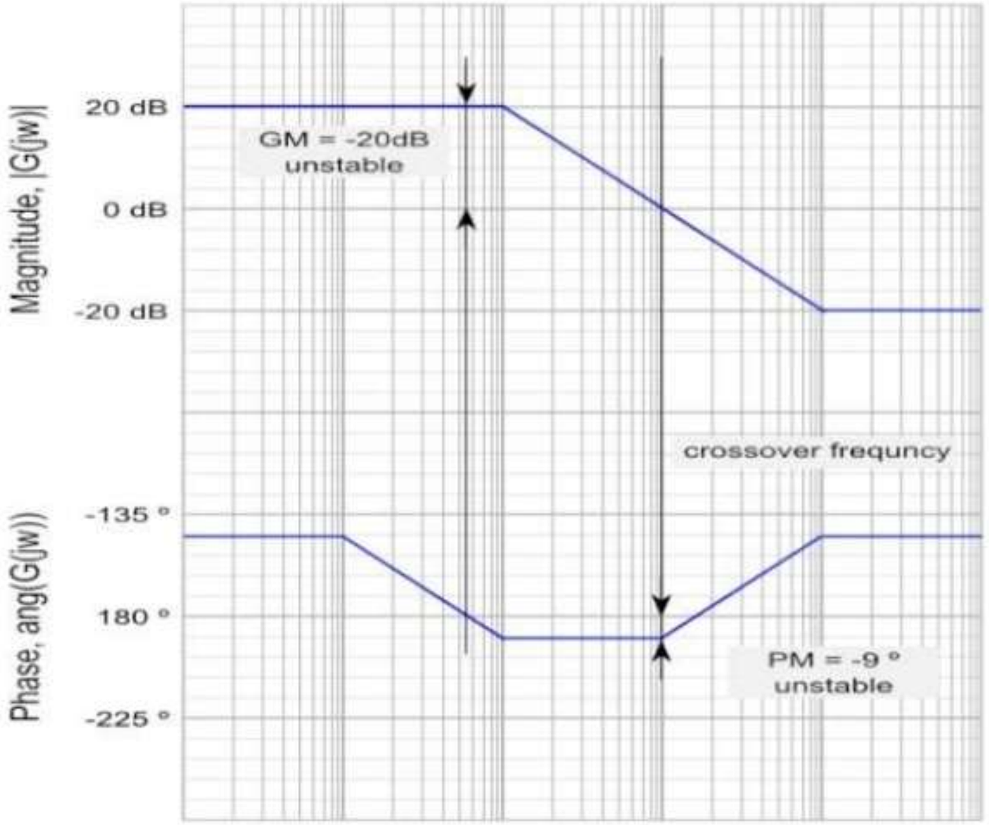
Gain & Phase

Gain Margin

- Factor by which system gain is increased to bring the system to the verge of stability
- **Gain crossover:** ωgc
- **The frequency at which the magnitude=1 in linear and 0 in dB.**

Phase Margin

- Additional phase lag required to ass system at ωgc to bring system to stability
- **Phase cross over :** ωpc
- **The frequency at which angle is -180 degre**

