SNS COLLEGE OF TECHNOLOGY

Coimbatore-21
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – LINEAR CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT III – FREQUENCY RESPONSE ANALYSIS

TOPIC 7,8- LEAD,LAG COMPENSATORS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- **INTRO-LEAD COMPENSATORS**
- RULES TO DESIGN PHASE LEAD COMPENSATION
- LEAD COMPENSATORS-REALIZATION,FREQUENCY RESPONSE
- ACTIVITY
- **INTRO-LAG COMPENSATORS**
- LAG COMPENSATORS-REALIZATION,FREQUENCY RESPONSE
- DETERMINATION OF $\omega_{m,}\ \varphi_m$
- SUMMARY

INTRO-LEAD & LAG COMPENSATORS

- The **lead compensator** is an electrical network which produces a sinusoidal output having phase **lead** when a sinusoidal input is applied. ... So, in order to produce the phase **lead** at the output of this **compensator**, the phase angle of the transfer function should be positive.

- The **Lag Compensator** is an electrical network which produces a sinusoidal output having the phase **lag** when a sinusoidal input is applied. ... So, in order to produce the phase **lag** at the output of this **compensator**, the phase angle of the transfer function should be negative

INTRO-LEAD COMPENSATORS

Three design rules for cascade compensator:

1.The system is stable with satisfactory steady-state error, but dynamic performance is not good enough.

Compensator is used to change medium and high frequency parts to change crossover frequency and phase margin.

2.The system is stable with satisfactory transient performance, but the steady-state error is large.

Compensator is used to increase gain and change lower frequency part, but keep medium and higher frequency parts unchanged.

3. If the steady-state and transient performance are either unsatisfactory, the compensator should be able to increase gain of the lower frequency part and change the medium and higher frequency parts.

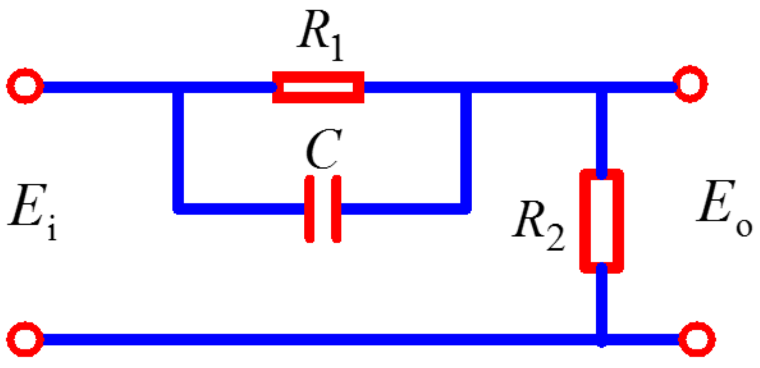
INTRO-LEAD COMPENSATORS

1. Transfer function：

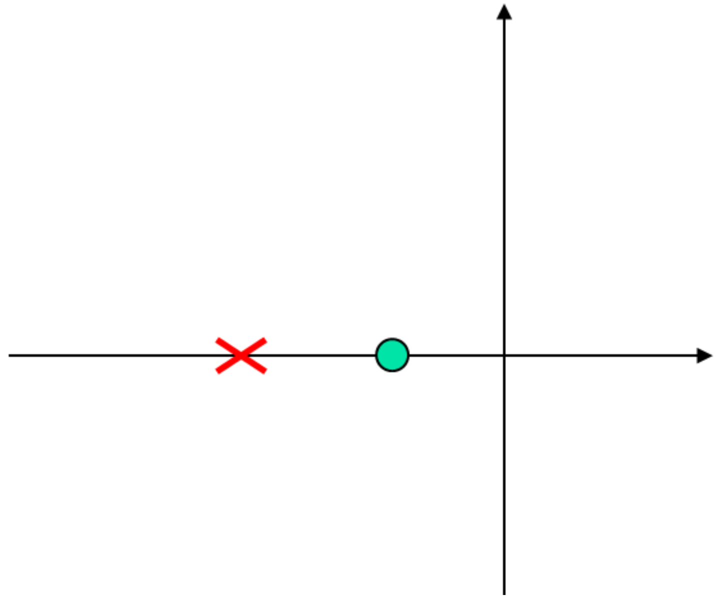
$$G_c(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{\alpha} \times \frac{1+\alpha Ts}{1+Ts}$$

