SNS COLLEGE OF TECHNOLOGY

Coimbatore-12
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT III – FREQUENCY RESPONSE ANALYSIS

TOPIC 1- FREQUENCY RESPONSE

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- WHAT IS FREQUENCY RESPONSE?
- FREQUENCY DOMAIN SPECIFICATIONS
- RESONANT PEAK
- EXAMPLES
- ACTIVITY
- RESONANT FREQUENCY
- BANDWIDTH.
- SUMMARY

WHAT IS FREQUENCY RESPONSE?

- The response of a system can be partitioned into both the transient response and the steady state response.
- We can find the transient response by using Fourier integrals.
- The steady state response of a system for an input sinusoidal signal is known as the **frequency response**.
- It will focus only on the steady state response.
- If a sinusoidal signal is applied as an input to a Linear Time-Invariant (LTI) system, then it produces the steady state output, which is also a sinusoidal signal.
- The input and output sinusoidal signals have the same frequency, but different amplitudes and phase angles.

WHAT IS FREQUENCY RESPONSE?

Let the input signal be –

- $r(t)=A\sin(\omega 0t)$
- The open loop transfer function will be – $G(s)=G(j\omega)$
- We can represent $G(j\omega)$ in terms of magnitude and phase as shown below.
- $G(j\omega)=|G(j\omega)|\angle G(j\omega)$
- Substitute, $\omega=\omega 0$ in the above equation. $G(j\omega 0)=|G(j\omega 0)|\angle G(j\omega 0)G$
- The output signal is $c(t)=A|G(j\omega 0)|\sin(\omega 0t+\angle G(j\omega 0))$

WHAT IS FREQUENCY RESPONSE?

A short introduction to the steady state response of control systems to sinusoidal inputs will be given.

Frequency domain specifications for a control system will be examined

Bode plots and their construction using asymptotic approximations will be presented.

In frequency response analysis of control systems, the steady state response of the system to sinusoidal input is of interest.

The frequency response analyses are carried out in the frequency domain, rather than the time domain.

It is to be noted that, time domain properties of a control system can be predicted from its frequency domain characteristics.

WHAT IS FREQUENCY RESPONSE?

- To carry out the same process in the frequency domain for sinusoidal steady state analysis, one replaces the Laplace variable s with

$s=j\omega$

in the input output relation $C(s)=T(s)R(s)$

with the result

$C(j\omega)=T(j\omega)R(j\omega)$

WHAT IS FREQUENCY RESPONSE?

- For the input and output described by

$r(t) = R\sin\omega t)$ $c(t) = C\sin(\omega t + \phi)$

the amplitude and the phase of the output can now be written as

$$C = R\,T(j\omega)$$

$$\phi = \angle T(j\omega)$$

WHAT IS FREQUENCY RESPONSE?

- Remember that for a complex number be expressed in its real and imaginary parts :

$z = a+bj$

- the magnitude is given by :

$$z = (a+bj)(a-bj) = a^2 + b^2$$

- the phase is given by :

$$\angle z = \tan^{-1} b$$

EXAMPLE 1FREQUENCY RESPONSE?

- For a system described by the differential equation

$$\ddot{x}' + 2\dot{x} = y(t)$$

determine the steady state response $x_{ss}(t)$ for a pure sine wave input

$y(t)=3\sin(0.5t)$

FREQUENCY RESPONSE – Example 2

- The transfer function is given by

$$\ddot{x} + 2\dot{x} = y(t)$$

$$T(s) = \frac{X(s)}{Y(s)} = \frac{1}{s(s+2)}$$

Insert $s = j\omega$ to get :

$$T(j\omega) = \frac{1}{j\omega\,(j\omega+2)}$$

For $\omega = 0.5$ [rad/s]:

$$T(0.5j) = \frac{1}{0.5j(0.5j+2)} = \frac{1}{-0.25+j}$$

WHAT IS FREQUENCY RESPONSE?

- The **amplitude** of the output sinusoidal signal is obtained by multiplying the amplitude of the input sinusoidal signal and the magnitude of $G(j\omega)$ at $\omega=\omega 0$
- The **phase** of the output sinusoidal signal is obtained by adding the phase of the input sinusoidal signal and the phase of $G(j\omega)$ at $\omega=\omega 0$
- Where, **A** is the amplitude of the input sinusoidal signal.
  $\boldsymbol{\omega_0}$ is angular frequency of the input sinusoidal signal.

We can write, angular frequency $\omega 0$ as shown below.

$$\omega 0=2\pi f0$$

Here, f0 is the frequency of the input sinusoidal signal.

Similarly, you can follow the same procedure for closed loop control system.

FREQUENCY DOMAIN SPECIFICATIONS

RESONANT PEAK, RESONANT FREQUENCY AND BANDWIDTH.