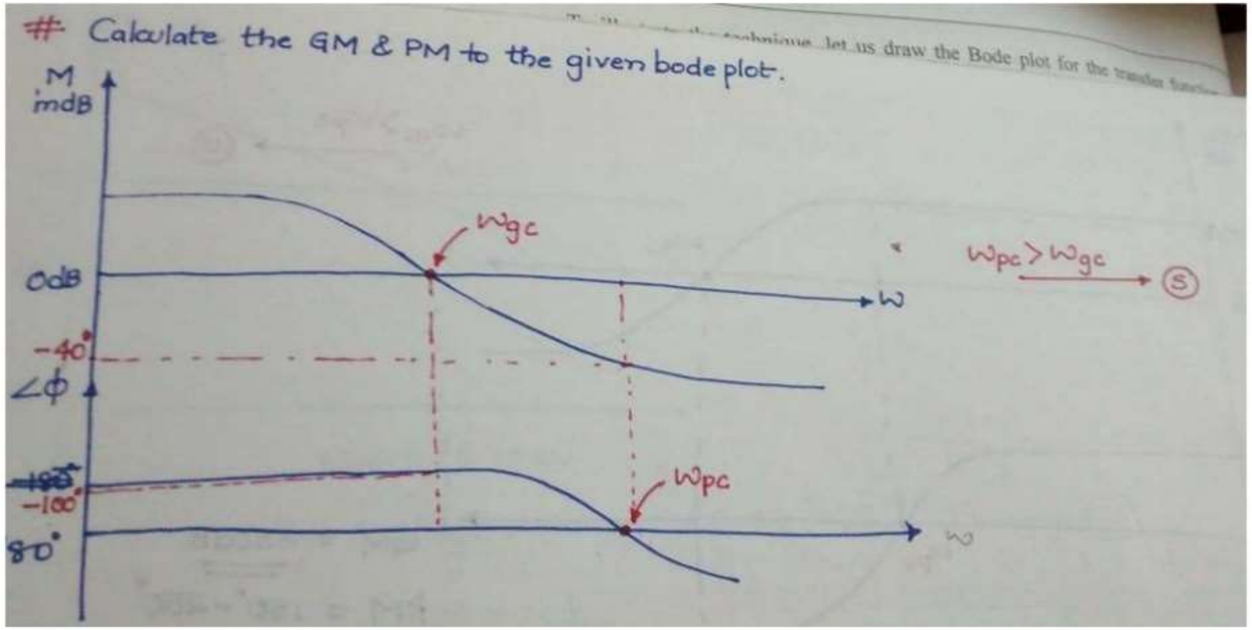


Magnitude, |G(jw)|
20 dB
0 dB
-20 dB
GM = -20dB
unstable
crossover frequncy
Phase, ang(G(jw))
-135 °
180 °
-225 °
PM = -9 °
unstable

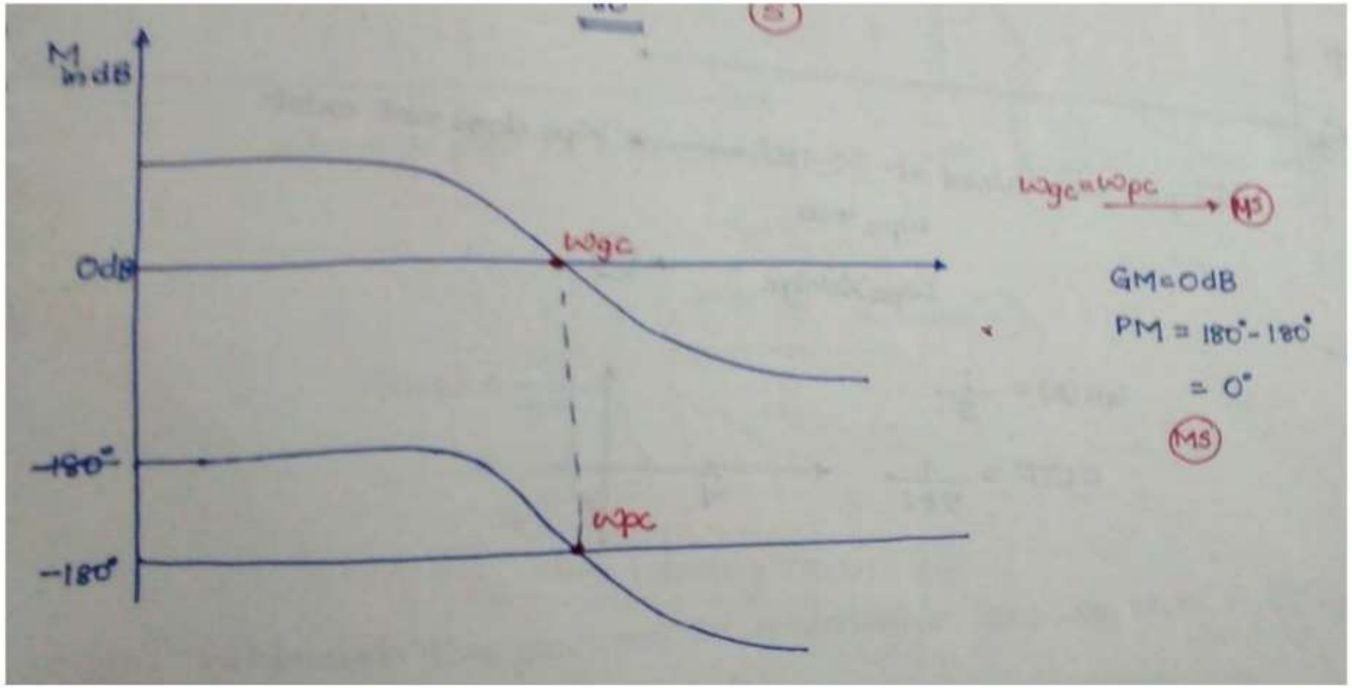
- **Gain Margin**
- The gain margin (GM) is the factor by which the gain is less than the neutral stability value. We can usually read the gain margin directly from the bode plot. This is done by calculating the vertical distance between the curve and the at the frequency.
- **Phase Margin**
- Another quantity related to determining stability margin is the phase margin (PM). This is a different way to measure how well stability conditions are met in a given system. Phase margin is determined by how much the phase of exceeds.
- The above figure shows that, **for a system to be stable, a positive PM is required**. From the figures, we can also see that **the GM indicates the amount that the gain can increase before a system becomes unstable**. The PM is calculated by measuring the difference between the and when crosses the circle . The stable case receives the phase margin's positive value.