where

$$\alpha = \frac{R_1 + R_2}{R_2} > 1, T = \frac{R_1 R_2}{R_1 + R_2} C$$



Passive Phase Lead Network



RULES TO DESIGN PHASE LEAD COMPENSATION

(1) Determine K to satisfy steady-state error constraint

(2) Determine the uncompensated phase margin γ_0

(3) estimate the phase margin φ_m in order to satisfy the transient response performance constraint

(4) Determine α

(5) Calculate ω_m

(6) Determine T

(7) Confirmation

LEAD COMPENSATORS

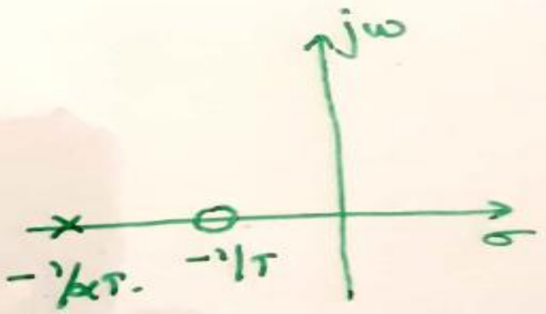
LEAD COMPENSATOR:

6

* A compensator having the characteristics of a lead n/w.
* Lead compensation increases the bandwidth, which improves the speed of response.
* It improves the transient response but small change in steady state accuracy.
* Basically a high pass filter.

S-plane representation:

$$G_c(s) = \frac{s+z_c}{s+p_c} = \frac{(s+1/T)}{\left(s+\frac{1}{\alpha T}\right)}$$



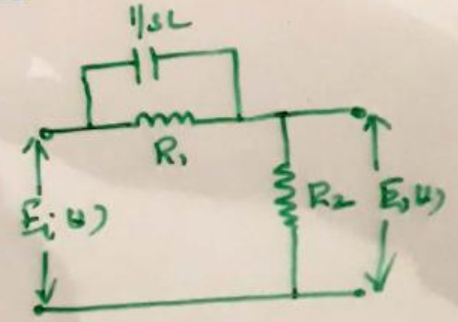
The zero of comp, $z_c = 1/T$.

The pole of comp. $p_c = 1/\alpha T$.

$\therefore T = 1/z_c$ & $\alpha = \frac{z_c}{p_c}$.

REALIZTION OF LEAD COMPENSATORS

Realization of Lead Compensator:



$$E_o(s) = E_i(s) \cdot \frac{R_2}{R_2 + \frac{\left(R_1 \times \frac{1}{sC}\right)}{\left(R_1 + 1/sC\right)}}.$$

$$E_o(s) = E_i(s) \cdot \frac{R_2}{R_2 + \frac{R_1}{(R_1Cs + 1)}}.$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\cancel{s+}\, R_2(R_1Cs+1)}{[R_1R_2Cs + R_2 + R_1]}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{s + \frac{1}{R_1C}}{\left[s + \left[\frac{1}{R_2/(R_1+R_2)}\right] \cdot \frac{1}{R_1C}\right]}$$

where, $T = R_1C$ & $\alpha = \frac{R_2}{R_1+R_2}$.

