$$\boxed{\begin{array}{l}\text{Type II : } R(x) = \sin ax \text{ (or) } \cos ax \\ \text{Replace } D^2 \to -a^2\end{array}}$$

Solve : $(D^2+4)y = \sin 3x$

Soln: The A.E is,

$$m^2+4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i = 0 \pm 2i$$

$$\alpha = 0, \ \beta = 2.$$

$$C.F = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

$$= e^{0x}(A\cos 2x + B\sin 2x)$$

$$C.F = A\cos 2x + B\sin 2x.$$

$$P.I = \frac{\sin 3x}{D^2+4}$$

$$\sin ax = \sin 3x$$

$$a = 3$$

$$D^2 \to -a^2 \to -3^2$$

$$= -9$$

$$= \frac{\sin 3x}{-9+4}$$

$$P.I = \frac{\sin 3x}{-5}$$

Hence the solution is

$$y = C.F + P.I$$

$$y = A\cos 2x + B\sin 2x - \frac{\sin 3x}{5}$$

(2) Find the P.I of $(D^2+1)^2 y = \sin 2x$

Soln:

$$P.I = \frac{\sin 2x}{(D^2+1)^2}$$

$\sin ax = \sin 2x$

$a = 2$

$D^2 \to -a^2 \to -2^2$

$= -4$

$$= \frac{\sin 2x}{(-4+1)^2}$$

$$\boxed{P.I = \frac{\sin 2x}{9}}$$

(3) Find the P.I of $(D^2+4)y = \cos 2x$

Soln:

$$P.I = \frac{\cos 2x}{D^2+4}$$

$\cos ax = \cos 2x$

$a = 2$

$D^2 \to -a^2 \to -2^2$

$= -4$

$$= \frac{\cos 2x}{-4+4}$$

$$= \frac{\cos 2x}{0} \quad \text{(failure)}$$

$$= \frac{x \cos 2x}{2D}$$

$$= \frac{x}{2} \cdot \frac{1}{D} (\cos 2x)$$

$$= \frac{x}{2} \cdot \frac{\sin 2x}{2}$$

$$\boxed{P.I = \frac{x \sin 2x}{4}}$$

Binomial expansion

1. $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$
2. $(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$
3. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$
4. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$