Crossover Frequency

- A gain of factor 1 (equivalent to 0 dB) where both input and output are at the same voltage level and impedance is known as unity gain. When the gain is at this frequency, it is often referred to as crossover frequency.
- Frequency-response design is practical because we can easily evaluate how gain changes affect certain aspects of systems. With frequency-response design, we can determine the phase margin for any value of without needing to redraw the magnitude or phase information. All we have to do is indicate where for certain trial values of



Calculate the GM & PM to the given bode plot.
M
in dB
0dB
-40°
∠φ
-180°
ωgc
ωpc
ω
ωpc > ωgc


M in dB
0dB
ωgc
-180°
ωpc
-180°
ωgc=ωpc → MS
GM=0dB
PM = 180°-180°
= 0°
MS

G(s)H(s)=2000(s+1)/s(s+10)(s+40)

8. Phase plot

Freq ω	$\angle\frac{1}{j\omega}$ $\phi_1 = -90^\circ$	$\angle 1 + j\omega$ $\phi_2 = tan^{-1}(\omega)$	$\angle\frac{1}{1+j\omega/10}$ $\phi_3 = -tan^{-1}(\omega/10)$	$\angle\frac{1}{1+j\omega/40}$ $\phi_4 = -tan^{-1}(\omega/40)$	Total angle Φ
0.5	-90	26.56	-2.86	-0.71	-67.1
1	-90	45	-5.71	-1.43	-52.14
2	-90	63.43	-11.31	-2.86	-40.73
5	-90	78.69	-26.56	-7.12	-45
10	-90	84.3	-45	-14.03	-64.73
20	-90	87.1	-63.43	-26.56	-92.89
40	-90	88.56	-75.96	-45	-122.4
80	-90	89.28	-82.87	-63.43	-147.02
200	-90	90	-87.13	-78.69	-165.82
500	-90	90	-88.85	-85.42	-174.27
1000	-90	90	-90	-87.7	-177.7

$$G(j\omega)H(j\omega) = \frac{6(1 + j\omega/4)}{j\omega(1 + j\omega)[1 + j0.3(\omega/6) - (\omega/6)^2]}$$