FREQ.RESPONSE OF LEAD COMPENSATORS

Frequency response :-

$$G_c(s) = \frac{s+1/T}{\left(s+\frac{1}{\alpha T}\right)} = \alpha\,\frac{(1+sT)}{(1+\alpha sT)}.$$

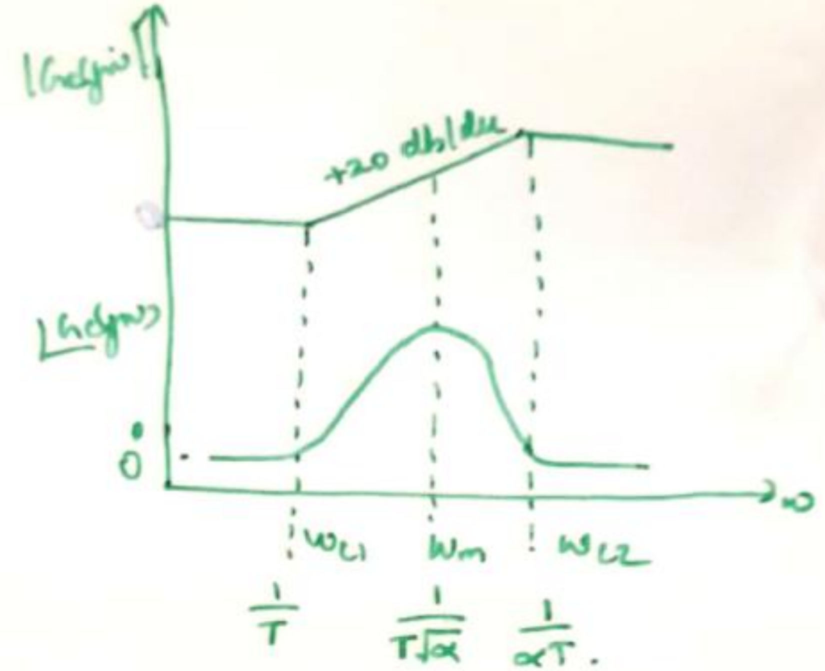
* Put $s = j\omega$,

$$G_c(j\omega) = \cancel{\alpha}\,\frac{(1+j\omega T)}{(1+j\omega\alpha T)} = \frac{\sqrt{1+(\omega T)^2}\;\angle \tan^{-1}\omega T}{\sqrt{1+(\omega\alpha T)^2}\;\angle \tan^{-1}\omega\alpha T}$$

* $\omega_{c1} = 1/T$ & $\omega_{c2} = 1/\alpha T$.

* $A = |G_c(j\omega)|\ db = 20\log\dfrac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega\alpha T)^2}}$

FREQ.RESPONSE OF LEAD COMPENSATORS



$$\phi = \tan^{-1}\omega T - \tan^{-1}\omega\alpha T.$$

As $\omega \to 0$; $\phi \to 0$

$\omega \to \infty$; $\phi \to 0$.

$$\omega_m = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{T}\cdot\frac{1}{\alpha T}} = \frac{1}{T\sqrt{\alpha}}.$$

FREQ.RESPONSE OF LEAD COMPENSATORS

Determination of ω_m & ϕ_m:

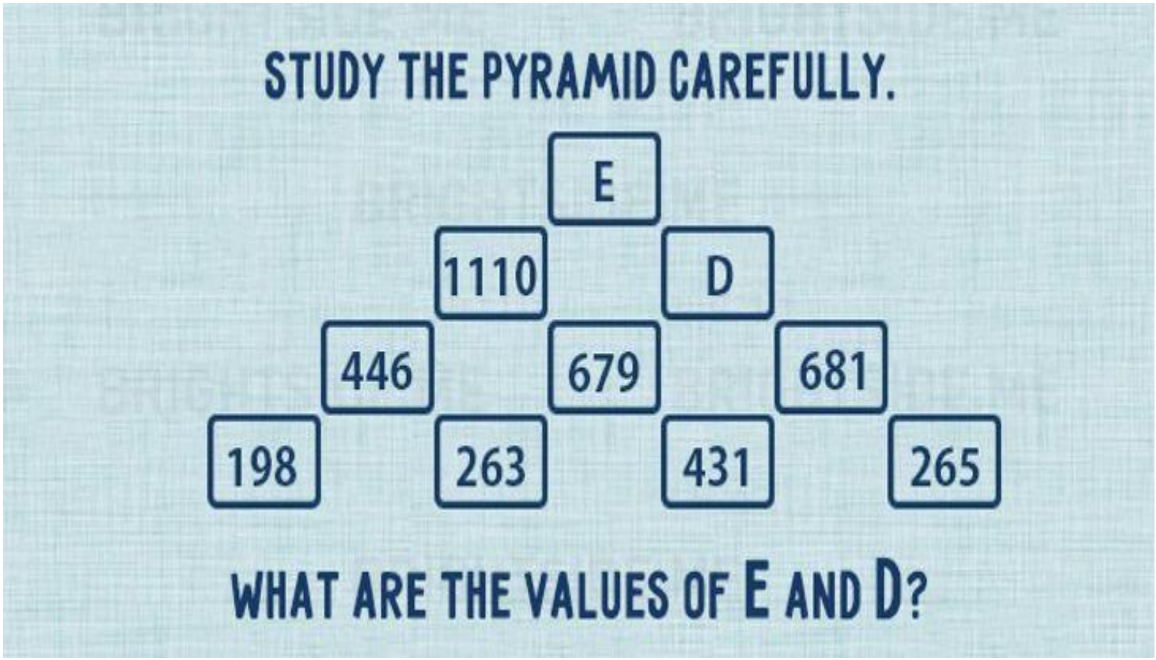
* $\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T$.

* Diff ϕ w.r.t. ω & equating $\frac{d\phi}{d\omega} = 0$.

$$\boxed{\omega_m = \frac{1}{T\sqrt{\alpha}} \qquad \phi_m = \tan^{-1}\frac{1-\alpha}{2\sqrt{\alpha}}}$$

ACTIVITY-PUZZLES

Which way this bus is driving?



ACTIVITY-PUZZLES-ANSWERS

1.The bus is moving to the left because the door is on the other side.

2.Answer: D = 1345; E = 2440.

The bottom numbers are connected to the upper level. First, add the numbers in the bottom line: 198 + 263 = 461.

Now you see that the number you got is greater than its neighbor above: 461 > 446. Subtract these numbers: 461 - 446 = 15.

If you check the rest of the pyramid, you'll get 15 in each case.

LAG COMPENSATORS

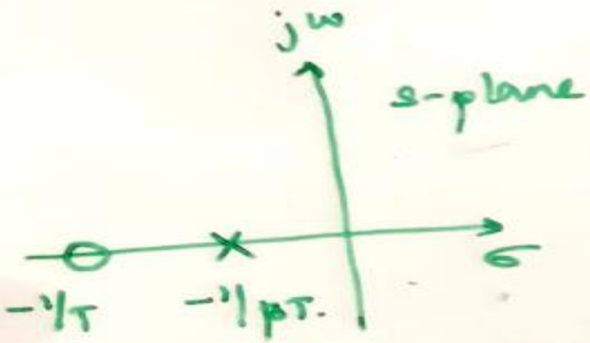
LAG COMPENSATOR:

→ Compensator having characteristics of lag networks.

→ Improves steady state performance but results in slower response due to reduced bandwidth.

→ Essentially a low pass filter.

S- plane representation:



$$G_c(s) = \frac{s+z_c}{s+p_c} = \frac{s+1/T}{s+1/\beta T}.$$

zero of lag comp, $z_c = 1/T$.