Consider the transfer function of the second order closed loop control system as,

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Substitute, $s = j\omega$ in the above equation.

$$T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n(j\omega) + \omega_n^2}$$

$$\Rightarrow T(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2j\delta\omega\omega_n + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2\left(1 - \frac{\omega^2}{\omega_n^2} + \frac{2j\delta\omega}{\omega_n}\right)}$$

$$\Rightarrow T(j\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\left(\frac{2\delta\omega}{\omega_n}\right)}$$

Let, $\frac{\omega}{\omega_n} = u$ Substitute this value in the above equation.

$$T(j\omega) = \frac{1}{(1 - u^2) + j(2\delta u)}$$

FREQUENCY DOMAIN SPECIFICATIONS

Magnitude of $T(j\omega)$ is -

$$M = |T(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\delta u)^2}}$$

Phase of $T(j\omega)$ is -

$$\angle T(j\omega) = -tan^{-1}\left(\frac{2\delta u}{1-u^2}\right)$$

RESONANT FREQUENCY

It is the frequency at which the magnitude of the frequency response has peak value for the first time. It is denoted by ωr .
At ω=ωr , the first derivate of the magnitude of T(jω) is zero.

Differentiate M with respect to u.

$$\frac{dM}{du} = -\frac{1}{2}\left[(1-u^2)^2 + (2\delta u)^2\right]^{\frac{-3}{2}} \left[2(1-u^2)(-2u) + 2(2\delta u)(2\delta)\right]$$

$$\Rightarrow \frac{dM}{du} = -\frac{1}{2}\left[(1-u^2)^2 + (2\delta u)^2\right]^{\frac{-3}{2}} \left[4u(u^2 - 1 + 2\delta^2)\right]$$

Substitute, $u = u_r$ and $\frac{dM}{du} == 0$ in the above equation.

$$0 = -\frac{1}{2}\left[(1-u_r^2)^2 + (2\delta u_r)^2\right]^{-\frac{3}{2}} \left[4u_r(u_r^2 - 1 + 2\delta^2)\right]$$

$$\Rightarrow 4u_r(u_r^2 - 1 + 2\delta^2) = 0$$

$$\Rightarrow u_r^2 - 1 + 2\delta^2 = 0$$

$$\Rightarrow u_r^2 = 1 - 2\delta^2$$

$$\Rightarrow u_r = \sqrt{1 - 2\delta^2}$$

Substitute, $u_r = \frac{\omega_r}{\omega_n}$ in the above equation.

$$\frac{\omega_r}{\omega_n} = \sqrt{1 - 2\delta^2}$$

$$\Rightarrow \omega_r = \omega_n \sqrt{1 - 2\delta^2}$$

RESONANT PEAK & ACTIVITY-GD

It is the peak (maximum) value of the magnitude of T(jω). It is denoted by Mr.

- Resonant peak in frequency response corresponds to the peak overshoot in the time domain transient response for certain values of damping ratio δ.

- So, the resonant peak and peak overshoot are correlated to each other.

At u=ur, the Magnitude of T(jω) is -

$$M_r = \frac{1}{\sqrt{(1-u_r^2)^2 + (2\delta u_r)^2}}$$

Substitute, $u_r = \sqrt{1-2\delta^2}$ and $1 - u_r^2 = 2\delta^2$ in the above equation.

$$M_r = \frac{1}{\sqrt{(2\delta^2)^2 + (2\delta\sqrt{1-2\delta^2})^2}}$$

$$\Rightarrow M_r = \frac{1}{2\delta\sqrt{1-\delta^2}}$$

BANDWIDTH

It is the range of frequencies over which, the magnitude of T(jω) drops to 70.7% from its zero frequency value.

At ω=0, the value of u will be zero. Substitute, u=0 in M.

$$M = \frac{1}{\sqrt{(1-0^2)^2 + (2\delta(0))^2}} = 1$$

Therefore, the magnitude of $T(j\omega)$ is one at $\omega = 0$.

At 3-dB frequency, the magnitude of $T(j\omega)$ will be 70.7% of magnitude of $T(j\omega)$ at $\omega = 0$.

i.e., at $\omega = \omega_B, M = 0.707(1) = \frac{1}{\sqrt{2}}$

$$\Rightarrow M = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\delta u_b)^2}}$$

$$\Rightarrow 2 = (1-u_b^2)^2 + (2\delta)^2 u_b^2$$

Let, $u_b^2 = x$

$$\Rightarrow 2 = (1-x)^2 + (2\delta)^2 x$$

BANDWIDTH

$$\Rightarrow x^2 + (4\delta^2 - 2)x - 1 = 0$$

$$\Rightarrow x = \frac{-(4\delta^2 - 2) \pm \sqrt{(4\delta^2 - 2)^2 + 4}}{2}$$

Consider only the positive value of x.

$$x = 1 - 2\delta^2 + \sqrt{(2\delta^2 - 1)^2 + 1}$$

$$\Rightarrow x = 1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}$$

Substitute, $x = u_b^2 = \frac{\omega_b^2}{\omega_n^2}$

$$\frac{\omega_b^2}{\omega_n^2} = 1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}$$

$$\Rightarrow \omega_b = \omega_n \sqrt{1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}}$$

BANDWIDTH

Bandwidth ωb in the frequency response is inversely proportional to the rise time tr in the time domain transient response.

SUMMARY

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THANK YOU