Factor	Corner frequency	Asymptotic log-magnitude characteristic	Phase angle characteris...
$\frac{6}{j\omega}$	None	Straight line with a slope of −20 dB/decade passing through the 20 log 6 = 15.56 dB point at $\omega = 1$.	Constant −90°.
$\frac{1}{1+j\omega}$	$\omega_1 = 1$	For $\omega < \omega_1 = 1$, it is a straight line of 0 dB. For $\omega > \omega_1 = 1$, it is a straight line with a slope of −20 dB/decade.	Phase angle varies fr... 0 to −90°. At $\omega = \omega_1$, $\phi = -45°$.
$1 + j\omega/4$	$\omega_2 = 4$	For $\omega < \omega_2 = 4$, it is a straight line of 0 dB. For $\omega > \omega_2 = 4$, it is a straight line with a slope of +20 dB/decade.	Phase angle varies fr... 0 to +90°. At $\omega = \omega_2$, $\phi = -45°$.
$\frac{1}{1 + j0.3\left(\frac{\omega}{6}\right) - \left(\frac{\omega}{6}\right)^2}$	$\omega_3 = 6$, $\xi = 0.15$	For $\omega < \omega_3 = 6$, it is a straight line of 0 dB. For $\omega > \omega_3 = 6$, it is a straight line with a slope of −40 dB/decade.	Phase angle varies fr... [illegible] to −180°. At $\omega = $ [illegible] $\phi = -90°$.

The resultant phase angle is

$$\phi = \angle \frac{6(1+j\omega/4)}{j\omega(1+j\omega)[1+j0.3(\omega/6)-(\omega/6)^2]}$$

i.e.

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{0.3\omega/6}{1-(\omega/6)^2} + \tan^{-1}(\omega/4)$$

* Imp.

The resultant phase at various frequencies is calculated and a smooth curve is drawn passing through those points.

$\omega = 1,\quad \phi = -90^\circ - 45^\circ - 2.94^\circ + 14.03^\circ = -123.91^\circ$

$\omega = 4,\quad \phi = -90^\circ - 76^\circ - 19.8^\circ + 45^\circ = -140.8^\circ$

$\omega = 5,\quad \phi = -90^\circ - 79^\circ - 39.3^\circ + 51.3^\circ = -157^\circ$

$\omega = 5.5,\quad \phi = -90^\circ - 79.7^\circ - 60^\circ + 54^\circ = -176^\circ$

$\omega = 6,\quad \phi = -90^\circ - 80.5^\circ - 90^\circ + 56.3^\circ = -204.2^\circ$

$\omega = 8,\quad \phi = -90^\circ - 82.87^\circ - 117^\circ + 63.4^\circ = -226.47^\circ$

G(s)H(s)=20(s+1)/s(s+5)(s^2+2S+10)

8. Phase plot

Freq. ω	$\angle\frac{1}{j\omega}$ $\phi_1 = -90^\circ$	$\angle 1 + j\omega$ $\phi_2 = tan^{-1}(\omega)$	$\angle\frac{1}{1+j\omega/5}$ $\phi_3 = -tan^{-1}(\omega/5)$	$\angle\frac{1}{1+j\omega/5-\omega^2/10}$ $\phi_4 = -tan^{-1}(\frac{\omega/5}{1-\omega^2/10})$	Total angle Φ
0.1	-90	5.71	-1.1458	-1.1469	-86.5821
0.2	-90	11.31	-2.29	-2.2998	-83.2805
1	-90	45	-11.31	-12.53	-68.8387
3	-90	71.56	-30.96	-80.53	-129.9364
4	-90	75.96	-38.66	-126.87	-179.5465
5	-90	78.69	-45	-146.31	-202.62
10	-90	84.29	-63.44	-167.47	-236.62
100	-90	89.42	-87.13	-178.85	-266.56


THANK
YOU