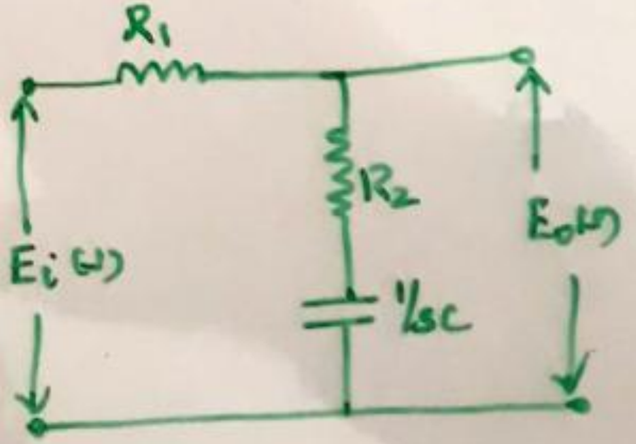
pole " " " $p_c = 1/\beta T$.

$\therefore T = 1/z_c$ & $\beta = \frac{z_c}{p_c}$.

LAG COMPENSATORS

Realization of Lag Compensator:

$$E_o(s) = E_i(s)\,\frac{(R_2 + 1/sC)}{(R_1+R_2+1/sC)}$$



$$\frac{E_o(s)}{E_i(s)} = \frac{CR_2\left(s+\frac{1}{CR_2}\right)}{C(R_1+R_2)\left[s+\frac{1}{C(R_1+R_2)}\right]}.$$

$$\frac{E_o(s)}{E_i(s)} = \frac{s+\frac{1}{R_2C}}{\left(\frac{R_1+R_2}{R_2}\right)\left[s+\frac{1}{[(R_1+R_2)/R_2]R_2C}\right]}.$$

$T = R_2C$

$\beta = \frac{R_1+R_2}{R_2}.$

LAG COMPENSATORS

Frequency Response of Lag Compensator:

$$G_c(s) = \frac{(s+1/T)}{\left(s+\frac{1}{\beta T}\right)} = \beta \frac{(1+sT)}{(1+s\beta T)}.$$

Put $s = j\omega$,

$$G_c(j\omega) = \frac{\beta(1+j\omega T)}{(1+j\omega\beta T)}.$$

* If dc gain of compensator is not desirable, β can be eliminated.

$$G_c(j\omega) = \frac{\sqrt{1+(\omega T)^2} \ \tan^{-1}\omega T}{\sqrt{1+(\omega\beta T)^2} \ \tan^{-1}\omega\beta T}.$$

LAG COMPENSATORS

* $\omega_{c1} = \frac{1}{\beta T}$ & $\omega_{c2} = 1/T$.

* $A = |G_c(j\omega)| = 20 \log \frac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega\beta T)^2}}$.

* At low freq. $\omega T \ll 1$ & $\omega\beta T \ll 1$.

  $\therefore A \approx 20 \log 1 = 0$.

* Freq. range from ω_{c1} to ω_{c2}, $\omega T \ll 1$ & $\omega\beta T \gg 1$.

  $\therefore A \approx 20 \log \frac{1}{\sqrt{(\omega\beta T)^2}} = 20 \log \frac{1}{\omega\beta T}$.

* At high freq. $\omega T \gg 1$ & $\omega\beta T \gg 1$.

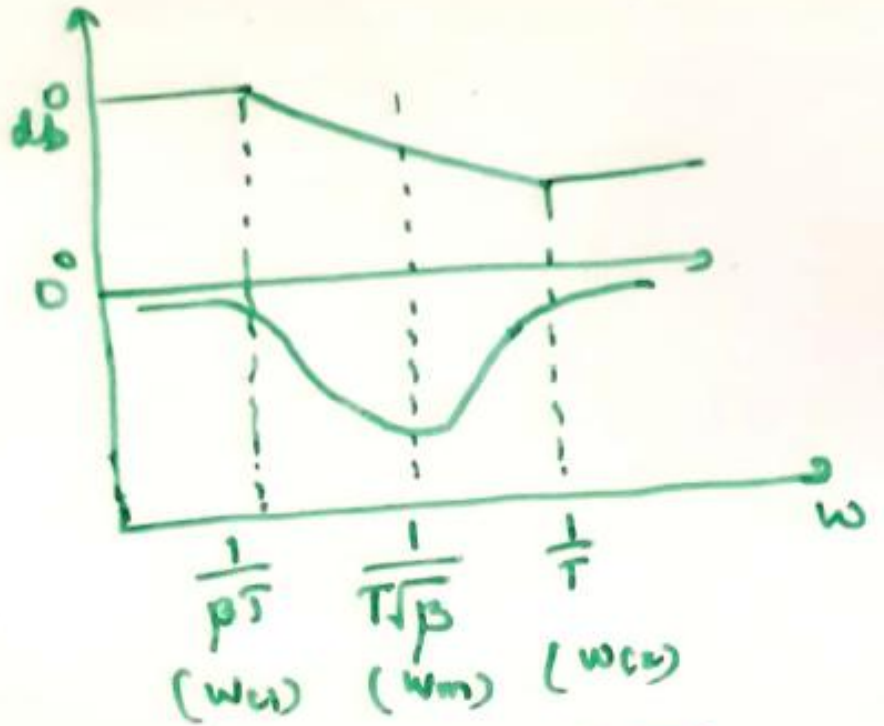
  $A \approx 20 \log \frac{\sqrt{(\omega T)^2}}{\sqrt{(\omega\beta T)^2}} = 20 \log 1/\beta$.

* $\phi = \tan^{-1}\omega T - \tan^{-1}\omega\beta T$.

  As $\omega \to 0$, $\phi = 0$

  As $\omega \to \infty$, $\phi \to 0$.

LAG COMPENSATORS



$$\omega_m = \sqrt{\omega_{c1}\,\omega_{c2}} = \sqrt{\frac{1}{\beta T}\cdot\frac{1}{T}} = \frac{1}{T\sqrt{\beta}}.$$

LAG COMPENSATORS

Determination of ω_m & ϕ_m:

* The freq. ω_m can be determined by diff. ϕ w.r.t. ω & equating $d\phi / d\omega = 0$.

$$\angle G_c(j\omega) = \phi = \tan^{-1}\omega T - \tan^{-1}\omega\beta T.$$

$$\frac{d\phi}{d\omega} = \frac{1}{1+(\omega T)^2} \cdot T - \frac{1}{1+(\omega\beta T)^2}\beta T.$$

$$\left[\frac{d}{d\theta}(\tan^{-1}\theta) = \frac{1}{1+\theta^2}\right].$$

* when $\omega \rightarrow \omega_m$ $\frac{d\phi}{d\omega} = 0.$

$$\frac{T}{1+(\omega_m T)^2} = \frac{\beta T}{1+(\omega_m \beta T)^2}.$$

$$1+(\omega_m \beta T)^2 = \beta[1+(\omega_m T)^2].$$

$$\boxed{\omega_m = \frac{1}{T\sqrt{\beta}}}.$$

LAG COMPENSATORS

We know that,

$$\phi = \tan^{-1}\omega T - \tan^{-1}\omega\beta T.$$

$$\tan\phi = \tan\left[\tan^{-1}\omega T - \tan^{-1}\omega\beta T\right].$$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}.}$$

$$\tan\phi = \frac{\omega T - \omega\beta T}{1+\omega^2 T^2 .\beta.} = \frac{\omega T(1-\beta)}{1+\beta(\omega T)^2}.$$

As $\omega \rightarrow \omega_m$ to $\phi \rightarrow \phi_m$.

$$\tan\phi_m = \frac{\omega_m T(1-\beta)}{1+\beta(\omega_m T)^2}.$$

$$\boxed{\phi_m = \tan^{-1}\left[\frac{1-\beta}{2\sqrt{\beta}}\right]}$$

SUMMARY